Influence of noise on the characterization of materials by terahertz time-domain spectroscopy

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We analyze the contributions of various error sources to uncertainty in the far-infrared optical constants (refractive index and absorption coefficient) measured by terahertz (THz) time-domain spectroscopy. We focus our study on the influence of noise. This noise study is made with a thick slab of transparent material for which the THz transmitted signal exhibits temporal echoes owing to reflections in the sample. Extracting data from each of these time-windowed echoes allows us to characterize the noise sources. In THz time-domain spectroscopy experiments in which photoswitches are used as antennae, the transmitting antenna constitutes the principal noise source. The uncertainty in the far-infrared optical constants can be strongly reduced when the extraction is performed with THz echoes that have encountered many reflections in the sample. © 2000 Optical Society of America [S0740-3224(00)01402-8]
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1. INTRODUCTION
For more than ten years, terahertz time-domain spectroscopy has been used to measure the optical constants of materials in the gigahertz–terahertz range. This technique has many advantages compared with classic far-infrared methods, the principal ones of which are rapidity of measurement and extremely wide bandwidth. The rapidity originates in the ultimate sensitivity of the coherent detection, whereas the large bandwidth is linked to the ultrashort duration of the electromagnetic pulses that are used to test the materials. Numerous materials have been studied, including semiconductors and dielectrics, superconductors, liquids, anisotropic crystals, organic materials, flames, gases, and thin films. In addition, time-resolved experiments can be performed.

In this paper we are interested in the precision of the optical material constants (refractive index and energy absorption coefficient) extracted from terahertz (THz) time-domain spectroscopy (TDS) measures. This precision is limited by systematic and random errors. The procedure that is used to extract the optical constants of a parallel slab of material is based on adjustment of the calculated transmission coefficient to the measured coefficient in the framework of a plane-wave model. Most of the research reported in the literature was performed with a quasi-planar THz beam at the sample location shaped by hyperhemispherical lenses associated with off-axis parabolic or elliptical mirrors. Such a system produces almost Gaussian beams, whose divergence has been measured to be a few tens of milliradians. For such beams, the transmission coefficient of a parallel slab differs from that in the plane-wave case by less than $5 \times 10^{-5}$. When parabolic mirrors are not used, the divergence of the beam can increase to 100 mrad at 250 GHz. The transmission coefficient’s difference from that in the plane-wave case remains smaller than $5 \times 10^{-4}$. Systematic errors are also induced by insufficient knowledge of the sample geometry (thickness, parallelism, and homogeneity) and parasitic reflections of the THz beam in the setup components. Usually, for flat samples, the thickness can be mechanically measured with a precision of 1%, which leads to a similar precision for the value of the refractive index. Moreover, the sample thickness, and consequently its optical constants, can be precisely determined from THz TDS experimental data. Indeed, the systematic error owing to thickness inaccuracy can be decreased to a few tenths of a percent. Other sources of systematic errors, such as phase errors owing to delay line misalignment (well known in Fourier-transform far-infrared spectroscopy), are negligible in THz TDS.

In this paper we treat the influence of random errors, i.e., the influence of noise, on the precision of the extracted optical constants. We focus our study on experiments performed with photoconducting antennae. Indeed, even if electro-optic techniques can sample the THz signal, photoconducting sampling is widely used in THz TDS because of its great sensitivity, which gives a detection dynamic larger than 50 dB. The experimental noise is obtained from a large set of measurements. We concentrate our study not on the physical origin of the noise, which was already studied by van Exter and Grischowsky, but mostly on the effect of this noise on the uncertainty of the far-infrared optical constants. Nevertheless, our approach leads to better knowledge of the various noise sources in the THz TDS experiment, mainly the emitter and detector noises, as well as of their dependence on signal amplitude. We show that the variance of the transmission coefficient modulus $\rho(\omega)$ of the sample can be written as

$$\sigma_\rho^2(\omega) = A(\omega)\rho^2(\omega) + B(\omega)\rho(\omega) + C(\omega),$$

where $\rho(\omega)$ is the calculated transmission coefficient and $A(\omega)$, $B(\omega)$, and $C(\omega)$ are functions of the incident THz signal with $\rho(\omega) = (1 - \rho)(1 - \rho^2)^{1/2}$.
where \( A(\omega) \), \( B(\omega) \), and \( C(\omega) \) are coefficients that depend on emitter, detector, and shot noises. To determine these three coefficients and therefore the contributions of the different noises to optical constant uncertainties, one needs to perform measurements with at least three different samples. As systematic and random errors linked to the sample and to its position, respectively, may vary from one measurement to another when different samples are used, we propose to use only one thick, transparent sample. For such a sample, the transmitted electromagnetic pulse exhibits well-separated temporal echoes that are due to backward and forward reflections in the sample. By temporal windowing it is possible to extract the sample material parameters, as well as the noise, from each echo. Thus the respective contributions to noise of emitter, detector, and shot noises could be determined by use of a single sample. We show that the relative noise can surprisingly be the smallest for the second or third temporal echo that has propagated over a longer path in the sample and not for the directly transmitted THz pulse. In this case, extracting the material parameters from such echoes strongly reduces the uncertainties in both the refractive index and the absorption coefficient.

