1. INTRODUCTION

An important operation in wavelength-division-multiplexing optical systems is to use a control optical beam at one wavelength to modulate a signal at another. A common approach, conversion of optical control and signal to electrical signals to effect the modulation, followed by backconversion to optical, however, puts severe limits on the operation speed. Consequently, for ultrafast signal processing, all-optical means are sought. Channel spacings in wavelength-division-multiplexing systems may be in the terahertz (THz) range. Thus optical/THz nonlinear mixing is of interest for wavelength-division-multiplexing applications. An advantage of using a THz field for mixing several optical fields (in contrast with mixing of optical beams by use of purely optical nonlinearities) lies in the possibility of using weak optical fields and relying instead upon the nonlinearities provided by the THz field, which may be strong. This case is particularly attractive for applications in communications technology where the optical power has to be weak to avoid nonlinear distortion of the transmitted signals.

Recently, the first experimental results were reported on the optical properties of semiconductor quantum wells (QWs) in the presence of a strong THz field that was oriented either in the QW plane or in the growth direction. The latter geometry is more likely to yield efficient optical/THz mixing since the excited electron and hole remain bound to one another owing to QW confinement. By growing asymmetric structures or by applying a bias field, one can generate odd-order sidebands that are forbidden in symmetric structures. The merger of THz/optical technologies is further facilitated by the recent advancement in design and manufacturing of THz ultrawide-band transmission channels, which can significantly improve the coupling of THz radiation to QWs.

So far, both in experimental and in theoretical papers, the effect of the THz field on the optical field with a single optical frequency was considered. Interesting effects can be observed if several optical beams with different frequencies are incident on a THz-field driven QW. In the linear optical regime, the response of the QW will be the superposition of the responses for each of the incident beams. For coherent optical fields the amplitudes must be added, and their superposition can give rise to some interference effects that may be used for controlling the optical transmission through the QW. To understand these effects, let us first consider an optical field incident on a QW in the absence of the THz field. If the optical field is in resonance with an exciton transition, it induces polarization in the QW, which in turn reemits light into the forward and backward directions. This re-emission and irreversible absorption in the QW result in a reduction of the transmitted energy compared with the incident energy. In the presence of a THz field, an excitonic state dressed by the THz field is formed. The dressed exciton has multiple resonant optical frequencies that differ from one another by a multiple of the THz frequency. An optical field tuned at any of these resonances excites dressed excitons that reemit light at the frequency of the incident light and the sidebands. Similar to the case without the THz field, this leads to a reduction in the transmitted signal at the incident frequency. If instead of one optical beam we use two (or possibly more) beams tuned at the optical resonances of the same dressed state, the amplitude of the resulting state will depend on the amplitudes and phases of the optical fields. This mixes the two beams. Under some conditions the two laser beams can even propagate through the QW without alteration of their intensity, giving rise to complete transparency. The key idea is that each of the laser beams excites the dressed excitonic state. We can choose the amplitude and phases of the laser beams in such a way that the two excitation amplitudes interfere destructively and produce no net excitation. It is important that this mutual transparency takes place even in the presence of optical dephasing in the QW.

This effect can also be used for coherent control of excitons by two laser pulses whose spectra do not overlap. It was experimentally demonstrated that one can use an...
optical pulse to create excitons and then coherently deexcite them after some time delay with a second optical pulse. In the frequency domain, the condition for complete destruction of excitons is that the total spectrum of the two pulses must have zero spectral density at the position of the exciton resonance. Dressed excitons have multiple resonant frequencies. So one can use a pulse at one frequency to excite dressed excitons and a delayed pulse at a different frequency to coherently deexcite them.

We also note that the two coherently laser beams need not come from two separate laser sources; they can be obtained as the field transmitted through another THz-modulated QW subjected to a monochromatic optical field. This will also provide their phase coherence.

The paper is organized as follows. First, we lay out a theoretical description of the interaction of an optical field with QW excitons dressed by a THz field. Second, we present numerically calculated absorption spectra in support of the theory. Third, we discuss the possibility of the coherent control of THz-dressed excitons.

