

beeswax (a group of marker compounds<sup>6</sup> that are not easily filtered out from mead).

The Homeric epics<sup>7,8</sup>, reflecting both Greek and Anatolian traditions of the eighth century BC and earlier, describe outdoor funeral banquets in which skewered and roast sheep and goat were served, together with a mixed fermented beverage (Greek *kykeon*)<sup>9</sup> similar to that in the Midas tomb. (Barley grains were added to *kykeon*, which may have been in the form of beer.) This beverage, in which other fruits such as apple and cranberry might have been used instead of grapes, had long been a traditional drink in Europe<sup>10</sup>, suggesting that the Phrygian population could have been of European extraction, perhaps from the Balkans or northern Greece.

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Game theory

## Losing strategies can win by Parrondo's paradox

In a game of chess, pieces can sometimes be sacrificed in order to win the overall game. Similarly, engineers know that two unstable systems, if combined in the right way, can paradoxically become stable. But can two losing gambling games be set up such that, when they are played one after the other, they becoming winning? The answer is yes. This is a striking new result in game theory called Parrondo's paradox, after its discoverer, Juan Parrondo<sup>1,2</sup>. Here we model this behaviour as a flashing ratchet<sup>3</sup>, in which

winning results if play alternates randomly between two games.

There are actually many ways to construct such gambling scenarios, the simplest of which uses three biased coins (Fig. 1a). Game A consists of tossing a biased coin (coin 1) that has a probability ( $p_1$ ) of winning of less than half, so it is a losing game. Let  $p_1 = 1/2 - \epsilon$ , where  $\epsilon$ , the bias, can be any small number, say 0.005.

Game B (Fig. 1a) consists of playing with two biased coins. The rule is that we play coin 2 if our capital is a multiple of an integer  $M$  and play coin 3 if it is not. The value of  $M$  is not important, but for simplicity let us say that  $M=3$ . This means that, on average, coin 3 would be played a

little more often than coin 2. If we assign a poor probability of winning to coin 2, such as  $p_2 = 1/10 - \epsilon$ , then this would outweigh the better coin 3 with  $p_3 = 3/4 - \epsilon$ , making game B a losing game overall.

Thus both A and B are losing games, as can be seen in Fig. 1b, where the two lower lines indicate declining capital. If we play two games of A followed by two of B and so on, this periodic switching results in the upper line in Fig. 1b, showing a rapid increase in capital — this is Parrondo's paradox. What is even more remarkable is that when games A and B are played randomly, with no order in the sequence, this still produces a winning expectation (Fig. 1b).

This phenomenon was recently proved mathematically<sup>1</sup> for a generalized  $M$  and analysed in terms of entropy based on Shannon's information theory<sup>3</sup>. We used the flashing Brownian ratchet<sup>4,5</sup> to explain the game by analogy. The flashing ratchet can be visualized as an uphill slope that switches back and forth between a linear and a sawtooth-shaped profile. Brownian particles on a flat or sawtooth slope always drift downwards, as expected. However, if we flash between the flat and saw-tooth slope, the particles are 'massaged' uphill. This is only possible if the sawtooth shape is asymmetrical in a way that favours particles spilling over a higher tooth.

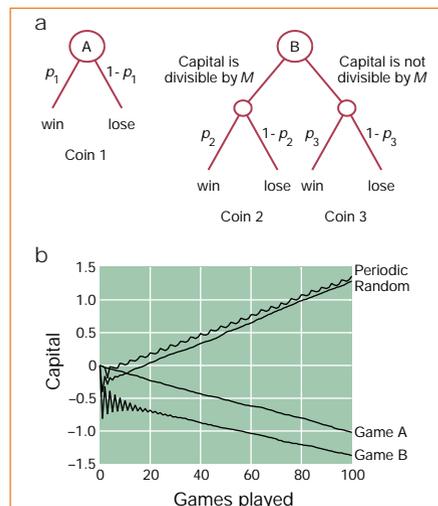
The flat slope is like game A, where the bias  $\epsilon$  is like the steepness of the slope. Game B is like the sawtooth slope, where the difference between coin 2 and coin 3 is like the asymmetry in the tooth shape. In the Brownian ratchet case, there are two types of slope, with falling particles, but when they are switched the particles go uphill. Similarly, two of Parrondo's games have declining capital that increases if the games are switched or alternated. The games can be thought of as being a discrete ratchet and are known collectively as a Parrondian ratchet.

Game theory is linked to various disciplines such as economics and social dynamics, so the development of Parrondian-like strategies may be useful, for example for modelling cases in which declining birth and death processes combine in a beneficial way.

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**Figure 1** Game rules and simulation. **a**, An example of two games, consisting of only three biased coins, which demonstrate Parrondo's paradox, where  $p_1$ ,  $p_2$  and  $p_3$  are the probabilities of winning for the individual coins. For game A, if  $\epsilon = 0.005$  and  $p_1 = 1/2 - \epsilon$ , then it is a losing game. For game B, if  $p_2 = 1/10 - \epsilon$ ,  $p_3 = 3/4 - \epsilon$  and  $M=3$  then we end up with coin 3 more often than coin 2. But coin 2 has a poor probability of winning, so B is a losing game. The paradox is that playing games A and B in any sequence leads to a win. **b**, The progress of playing games A and B individually and when switching between them. The simulation was performed by playing game A twice and game B twice, and so on, until 100 games were played; this is indicated by the line labelled 'Periodic'. Randomly switched games result in the line labelled 'Random'. The results were averaged from 50,000 trials with  $\epsilon = 0.005$ .

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