The balance of this paper is divided into three sections. Section 2 is devoted to theory. We derive the uncertainties that arise from noise, i.e., from random phenomena. Each term \( p \) of this summation corresponds to a particular temporal echo, whereas the directly transmitted signal corresponds to \( p = 0 \). If the temporal echoes are separated enough to permit time windowing, i.e., if the recorded signal drops to zero between two echoes, the extraction of material parameters can be performed from any echo \( p \). Therefore it is possible to derive from relation (1) the modulus and the argument of the sample transmission coefficient \( T(p, \omega) \) for echo \( p \):

\[
\rho(p, \omega) = |T(p, \omega)| = \frac{4(n^2 + \kappa^2)[(n - 1)^2 + \kappa^2]^p}{[(n + 1)^2 + \kappa^2]^{p+1}} \times \exp\left[-\kappa(2p + 1) \frac{\omega l}{c}\right], \tag{2}
\]

\[
\varphi(p, \omega) = \arg[T(p, \omega)] = -[(2p + 1)n - 1] \frac{\omega l}{c} - 2p \times \arctan\left(\frac{2\kappa}{n^2 + \kappa^2 - 1}\right) - \arctan\left(\frac{\kappa}{n(n + 1) + \kappa^2}\right). \tag{3}
\]

For low to moderately absorbing materials (i.e., for most materials\(^{23}\)), \( \kappa \ll n \). This condition is required by the very existence of different observable echoes in the THz signal.\(^{19} \) Thus the refractive index and the energy absorption coefficient \( (\alpha = 2\omega\kappa/c) \) of the material can easily be deduced from the transmission coefficient’s modulus and argument:

\[
n = \left[1 - \frac{c}{\omega l} \varphi(p, \omega)\right] / (2p + 1), \tag{4}
\]

\[
\alpha = \frac{-2}{(2p + 1)l} \ln\left[\frac{(n + 1)^{2p+2}}{4n(n - 1)^{2p}}\right] \rho(p, \omega). \tag{5}
\]

B. Uncertainties in Optical Constants Induced by Random Errors

The complex refractive index of the sample is determined, from the experimental data \( \rho(p, \omega) \) and \( \varphi(p, \omega) \), by use of relations (4) and (5). Here we are interested in the uncertainties that arise from noise, i.e., from random phenomena. We derive the errors \( \Delta n \) and \( \Delta \alpha \) by calculating the complete differentials of relations (4) and (5) without considering any systematic error in sample thickness \( l \):

\[
\Delta n = \frac{c}{(2p + 1)\omega l} \Delta \varphi(p, \omega), \tag{6}
\]

\[
\Delta \alpha = \frac{2}{(2p + 1)l} \left[\frac{(n - 1)^2 - 4pn}{n(n^2 - 1)} \Delta n + \frac{\Delta \rho(p, \omega)}{\rho(p, \omega)}\right]. \tag{7}
\]

Errors \( \Delta \rho(p, \omega) \) and \( \Delta \varphi(p, \omega) \) are obtained from the standard deviation of the experimental transmission coefficient \( T_{\exp}(p, \omega) \). Let us write \( T_{\exp}(p, \omega) \) as the summation of the coefficient’s average value \( T(p, \omega) \) and a random contribution \( T_N(p, \omega) \) that is due to noise:

\[ T_{\exp}(p, \omega) = T(p, \omega) + T_N(p, \omega). \]
\[ T_{\text{exp}}(p, \omega) = T(p, \omega) + T_N(p, \omega) = \rho(p, \omega) \exp(j \varphi(p, \omega)) + \rho_N(p, \omega) \exp(j \varphi_N(p, \omega)) \times \exp(j \varphi_N(p, \omega)), \] (8)

where the subscript \( N \) is for noise. Considering a noise level much lower than the signal \([\rho_N(p, \omega)/\rho(p, \omega) \ll 1]\), we derive from relation (8) the modulus \( \rho_{\text{exp}}(p, \omega) \) and the argument \( \varphi_{\text{exp}}(p, \omega) \) of \( T_{\text{exp}}(p, \omega) \):

\[ \rho_{\text{exp}}(p, \omega) \equiv \rho(p, \omega) + \rho_N(p, \omega) \cos[\varphi(p, \omega) - \varphi_N(p, \omega)], \] (9)

\[ \varphi_{\text{exp}}(p, \omega) = \arctan \left[ \frac{\rho(p, \omega) \sin[\varphi(p, \omega)] + \rho_N(p, \omega) \sin[\varphi_N(p, \omega)]}{\rho(p, \omega) \cos[\varphi(p, \omega)] + \rho_N(p, \omega) \cos[\varphi_N(p, \omega)]} \right]. \] (10)