2. INTERACTION OF OPTICAL PULSES WITH TERAHERTZ-FIELD DRIVEN QUANTUM WELLS

We will now turn to the analysis of how optical fields interact with THz-field driven QWs. The optical properties of a QW are determined by its electronic states, which can be characterized in the Schrödinger picture by a time-dependent wave function,

$$|\Psi(t)\rangle = |0\rangle + C(t)|\text{exc}\rangle,$$

where $|0\rangle$ is the crystal ground state and $|\text{exc}\rangle$ is the optically excited state with $C(t)$ being its amplitude. The wave functions $|0\rangle$ and $|\text{exc}\rangle$ include the time dependence associated with their energies. The THz field does not change the crystal ground state since its frequency is far below the bandgap and its strength is not sufficient to cause tunneling ionization. All the effects of the THz field are included in the excited state $|\text{exc}\rangle$. Such a state is called dressed state. Depending on the parameters of the THz field and QW material parameters, the excited state will have different forms. We will make several assumptions that will allow us to obtain an approximate (but nonperturbative) form of the dressed state for a THz field that is not in resonance between any pair of subbands.

We assume that the QW is biased by $F_{dc}$. The bias field changes the single-particle envelope functions for electrons $\varphi_e(z)$ and holes $\varphi_h(z)$, giving rise to a dipole moment associated with $\varphi_{e,h}(z)$. A nonresonant THz field $F_{ac}\cos(\Omega t)$, where $F_{ac}$ is the amplitude and $\Omega$ is the frequency, polarized in the QW growth direction, gives rise to adiabatic modulation of the energy of the electron and hole single-particle states without significant alteration of the wave functions $\varphi_{e,h}(z)$. Since the dipole moment associated with $\varphi_e(z)$ is much smaller compared with that for $\varphi_h(z)$, we assume that only the energy of the hole is modulated (see Fig. 1). The envelope function for an optically excited exciton is

$$\Psi(r_e, r_h, t) = \frac{F(p)}{\sqrt{S}} \varphi_e(z_e) \varphi_h(z_h) \sum_n a_n \exp(-i\omega_n t),$$

with $\omega_n = \omega_{exc} + n\Omega$, $n = 0, \pm 1, \ldots$.

The interaction with the optical field is described by the Hamiltonian $\mathcal{H}_\text{int} = -\mathcal{D}E(t)$, where $\mathcal{D}$ is the interband dipole-moment operator and $E$ is the optical field at the QW location. The Schrödinger equation yields

$$\frac{dC(t)}{dt} = -E(t) \sum_n d_n \exp(i\omega_n t) - \Gamma_{ac} C(t),$$

where $d_n = \sqrt{S}g \langle d_e F(0) a_n \rangle$, $g = \int dz \varphi_e(z) \varphi_h(z)$ is the overlap integral between the hole and the electron states, and $\Gamma_{ac}$ is the phenomenological scattering rate. The electric field in Eq. (4) differs from the incident field, which for simplicity we assume to be incident normally on the QW. We can integrate Maxwell’s equation across the
QW and obtain the boundary conditions that match the incident $E_0(t)$, reflected $E_r(t)$, transmitted $E_t(t)$ fields, and QW polarization $P(t)$,

$$E_r(t) = \frac{2i \pi \omega_{\text{ext}}}{c \sqrt{\varepsilon_s}} P(t), \quad E_t(t) + E_0(t) = E_r(t), \quad (5)$$

where the QW polarization

$$P(t) = (1/S)C(t) \sum_n d_n \exp(-i\omega_nt). \quad (6)$$

The optical field in Eq. (4) coincides with $E_t(t)$. Using Eqs. (5) and (6), we obtain, from Eq. (4),

$$\frac{dC}{dt} = \frac{i}{\hbar} E_0(t) \sum_n d_n \exp(i\omega_nt) - (\Gamma_{\text{rad}} + \Gamma_{\text{sc}})C, \quad (7)$$

where we introduced the radiative decay rate $\Gamma_{\text{rad}} = 2\pi \omega_{\text{ext}} d^2_1 \varepsilon^2 f(0) g^2/(c \hbar \sqrt{\varepsilon_s})$. The first term on the right-hand side of Eq. (7) describes excitation of the dressed state; the second describes its radiative and nonradiative decay. The radiative decay rate is identical to the one in the absence of the THz field since the THz field does not change the electron and hole single-particle wave functions. In deriving Eq. (7), we also used the condition that the exciton decay rate (both radiative and nonradiative) is slower than the THz period, i.e., $\Gamma_{\text{sc}} + \Gamma_{\text{rad}} \ll \Omega$. This is the condition when it is legitimate to speak of formation of the dressed states.

