ON THE VELOCITY OF PULSED BEAMS

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The problem of the superluminal wave propagation has investigated and discussed by many researchers in recent years. Actually there are at least three different questions that concern superluminality. At first what is it the velocity of a beam? Then why is the superluminality calculated only for pulsed beams with comparable spatial width and length? At least how numerous experimental measurements of the superluminal wave propagation may be interpreted?

Of course, the causality is the main problem of superluminality. If the velocity of a pulse or its part may exceed the speed of light it seems that information transfer may be faster than c. On the analysis of general principles it was shown recently [1] that useful superluminal information transfer is strictly prohibited. The word “useful” means that advance must be larger than the interval over which the input pulse is defined. It follows that the group velocity or other type of velocity, e.g. of the part of a pulse, can be larger than c. The goal of this article is to find the appropriate speed parameter to characterize short pulse beam propagation and analysis of possible mistakes in simulations and experiments.

Experimentally the exceeding of the speed of light in vacuum was measured for pulse propagation through media with anomalous dispersion [2], in optical systems with frustrated total internal reflection or in the presence of evanescent waves (see e.g. review [1]). This type of superluminality has the quantum-mechanical nature and media may be described by effective refractive index that is smaller than unity [3]. In the free space experimentally measured superluminality has been observed in the case of Bessel beams propagation [4]. In this experiment the annular source was used to generate the pulsed beam and its velocity was defined as \( v = \frac{\partial z}{\partial t} \), where \( \partial z \) is the distance between two consistent positions of the detector. Then \( v \) is the local speed for specified position. If the mean value \( v_\mu = z_l/t < c \) then \( v_\mu \) only comes to \( c \) with distance of propagation of the beam that isn’t finally formed. This phenomenon cannot be considered as superluminal because the beam size in all this zone of “superluminality” remains compatible with the wavelength and all measurements was performed in the near field where we have no the beam. The difference between optical lengths of ways for two consistent on-axis positions is less than the distance between them. According [1] this may be considered as a typical useless superluminal information transfer. So, the pure superluminality means that pulsed beam propagates in the free space between moments of generation and detection with the speed that exceeds the speed of light in vacuum \( c \). The time between these moments must be more than duration of a pulse. This phenomenon has been found only in simulations for short time-space pulses [5-8].

Nearly all recent simulations of the short pulse beam propagation are based on the first Rayleigh-Sommerfeld integral formula or on expressions that follows from it after substitution of initial field specified in the near-field zone. By the most part Gaussian distribution is used as initial and group velocity \( v_g \) as a speed parameter [5,6,8]. For example, in [8] it is shown that for Gaussian beam the ratio \( v_g/c \) reaches the maximum of \((1 - \theta_0^2/16)^{1/2}\) in the near-field zone at a distance \( 3^{0.5} z_0 \). The divergence of a beam \( \theta_0 = 8M^2/ld \) and for Gaussian distribution with \( d = 2\lambda \) it was obtained \( v_g/c = 1.0064 \). This result has
proved by comparing with “exact” on-axis group velocity [9]. In this simulation it was used the vectorial form of the first Rayleigh-Sommerfeld integral formula with initial Gaussian distribution at the focal plane.

To use any type of integral formulas functions in the integrand have to be solutions of Helmholtz equation [10,11]. Gaussian distribution satisfies to this condition only in the paraxial case with approximations $kq \gg 1$ and $x/q << 1$, where $q=|z-z_0-iq_R|$, where $z_0$ is the Rayleigh distance, $z_0$ - the waist position. For mentioned above parameters and $q_{min}=z_R$ we have $kq_{min}=2\pi^2$ and $x/q_{min}$ $\pi^{-1}$. These values are much more than calculated exceeding of $c$. The error due to nonparaxiality may be estimated directly by comparing of beam diameters calculated using the first Rayleigh-Sommerfeld integral formula and rigorous solution that is

$$E_z(r,\theta) = \sum_{n=-\infty}^{\infty} i^n C_n J_n(kr)\exp(in\theta) \tag{1}$$

where $C_n$ are coefficients of Fourier expansion of the far-field angular distribution, $J_n$ are Bessel functions. The other problem of simplified integral formulas is the assumption that only the field distribution or its normal derivative can he used as initial. Really for sources of a size about a wavelength both this values must be specified simultaneously and the general Kirchhoff integral formula.

The relative difference between beam diameters is shown in Fig.1, where $\varepsilon=(d_1-d_2)/d_1$, $d_1$ is a diameter calculated with Eq.(1), $d_2$ - diameters that are calculated with the first (solid curves) and second (dashed) Rayleigh-Sommerfeld integral formulas. Curves 1 and 3 corresponds to the distance $z_R$ from the origin, 2 and 4 - to $z=5z_R$. The error between rigorous and integral solutions at $\theta=0.6$ (our example) is from 2.3% to 5%, i.e. larger than exceeding of speed of light in [8]. The shape error between pure Gaussian and focal distribution is less than 0.7%. Thus the approximation errors in this case are more than the value of superluminality has found [5-8].

The next possible source of errors is using of a group velocity to characterize the speed of a pulse. It is well known that the concept of the group velocity as $v_g=1/\langle |\text{grad}(r_\omega)| \rangle$ was introduced for quasi-monochromatic wave with $\Delta \omega/\omega_0 << 1$ [10,11]. But femtosecond pulses considered in [5-8] contains from a part of a cycle to several cycles and this inequality is evidently broken. Also the wave packet can change its shape along a propagation path even if there are no dispersion or approximation mistakes. Then the maximum of pulse envelope can move slightly to the leading or back edge of a pulse and the group velocity will differ from $c$. This is not something special because the spatially induced group velocity dispersion may be considered as a result of diffraction of a short pulse [12]. The other problem of results in [5-9] is factorization of field distribution to time and space functions. Then the group velocity they calculate does not depend on the pulse duration. This conflict with theoretical result [10,11] that in non-dispersive media continuous in time beam propagates with the speed of light $c$. 

![Fig.1](image-url)
The group velocity can be defined uniquely only for spectral distribution of a simple shape, e.g. Gaussian. Real lasers usually generate the set of transverse modes with crossing and sometimes separated spectra in frequency domain. Then in the case of a short pulse and existence of mode competition the total space-frequency distribution is much more complex. At the same time the parameter to characterize the beam position is standardized [12]. There are centroid coordinates $R_c(t)$

$$R_c(t) = P_0^{-1} \int S(r,t) d^3r$$

(2)

where $S(r,t)$ is the Poynting vector, $P_0$ is the total power. The beam velocity will be the time derivative $v_b = \frac{dR}{dt}$. For short pulses this definition is more preferable than group velocity at least by two reasons. At first Eq.(2) is a general analytical expression and limitation is only such that the beam may be characterized by the Poynting vector. The next is that $R_c(t)$ has statistical nature and can be used for any temporal or spatial distribution. The remark of J.A. Stratton that the "center of gravity" of a beam becomes indefinite with diffraction [11] is correct only partially because in practice all beam position parameters will depend on the pulse shape and diffraction. But centroid coordinates has advance as a mean statistical value. Thus, the superluminality in the free space. It is shown that for free space superluminal propagation is by the most part results of different mistakes.

REFERENCES