The root mean square (rms) values of corresponding noises are deduced from \( M \) measurements. The variance of \( \rho(p, \omega) \) is given by

\[ \sigma^2_{\rho}(p, \omega) = \frac{1}{M} \sum_{i=1}^{M} \left[ \rho_{\text{exp}}(p, \omega) - \rho(p, \omega) \right]^2 \]

\[ = \frac{1}{M} \sum_{i=1}^{M} \rho^2_N(p, \omega) \cos^2[\varphi(p, \omega) - \varphi_N(p, \omega)]. \] (11)

Assuming that the noises on the modulus and on the argument are not correlated, and with \( \varphi_N(p, \omega) \) being a random variable uniformly distributed on the \([0, 2\pi]\) interval, we get

\[ \sigma^2_{\rho}(p, \omega) = \langle \rho^2_N(p, \omega) \rangle \langle \cos^2[\varphi(p, \omega) - \varphi_N(p, \omega)] \rangle \]

\[ = \langle \rho^2_N(p, \omega) \rangle / 2 \]

\[ \Rightarrow \sigma_{\rho}(p, \omega) = \Delta \rho(p, \omega) = \left[ \langle \rho^2_N(p, \omega) \rangle / 2 \right]^{1/2}, \] (12)

where \( \langle \rangle \) denotes the average value. Considering again a noise level much lower than the signal, we deduce the variance of \( \varphi(p, \omega) \):

\[ \sigma^2_{\varphi}(p, \omega) = \frac{1}{M} \sum_{i=1}^{M} \arctan^2 \left[ \frac{\sin[\varphi(p, \omega) - \varphi_N(p, \omega)]}{\rho(p, \omega) / \rho_N(p, \omega) + \cos[\varphi(p, \omega) - \varphi_N(p, \omega)]} \right] \]

\[ = \frac{\sin^2[\varphi(p, \omega) - \varphi_N(p, \omega)]}{\left[ \rho(p, \omega) / \rho_N(p, \omega) \right]^2} \]

\[ \Rightarrow \sigma_{\varphi}(p, \omega) = \Delta \varphi(p, \omega) = \left[ \langle \rho^2_N(p, \omega) \rangle / 2 \rho(p, \omega) \right]^{1/2}. \] (13)

Using relations (6) and (7) together with relations (12) and (13), we obtain

\[ \Delta n = \frac{c}{(2p + 1) \omega l} \frac{\sigma_{\varphi}(p, \omega)}{\rho(p, \omega)}, \] (14)

\[ \Delta \alpha = 2\Delta n \left[ \frac{\omega}{c} + \frac{1}{(2p + 1) l} \frac{(n - 1)^2 - 4pn}{n(n^2 - 1)} \right]. \] (15)

Thus \( \Delta n \) and \( \Delta \alpha \) are directly related to \( \sigma_{\rho}(p, \omega) \). \( \sigma_{\rho}(p, \omega) \) is experimentally derived from a set of several measurements with the same sample, as explained in the next subsection.

### C. Expression of Noise in the Modulus of the Sample Transmission Coefficient

In THz TDS the transmission coefficient of a sample is experimentally determined from two temporal measurements: the reference measurement \( R(t) \), without the sample, and the sample measurement \( S(t) \), with the sample placed in the setup. We consider here that we keep only the \( p \)th echo of the signal. Both signals are noisy, and thus we write them as \( R(t) + R_N(t) \) and \( S(t) = S_N(t) \), respectively, where \( R(t) \) and \( S(t) \) are the signals without noise (mean value) and \( R_N(t) \) and \( S_N(t) \) are the noise contributions. The experimental transmission coefficient \( T_{\text{exp}}(p, \omega) \) corresponds to the sample signal spectrum divided by the reference spectrum:

\[ T_{\text{exp}}(p, \omega) = \frac{S(p, \omega) + S_N(p, \omega)}{R(\omega) + R_N(\omega)}. \] (16)

Considering that the rms noise values are much smaller than the signal mean values, we can take a limited expansion of relation (16) into series:

\[ T_{\text{exp}}(p, \omega) \equiv T(p, \omega) + \frac{S_N(p, \omega) - T(p, \omega) R_N(\omega)}{R(\omega)}, \] (17)

where \( T(p, \omega) = S(p, \omega) / R(\omega) \) is the complex transmission coefficient without noise, which should correspond to the theoretical coefficient. Thus \( T_{\text{exp}}(p, \omega) \) exhibits a noise term \( \left[ S_N(p, \omega) - T(p, \omega) R_N(\omega) \right]/R(\omega) \)

\[ = \rho(p, \omega) \exp(j \varphi_N(p, \omega)) \] whose modulus variance is given by

\[ \sigma^2_{\rho}(p, \omega) = \sigma^2_{\rho}(p, \omega) \left[ \frac{R(\omega)}{R(\omega)} \right] + \rho^2(p, \omega) \sigma^2_{\rho}(p, \omega), \] (18)
where \( \sigma_R^2(\omega) \) and \( \sigma_S^2(\omega) \) are the variances of the moduli of the reference and the sample signals, respectively.

