MRFD method for numerical solution
of wave propagation in layered media with
general boundary condition

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A fast adaptive wavelet-based method, called multiresolution finite difference (MRFD), is proposed to simulate the wave propagation in multilayered media with general boundary. It is a promising method for complex media because of its robustness and small computational burden. Numerical results derived from geophysics exploration show the effectiveness and potential of the method.

Introduction: Conventional numerical methods such as finite difference (FD), finite element and spectral methods, have been successfully applied for wave propagation. However, it is difficult to obtain an efficient solution with large gradient, boundary layer and dramatic variety.

The theory of wavelet transform (WT) has developed rapidly during the past few years and has been applied successfully in many different fields [1 - 5]. Various papers on partial differential equations (PDEs) based on wavelet multiresolution analysis (MRA) have been published [6 - 9], but solutions of PDEs are not always smooth.

The wavelet-based method allows a good approximate representation of various functions and operators to be efficiently obtained due to the vanishing moments, localization, and MRA of the wavelet. High resolution computations are performed only in regions where sharp transitions occur. Thus, the computational efficiency and memory requirement can be optimised. Moreover, by using WT it is possible to detect singularities and irregular structure. However, at present, most wavelet algorithms can handle periodic boundary conditions (BC) and uniformity media. The effective treatment of general BC is still unresolved, especially the bounded region problem, even though different approaches for dealing with this problem have been studied [9].

This Letter focuses on the simulation of wave propagation in complex media, using FD and Daubechies' compactly supported orthogonal WT to discretise the time and space dimension of wave equation, respectively. It is a fast adaptive algorithm owing to the difference operator and solution vector being sparse in the wavelet domain. The scheme handles general BC.

Solving wave propagation using MRFD scheme: (1) MRFD scheme for wave propagation: We consider the following one-dimensional wave propagation problem in inhomogeneous dissipative media with general BC and initial condition in geophysical exploration.

\[ u_{xx} - \mu_0 \sigma(x) u_t = \frac{1}{v^2(x)} u_{tt} = s(x,t) \]

\[ u_t(0,t) = 0 = \frac{1}{v(x)} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} = 0 \quad x \in \Omega_4 \]

\[ u(x,0) = u_t(x,0) = 0 \]

Here, \( u(x, t) \) is displacement, \( s(x, t) \) is seismic wavelet, \( \nu(x) \) is velocity, and \( \Omega_4 \) denotes absorbing boundary region in the bottom. Assuming FD, Daubechies WT is used to discretise the time and space variable, respectively. Let \( \Delta t^2 \) be \( k(x) \), \( \Delta x^2 \) be \( p(x) \), and \( u(x, t) \) represent the wavelet approximation of the solution from scale zero to scale J. We then obtain:

\[ u_{j+1}^{x}(x_m) = \frac{2u_{j}^{x}(x_m)}{1 + k(x_m)\sigma(x_m))} - \frac{u_{j-1}^{x}(x_m)}{1 + k(x_m)\sigma(x_m))} \]

\[ + \frac{\Delta t k(x) u_{j+1}^{x}(x_m)}{\mu_0(1 + k(x_m)\sigma(x_m))} - \frac{k(x_m)u_{j}^{x}(x_m)}{\mu_0(1 + k(x_m)\sigma(x_m))} \]

\[ u_{j+1}^{x}(0) = u_{j-1}^{x}(0) = 0 \]

\[ u_{j+1}^{x}(x_m) = u_{j}^{x}(x_m) - p(x_m)u_{j+1}^{t}(x_m) \]

\[ u_{j}^{t}(x_m) = 0 \]

where \( x_m^j = A2^{-j}m \), \( m = 0, 1, \ldots, N / 2^j - 1, j \in (0, J) \). \( A, N \) denote

References