Equation (7) allows us to find the amplitude $C(t)$ of the dressed state (1) and then, using Eqs. (5) and (6), the reflected and transmitted fields. It follows from Eq. (7) that the dressed state is excited only if the incident field coincides with one of the resonant frequencies $\omega_n$. Let us take the incident field to be $E_0(t) = E_m \Theta(t)$ $\times \exp(-i\omega_nt)$ with $\Theta(t)$ the step function and $m$ fixed. By solving Eq. (7) and using Eqs. (6) and (5), we obtain that, after transient processes are completed for $t \gg (\Gamma_{\text{sc}} + \Gamma_{\text{rad}})^{-1}$, the reflected field becomes $E_r(t) \rightarrow E_m \alpha_m \Gamma_{\text{rad}}/\Gamma_{\text{rad}} + \Gamma_{\text{sc}} + \Gamma_{\text{rad}} + \Gamma_{\text{sc}}) \sum_n \alpha_n \exp(-i\omega_nt)$. The reflected field contains components at the incident frequency $\omega_n$ as well as the sidebands at $\omega_n$ with $n \neq m$. The reflection and irreversible absorption reduce the transmitted signal at $\omega_m$ whose amplitude is $[1 - \alpha_m^2 \Gamma_{\text{rad}}/(\Gamma_{\text{rad}} + \Gamma_{\text{sc}})] E_m < E_m$. We note that in the limit of vanishing amplitude of $F_{\text{sc}}$, the transmitted field will be reduced only if the frequency of the incident field $\omega_n = \omega_{\text{ext}}$, which gives $E_t = [1 - \Gamma_{\text{rad}}/(\Gamma_{\text{rad}} + \Gamma_{\text{sc}})] \times E_m$.

We now assume the incident field has two components at frequencies $\omega_{n_1}$ and $\omega_{n_2}$: $E_0(t) = E_{n_1} \exp(-i\omega_{n_1}t) + E_{n_2} \exp(-i\omega_{n_2}t)$ with some fixed integers $n_1$ and $n_2$, and constant complex amplitudes $E_{n_1}$ and $E_{n_2}$. Each of the components in the absence of the other exciton polarization in the QW, which reduces its transmission owing to reflection and absorption. Let us find the condition when the two fields pass through the QW without alteration of their intensity. There are several alternative ways in which this can be done. We can consider the amplitude of the reflected signal at a resonant frequency $\omega_l$, with some arbitrary $l$ that may or may not coincide with the frequencies of the incident light. The first field excited the reflected signal at $\omega_l$ with amplitude that varies as $\sim a_n \alpha_n E_{n_1}$, while the second field gives $\sim a_n \alpha_n E_{n_2}$. The sum gives zero if $a_n \alpha_n E_{n_1} + a_n \alpha_n E_{n_2} = 0$. Alternatively, one can look at the excitation term in Eq. (7). If we average this term over the THz period, we obtain that, under the above condition, the driving term is zero. Even though the optical field coincides with the optical resonances and has finite amplitude at the QW location, it does not induce any net polarization in the QW. As a result, no reflection and absorption occur. The incident optical field $E_0(t)$ with two frequency components pass through the QW without alteration of its intensity. The fact that the two fields are in resonance with the dressed state makes this situation different from the effect of electromagnetically induced transparency, where the transparency is achieved when the probe beam is detuned from the dressed-state resonances.

3. NUMERICAL RESULTS

To illustrate the applicability of our theoretical treatment to realistic QWs, we studied the optical absorption in a 15-nm GaAs QW using the semiconductor Bloch equations in the linear regime with respect to the optical field strength. The effects of the growth-direction polarized THz field are treated nonperturbatively. The use of semiconductor Bloch equations also allows one to account for both electron–hole bound and continuum states. The material parameters and numerical details are given in Ref. 4. Our findings are summarized in Fig. 2. In the absence of the THz field, one can clearly see two absorption peaks at $\sim -18$ meV and $-4$ meV due to HH1–CB1 (heavy-hole–conduction band) and HH2–CB1 transitions, respectively. In addition to the excitonic peaks, there is a continuum due to excitation of unbound electron–hole pairs. The half-width at half-maximum of the excitonic peaks is $\sim 0.5$ meV (which corresponds to the dephasing time of polarization 1.3 ps). In the presence of a THz field $F_{\text{sc}} \cos(\Omega t)$, one observes the breakup of the lowest excitonic peak into several peaks. These reso-