D. Contributions of the Noise Sources to \( \sigma_{S}^2(p, \omega) \)

Any signal recorded in a THz TDS experiment encounters three types of noise (which are given thereafter in terms of power). The first type is the noise of the emitting antenna, \( \sigma_R^2(\omega) \), which is transmitted from the emitter to the detector [thus it is multiplied by \( \rho^2(\omega) \)]. The second noise, \( \sigma_{sh}^2(\omega) \), corresponds to the shot noise in the detector, and it is proportional to the recorded current \( |R(\omega)|^2 S(p, \omega) \). The third contribution, \( \sigma_d^2(\omega) \), gathers all the other noises in the detector that are a priori signal independent (Johnson noise, amplification noise, laser noise, thermal noise, the zero-THz-field photocurrent contribution, etc.). Thus we write the variance of the modulus of any recorded signal as

\[
\sigma_{\text{signal}}^2(\omega) = \rho^2(\omega)\sigma_R^2(\omega) + \sigma_{sh}^2(\omega) + \sigma_d^2(\omega),
\]

from which we deduce the following relations for the reference \( [\rho(\omega) = 1] \) and sample signals:

\[
\sigma_{R}^2(\omega) = \sigma_{R}^2(\omega) + 2eR(\omega)\Delta f + \sigma_{S}^2(\omega)
\]

\[
\sigma_{S}^2(p, \omega) = \rho^2(p, \omega)\sigma_{R}^2(\omega) + 2eS(p, \omega)\Delta f + \sigma_{d}^2(\omega),
\]

where \( e \) is the electron charge and \( \Delta f \) is the noise bandwidth \( (\Delta f = 1/2\tau \) for detection with a 6-dB/octave filter and a time constant \( \tau \). From relations (18) and (20) we obtain

\[
\sigma_{R}^2(\omega) = \sigma_{R}^2(\omega) + 2eR(\omega)\Delta f + \sigma_{S}^2(\omega)
\]

\[
\sigma_{S}^2(p, \omega) = \rho^2(p, \omega)\sigma_{R}^2(\omega) + 2eS(p, \omega)\Delta f + \sigma_{d}^2(\omega),
\]

Figure 1 sums up the results derived in this theoretical section. We calculate the optical constants from the experimental data, using relations (4) and (5). Their uncertainties, given by relations (14) and (15), depend on \( \sigma_{\rho(p, \omega)}^2 \), which can be obtained from the standard deviation of several data recorded with the same sample. On the other hand, \( \sigma_{\rho(p, \omega)}^2 \) is related to the sample transmission coefficient by relation (21). Fitting the experimental \( \sigma_{\rho(p, \omega)}^2 \) values with relation (21) allows us to get the unknown noise parameters \( A(\omega), B(\omega), \) and \( C(\omega) \) and thus to get more information on the various noise sources in THz TDS experiments.

3. EXPERIMENTAL RESULTS

A. Setup and Experimental Procedure

We use a typical THz TDS setup with similar transmitting and receiving low-temperature-grown–GaAs photo- switch antennae. The THz beam radiated by the transmitting antenna is first collimated and then focused on the receiving antenna by silicon extended hemispherical lenses; the distance between the two antennae is \( \sim 20 \text{ cm} \). The photoswitch gap is 6 \( \mu \text{m} \). The antennae are excited at \( \lambda = 800 \text{ nm} \) by a pulsed mode-locked Ti:sapphire laser at an 82-MHz repetition rate. At the antennae location, the impinging laser beam average power is 7 mW and the pulse duration is \( \sim 120 \text{ fs} \). The emitting antenna is 9-V biased with a battery. The detected current is amplified and transformed into a voltage by a current–voltage converter, and then it is read with a lock-in amplifier. The data presented here are recorded with a lock-in time constant of 100 ms. To determine the standard deviation of the modulus of the sample transmission coefficient, we record each temporal signal 12 times in similar experimental conditions.

B. Experimental Results: Uncertainties in the Optical Constants

First we experimentally check the validity of expressions (14) and (15) for uncertainties \( \Delta n \) and \( \Delta \alpha \). For this purpose we use a 480-\( \mu \text{m} \)-thick wafer of \( n \)-doped silicon, which exhibits a resistivity of 30 \( \Omega \cdot \text{cm} \). Figure 2 shows a typical temporal signal transmitted by the sample, together with the reference signal (recorded without the sample). One can observe five pulses in the transmitted temporal signal. The first one corresponds to the directly transmitted THz pulse (echo 0), whereas the other echoes (1–4) have encountered backward and forward reflections in the wafer and therefore are time delayed. As the signal drops to zero between two consecutive pulses, we can time window the recorded data and extract the silicon parameters from each echo, as explained in Subsection 2.A. Figure 3 shows the modulus \( |\rho(p, \omega)| \) of the complex transmission coefficient obtained for each
time-windowed pulse, together with its standard deviation $\sigma_r(p, \omega)$ (error bars). One observe that the signal level decreases as the echo number increases, because of the Fresnel reflection losses at the sample faces and intrinsic absorption of the material. Each curve is drawn over a frequency range for which the signal-to-noise ratio, for the sample signal, remains greater than 1. Indeed, as the signal strength decreases with the echo number $p$, the useful bandwidth decreases strongly as well.

The refractive index and the absorption coefficient of this silicon sample, extracted from the directly transmitted THz pulse, are presented in Fig. 4, together with their standard deviations. Our values are in excellent agreement with the data published by Grischkowsky et al.\textsuperscript{5} The corresponding uncertainties $\Delta n$ and $\Delta \alpha$ are plotted in Figs. 5(a) and 5(b), respectively. The circles correspond to the standard deviation of 12 measurements. The curves are calculated with both relations (14) and (15), with $\sigma_r(p, \omega)$ extracted from Fig. 3. For sake of legibility, only data that correspond to echoes 0 and 2 are presented: Similar results are obtained for the other echoes. One can notice the good agreement between the two ways of achieving $\Delta n$ and $\Delta \alpha$, demonstrating the validity of relations (14) and (15). Uncertainty $\Delta n$, which is due to noise, is very low. Indeed, $\Delta n$ remains smaller than 1% over the full range 100–1500 GHz. The uncertainty increases dramatically for the low-frequency range, because, for frequencies smaller than 70 GHz our THz beam is no longer collimated by the silicon lens. In the high-frequency range, the uncertainty increases continuously, as the THz signal radiated by our antenna carries increasingly less energy over 1 THz. The absorption coefficient cannot be derived with good precision, as we use a thin sample ($l = 480 \mu$m) and as the absorption is weak in such 30-$\Omega \cdot \text{cm}$ silicon. Nevertheless, when one is observing the low noise level on the temporal curves (see Fig. 2), it could be thought that extracting $\alpha$ from slow temporal scans with a time constant of 100 ms would lead to a much smaller uncertainty. Unfortunately, recording the two reference and sample temporal signals takes approximately a quarter of an hour, and the measured transmission coefficient is thus affected by the long-term intensity noise of the laser. For example, a 1% variation in laser power in recording the reference and the sample signals leads to a 2% error in the transmission coefficient of the sample. For $al \approx 0.1$ (this is the case here), this 2% error induces an uncertainty of $\sim 50\%$ for the absorption coefficient.

The second remarkable feature seen in Fig. 5 concerns the dependence of the uncertainties on echo number $p$. For example, in the 200–1200-GHz range, $\Delta n$ is $\sim 0.01$
(≈0.3%) for echo 0, and it decreases to 0.002 (≈0.06%) for echo 2. Similar behavior is observed for the absorption coefficient. This behavior can be deduced from the theory given above, and we study it in detail at the end of this paper.

C. Experimental Study of Noise

From the preceding experimental results one can derive the contributions of the various noises, i.e., noise parameters \( A(\omega), B(\omega), \) and \( C(\omega) \) of relation (21). For each angular frequency value \( \omega \) we extract \( \rho(p, \omega) \) and \( \sigma_p(p, \omega) \) from Fig. 3: We obtain five couples of values, which correspond to the first five echoes \( (p = 0 \ldots 4) \) of the THz pulse transmitted by the sample. Then we plot these five couples of values on a graph as \( \sigma_p \) versus \( \rho \) (see Fig. 6).

Fitting this plot by using relation (21) leads to the coefficients \( A(\omega), B(\omega), \) and \( C(\omega) \). The value of coefficient \( A(\omega) \) is well determined because it is extracted from the high-value asymptote of \( \rho \). \( C(\omega) \) is derived from the low-value asymptote of \( \rho \), and thus its determination is less precise than for \( A(\omega) \). Finally, as the value of \( B(\omega) \) modifies only the bend of the curve, only its order of magnitude can be known. This procedure is repeated for all the frequencies in the useful frequency range (100–550 GHz). Figure 7 shows the dispersion of the values of \( A(\omega), B(\omega), \) and \( C(\omega) \). \( A(\omega) \) is almost constant over the frequency range studied, and it is, respectively, 2 and 4 orders of magnitude larger than \( B(\omega) \) and \( C(\omega) \).

The expression for \( A(\omega) \) can be written from relation (21) as a function of \( B(\omega) \) and \( C(\omega) \):

\[
A(\omega) = 2\sigma_p^2(\omega)/|R(\omega)|^2 + B(\omega) + C(\omega). \quad (22)
\]

As \( B(\omega) \) and \( C(\omega) \) are negligible compared with \( A(\omega) \), the emitter noise \( \sigma_{e}(\omega) \) dominates all the other noise contributions. From the value of \( A(\omega) \) and relation (22), we deduce the relative emitter noise \( \sigma_{e}(\omega)/|R(\omega)| \), which has a constant value of 2.24 ± 0.07% over the useful frequency range. Thus the emitter noise is directly proportional to the signal \( |R(\omega)| \), and therefore its physical origin can be attributed neither to the Johnson noise nor to the shot noise in the transmitting antenna. On the other hand, it can be linked to the relative laser intensity noise, whose long-term value is 2%.