![Fig. 2. Optical absorption of a QW biased by a 50-kV/cm dc field in the absence of the THz field (dashed curve) and in the presence of a 10-kV/cm THz field with $\hbar \Omega = 3$ meV (solid curve). The energy is given with respect to the bandgap energy of an unbiased QW. The inset shows the absorption of the two-color optical field as a function of the phase $\phi$ between the two spectral components tuned to the peaks indicated by the arrows (the data points show the calculated absorption, and the curve is a smooth fit).](image-url)
nances comprise the Floquet state formed near the band edge and described by Eqs. (2) and (3). At any of these resonances, a monochromatic optical field experiences significant absorption. Let us now take a two-color optical field: \( E(t) = E_1 \exp(-i \omega_1 t) + E_2 \exp(-i \omega_2 + i \phi) \) with \( E_{1,2} \) real amplitudes of the spectral components and \( \phi \) the phase between them. The frequencies \( \omega_1 \) and \( \omega_2 \) are chosen to coincide with two absorption peaks: \( \hbar \omega_1 = E_g - 18.2 \text{ meV} \) and \( \hbar \omega_2 = E_g - 21.2 \text{ meV} \), with \( E_g \) the bandgap energy at zero bias. The frequency difference between the two spectral components is exactly \( \Omega \). Each of the fields \( E_1 \) and \( E_2 \) will experience absorption in the absence of the other. The absorption is simply proportional to the value of the absorption coefficient (\( A_1 = 6.04 \times 10^{-3} \) and \( A_2 = 4.07 \times 10^{-3} \)) at the corresponding frequencies (see Fig. 2, solid line). Let us take the amplitude ratio \( E_2/E_1 = A_1/A_2 = 1.48 \). Since the heights of absorption peaks are proportional to the coefficients \( d_n \), which enter Eq. (7), this ratio is expected to provide a complete transparency at some particular phase between the beams.

The inset in Fig. 2 shows that the total absorption of this two-color field varies substantially with the phase \( \phi \). The maximum value of the absorption is a factor of \( \sim 8 \) greater than the minimum value. Even though the absorption never completely disappears, owing to finite dephasing \( \Gamma_{xc} \), it does drop off quite significantly for \( \phi = \pi \). This corresponds to the case when the two spectral components \( E_1 \) and \( E_2 \) excite the THz-dressed exciton state in opposite phases, thus producing no net exciton absorption.

As we already mentioned, excitons can be coherently controlled by a pair of phase-locked pulses. The condition for complete destruction of the exciton population after the passage of the pair of pulses is the zero total spectral density at the resonant frequency. We will now generalize this condition for the case of the THz-dressed excitons. Apparently, after excitation by a time-dependent optical field, the QW will be returned to the ground state if the time integral of the right-hand side in Eq. (7) is zero. Let us assume that both dephasing rates are slow compared with the duration of the incident optical field. In this case the desired condition is

\[
\int_{-\infty}^{+\infty} dt \left[ E_0(t) \sum_n d_n \exp(i \omega_n t) \right] = 0. \tag{8}
\]

In the case of an undriven QW, this condition gives zero total intensity at the position of the optical resonance. Apparently, in the case of a THz-driven QW, the spectrum of \( E_0(t) \) with zero intensities at the positions of all optical resonances also gives complete destruction of the exciton population. However, it is more important that the spectrum of \( E_0(t) \), which does not give zero at any of the optical resonances, also may provide complete destruction of exciton population. To demonstrate this, let us assume that we use a pair of pulses with different carrier frequen-

5. CONCLUSION

To conclude, we have shown how two coherent optical beams with different frequencies can be mixed in a QW driven by a THz field. We found that under certain conditions on the amplitude and phases of the beams, the beams pass through the QW without alteration of their intensities, even though each of the beams in the absence of the other experiences attenuation. We presented numerical results that confirmed that the transparency can be achieved in realistic QWs at reasonable strengths of THz field. If instead of two cw field we take two pulses and separate them in time, one can optically create THz-dressed excitons with the first pulse at one frequency and destroy them with the second pulse at a different frequency. While we explicitly focused on the case of nonresonant THz fields, the qualitative results are essentially independent of how the dressed states are created. For example, one can use THz fields in resonance with the QW intersubband transitions. Once the dressed states are known, one can easily investigate the mixing of the optical fields in the driven QW using the approach presented in the paper.
ACKNOWLEDGMENTS
This study was supported by the Office of Naval Research and the National Science Foundation through ECS-0072986 and DMR-0073364 and by the State of Georgia through the Yamacraw program.

REFERENCES