The coefficient \( B(\omega) = 2e\Delta f/|R(\omega)| \) can be directly calculated from the reference spectrum modulus \( |R(\omega)| \) (see Appendix A) and the noise bandwidth \( \Delta f \). In the present case we have used a lock-in amplifier with a time constant of \( \tau = 100 \) ms and a 6-dB/octave filter, corresponding to a noise bandwidth of \( \Delta f = 5 \) Hz. The solid curve in Fig. 7 represents this calculated value of \( B(\omega) \), whereas the circles are derived from relation (21). Both approaches give the same order of magnitude for \( B(\omega) \) (keep in mind that \( B(\omega) \) is derived from relation (21) with poor precision).

As one can see from Fig. 7, the noise parameter \( C(\omega) = \sigma_p^2(\omega)/|R(\omega)|^2 \) varies, in a first approximation, as \( 1/|R(\omega)|^2 \) (dashed curve). Therefore the overall noise (except shot noise) \( \sigma_{s}(\omega) \) in the detector is frequency independent. As the actual spectrum modulus \( |R(\omega)| \) has been measured, we deduce, from relation (21), the average detector noise power \( \langle \sigma_{s}^2(\omega) \rangle : \langle (\sigma_{s}^2(\omega)) \rangle^{1/2} \approx 800 \) fA (see Appendix A). Because the THz signal is strongly frequency dependent and \( \sigma_{s}(\omega) \) is not, this detector noise is signal independent. We could find its rms value again by calculating the noise level that is present in the temporal records, before the signal. We have measured a rms noise value of 900 fA on both reference and sample records, in good agreement with the 800-fA value determined from the noise study.

Let us now turn our attention to the physical origin of this detector noise. One of its causes is the Johnson noise of the mean photo switch gap resistance \( R_{\text{switch}} \equiv 550 \) kΩ of the receiving antenna under illumination. Expressed in terms of current, the rms value of this noise is given by

\[
\sigma_{s} = \frac{4k_{B}T\Delta f}{R_{\text{switch}}}^{1/2}, \quad (23)
\]

where \( k_{B} \) is the Boltzmann constant and \( T \equiv 300 \) K is the room temperature. For the noise bandwidth \( \Delta f \equiv 5 \) Hz, we obtain \( \sigma_{s} \approx 390 \) fA (25% of the detector noise power). The data acquisition system constitutes another noise source. Thus, replacing the receiving antenna by an open circuit, we measured a rms noise current value of 500 fA (40% of the detector noise power). One of the possible origins of the remaining noise is the laser. As was already shown by van Exter and Grischowsky, the contribution that is due to the other sources of noise.
Uncertainties due to both Johnson noise and data acquisition noise are general. The difference between relative optical path in the material increases with the thermial noise, zero-THz-field photocurrent owing to the Schottky junction located at the metal–semiconductor interface of the photoswitch, etc.) is much weaker than the one that is due to both Johnson noise and data acquisition system noise.

Let us comment about the noise parameters. Relation (21) gives general expressions for $A(\omega)$, $B(\omega)$, and $C(\omega)$ that are valid for any THz TDS experiment, including a detection system based either on photoconductive sampling or on electro-optic sampling. Of course, each THz TDS setup exhibits its own characteristics, such as its spectrum and laser intensity noise. Therefore the specific values of the noise parameters given in this paper apply only for our setup. However, in most published works, the spectra, as well as the measurement time constant and the current amplitude of the detected signals, are comparable, at least in terms of order of magnitude. Thus the effects of the various noise parameters on the precision of $\rho$, described hereafter and summarized in Fig. 8, are general. The difference between $\rho$ and $\sigma_\rho$ represents a dynamic of the measurement. For transparent materials it is limited only by the emitter noise because the detected signal is strong. However, for absorbing materials the detected signal is weak and is strongly affected by the detector noise (noise floor) and not by the emitter noise, which, nevertheless, constitutes the main noise source. Depending on the values of the emitter and detector noises, the quantum noise limit (shot noise) can be reached for moderately absorbing materials. In that case, the ultimate dynamic of measurement is attained.

4. OPTIMIZATION OF THE EXTRACTION PROCEDURE

Uncertainties $\Delta n$ and $\Delta a$ depend on echo number $p$, on material parameters $n$ and $a$, and on sample thickness $l$, as indicated in relations (14) and (15). Indeed, the effective optical path in the material increases with $p$; thus the uncertainties of the optical constants should be reduced, at least for moderately absorbing materials. On the other hand, because of the Fresnel losses at the sample faces and of the intrinsic absorption of the material, the strength of the THz signal and consequently the signal-to-noise ratio decrease with $p$. The two effects are opposite, leading to an optimum value of $p$ that minimizes the uncertainties for a given couple $(n, a)$. An analytic expression of such an optimum $p$ value cannot easily be derived, as it involves too many relations [(4), (14), (15), and (21)]. We have calculated it numerically, taking $A(\omega) = 10^{-3}$, $B(\omega) = 10^{-5}$, and $C(\omega) = 10^{-7}$ (see Fig. 7). The result appears in Fig. 9(a), where the optimum $p$ value is shown versus $n$ and $a$. The contour lines correspond to actual pulse echo numbers (integer values of $p$). The directly transmitted pulse ($p = 0$) contour line merges into the $n = 1$ straight line. For very low-index materials ($n < 1.4$, corresponding to a weak Fresnel reflection) or strongly absorbing samples ($a l > 2 - 3$), the temporal echoes are weak, and thus it is preferable to extract the optical constants from the directly transmitted pulse. On the contrary, for weak-absorption ($a l < 1$) and medium-to-high-index ($n > 2$) materials, using a high-$p$-number echo reduces the uncertainties in the extracted optical constants. In the case of the silicon wafer studied previously, $n = 3.43$ and $a l = 0.1$ at 400 GHz: The optimum $p$ value lies between 2 and 3 [see Fig. 9(a)]. We define the gain (uncertainty reduction) as the ratio of the uncertainties observed when the directly transmitted THz pulse and the optimal echo number, respectively, are used. The calculated gain is plotted versus $n$ and $a l$ in Fig. 9(b). The gain could reach a value of 8 for high-index and weakly absorbing samples (e.g., LiNbO3). For the silicon wafer studied here, the plot indicates a gain value of approximately 4–5, which corresponds to the reduction of noise observed experimentally when data are extracted from pulse 2 instead of pulse 0 (see Fig. 5).

Figure 10 shows $\Delta n$ and $\Delta a$ measured (circles) for the previously described silicon wafer at the frequency of 400 GHz and calculated (solid curves) from the previous rela-
tions (14), (15), and (21) together with the values of noise parameters $A_v$, $B_v$, and $C_v$ that we used to draw Fig. 9. One can clearly observe that both $D_n$ and $D_a$ are strongly reduced (by ~4) when the optical constants are extracted from echo 2 or 3 instead of from the directly transmitted pulse. Similar results are obtained at other frequencies. Let us notice that the uncertainties in the refractive index and in the absorption coefficient are given, respectively, by $D_n/n = 0.04\%$ and $D_a l = 0.015$ for $p = 2$ at 400 GHz. At this level the systematic errors, such as an erroneous sample thickness, become the main source of inaccuracy in this kind of experiment.

5. DISCUSSION AND CONCLUSION

Using the multiple reflections of the THz beam in a sample under test, we are able to determine the contributions of various noise sources to uncertainties in the extracted optical constants of the sample. In such a THz TDS experiment with photoswitches as antennae, the noise originates mainly from the emitter. As it could be attributed to the long-term laser intensity noise, it is directly proportional to the THz signal. It remains limited to a few percent of the signal amplitude, and it restricts only the dynamic of the measurement. A reduction of this emitter noise can be obtained either by use of a low-noise solid-state laser to pump the mode-locked Ti:Sapphire laser (we are now using an argon-ion laser) or by averaging of a large number of rapid scans of the THz signal with and without the sample instead of making two slow scans. Such a reduction in the emitter noise may lead to improved accuracy in measurement of the optical constants if the contribution of the systematic errors is less than that which is due to noise. The detector noise is almost frequency independent (white noise), and its influence on the precision of the sample optical constants is a few orders of magnitude weaker than that of the emitter noise. However, the detector noise constitutes the noise floor, which forbids the characterization of highly absorbing materials. A reduction of this detector noise, mainly as a result of the noise of the data acquisition system, would permit the characterization of more highly absorbing materials. However, it would not lead to improved precision for the extracted optical constants in transparent materials.

When one is recording and averaging several measures, the uncertainties that are due to noise become rather low. In the case of the 30-$\Omega \cdot$ cm-resistivity silicon wafer studied here, the uncertainty in the refractive-index value has been decreased to 0.2% over the 100–1200-GHz frequency range and even more to 0.04% at 400 GHz, where the THz signal is the strongest. For such low values of uncertainty, the errors that are due to noise become smaller than the contribution of systematic errors: sample thickness, plane-wave approximation for the extraction procedure, parasitic reflections of the THz beam in the setup components, and so on. Let us remark that most of these systematic errors will also be encountered when other characterization techniques are used, either in the time domain or in the frequency domain. This study confirms that THz time-domain spectroscopy is a reliable and precise tool for characterization of the electromagnetic properties of any material in the far-infrared region. Moreover, the noise analysis presented in this paper, and to some extent the procedure for reduction of uncertainty in optical constants, is general and can be applied to any experiment on characterization of physical constants in the time domain.25

APPENDIX A

1. Calculation of the Actual Reference Spectrum

The important point here is to calculate the reference spectrum with care. For this purpose, one has to remember that the actual THz signal is periodic, with the same period $T_0$ as the laser pulses. The reference spectrum is a discrete spectrum whose components are equally separated in frequency by a frequency step $1/T_0$. Generally, reference and sample signals are recorded on the smallest time window $T_{\text{record}}$ for which no truncation of the signals is made (see Fig. 11). To keep a single echo, we replace
both reference and sample signals with zeros outside time windows of duration \( T_{\text{signal}} \). Consequently, the noise remains present only in the duration \( T_{\text{signal}} \). To improve the frequency resolution of the measurement, before fast Fourier transformation (FFT) of the signals we extend the record time window \( T_{\text{record}} \) by adding zeros at the end of the recorded data up to duration \( T_{\text{eff}} \). Even if the spectrum is not calculated on the actual signal period \( T_0 \), we can obtain an excellent approximation of the actual values of \( |R(\omega)| \) by using Parseval’s theorem (we assume here that the FFT algorithm satisfies Parseval’s theorem)\(^{26}\):

\[
T_0 \sum_m |R_m(\omega = 2 \pi n/T_0)|^2 = \int_0^{T_0} R(t)^2 dt = \int_0^{T_{\text{eff}}} R(t)^2 dt = T_{\text{eff}} \sum_n |\bar{R}_n(\omega = 2 \pi n/T_{\text{eff}})|^2, \quad (A1)
\]

where \( \bar{R}_n \) is the FFT-calculated component of the reference spectrum at frequency \( n/T_{\text{eff}} \) and \( R_m \) represents the actual value of the reference spectral component at frequency \( m/T_0 \). The number of spectral components per unity of frequency is reduced by a factor \( T_0/T_{\text{eff}} \) when one considers the time window \( T_{\text{eff}} \) instead of \( T_0 \) for calculation of the reference spectrum. We then deduce that

\[
|R(\omega = 2 \pi n/T_{\text{eff}})| \approx |\bar{R}_n|(T_{\text{eff}}/T_0). \quad (A2)
\]

Thus the modulus of the \( n \)th FFT component \( |\bar{R}_n| \) is to be multiplied by \( T_{\text{eff}}/T_0 \) to yield a good estimation of the actual modulus value of the reference spectrum at the particular frequency \( n/T_{\text{eff}} \). In the present study, the different durations are \( T_0 = 12.3 \) ns, \( T_{\text{record}} = 70 \) ps, \( T_{\text{signal}} \approx 13 \) ps, and \( T_{\text{eff}} = 4 \times T_{\text{record}} = 280 \) ps.

2. Calculation of the Average Detector Noise Power

From Parseval’s theorem we get\(^{26}\)

\[
\langle \sigma_d^2(t) \rangle = \frac{1}{T_0} \int_0^{T_0} \sigma_d^2(t) dt = \sum_m \langle \sigma_d^2(\omega = 2 \pi m/T_0) \rangle. \quad (A3)
\]

Moreover, detector noise power \( \sigma_d^2(\omega) \) is related to noise parameter \( C(\omega) \) by means of relation (21). As detector noise is white noise, its average power is uniformly temporally distributed. As explained in Appendix A.1, the detector noise has been suppressed outside a time window of duration \( T_{\text{signal}} \): The noise parameter \( C(\omega) \) is then strongly underestimated by a factor \( T_0/T_{\text{signal}} \). Thus we obtain from relation (21)

\[
\langle \sigma_d^2(t) \rangle = \sum_m |R_m(\omega = 2 \pi m/T_0)|^2 \times \frac{T_0}{T_{\text{signal}}} C(\omega = 2 \pi m/T_0). \quad (A4)
\]

When we use relation (A2) and take into account the reduction of the spectral components density when we consider \( \bar{R}_n(\omega = 2 \pi n/T_{\text{eff}}) \) instead of \( R_m(\omega = 2 \pi m/T_0) \), relation (A4) leads to

\[
\langle \sigma_d^2(t) \rangle = \frac{T_{\text{eff}}}{T_{\text{signal}}} \sum_n |\bar{R}_n|^2 C(\omega = 2 \pi n/T_{\text{eff}}). \quad (A5)
\]

As noise parameter \( C(\omega) \) varies inversely proportionally to the reference power spectrum (see Fig. 7), the summation in relation (A5) simplifies to

\[
\langle \sigma_d^2(t) \rangle = \frac{T_{\text{eff}}}{T_{\text{signal}}} \bar{N} \langle |\bar{R}(\omega)|^2 C(\omega) \rangle, \quad (A6)
\]

where \( N \) represents the number of FFT components of the reference spectrum and the average value \( \langle |\bar{R}(\omega)|^2 C(\omega) \rangle \) is evaluated on the useful bandwidth from Fig. 7. Here \( N = 2048 \) and \( \langle |\bar{R}(\omega)|^2 C(\omega) \rangle = 13 \) fA\(^2\), leading to \( \langle \sigma_d^2(t) \rangle = 800 \) fA.

REFERENCES AND NOTES


23. See, for example, E. D. Palik, ed., Handbook of Optical Constants of Solids (Academic, Orlando, Fla., 1985). Nevertheless, this is not the case for highly absorbing materials, such as superconducting thin films, for which another simplification holds.8 The general case does not lead to analytical expressions but can be numerically treated by a reliable and fast method.22
24. Indeed, the Johnson noise is linked to the photoswitch resistance, which varies under laser illumination. Nevertheless, because of the slight decrease of the mean photoswitch resistance induced by the laser illumination, the Johnson noise is rather independent of the signal. Moreover, the shot noise is proportional to the square root of the signal amplitude and not to its amplitude.
25. Both noise analysis and uncertainty reduction procedures have been applied with time-domain reflectometry equipment in our laboratory for the characterization of guided structures in the microwave domain. First results confirm the possibility of extending these two methods to another area of time-domain characterization.