Demonstration of negative group delays in a simple electronic circuit

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We present a simple electronic circuit that produces negative delays. When a pulse is sent to the circuit as input, the output is a pulse with a similar waveform that is shifted forward in time. The advance time or negative delay can be increased to the order of seconds so that we can observe the advance with the naked eye by observing two light emitting diodes that are connected to the input and the output. The negative group delay in the electronic circuit shares the same mechanism with superluminal light propagation, where the group velocity exceeds the speed of light or even becomes negative. © 2002 American Association of Physics Teachers. [DOI: 10.1119/1.1503378]

I. INTRODUCTION

Brillouin and Sommerfeld investigated the propagation of a light pulse in a dispersive medium described by the Lorentz model. They pointed out that in the region of anomalous dispersion, the group velocity is larger than c, the speed of light in a vacuum, or even be negative. Anomalous dispersion occurs near the center of absorption lines. For superluminal group velocities (v_g > c), the transit time of the light envelope through the medium is smaller than in vacuum with the same length. For negative group velocities, the envelope leaves before it enters the medium.

Chu and Wong demonstrated experimentally that the light in a Ga:N crystal can be propagated at such extraordinary group velocities (v_g > c or v_g < 0). But in this type of experiment, it is inevitable that the shape of the light pulse is strongly distorted due to the absorption. The standard definition of the group velocity, \( v_g = d\phi/dk \) (\( \phi \) is the angular frequency and \( k \) is the wave number), which describes the velocity of the light envelope or peak, loses its physical meaning if the waveform is strongly distorted. A new definition of group velocities that consider the distortion of the spectrum caused by the absorption was proposed recently.\(^3\)

Wang et al.\(^4\) realized anomalous dispersion without absorption using gain-assisted linear dispersion and demonstrated light propagation at negative group velocities.\(^5,6\) For light pulse propagation in an absorption-free medium, the standard definition of the group velocity describes the propagation speed of the wave form or its envelope.

Pulse propagation with superluminal or negative group velocities is counterintuitive and is likely to cause many misunderstandings. However, it is the direct result of the interference between waves with different frequencies and is consistent with relativistic causality. Anomalous dispersion with no absorption induces phase shifts that depend on the frequency, thereby enhancing the front part of the pulse by constructive interference and canceling the rear part by destructive interference. The pulse shape is maintained if the phase shift is a linear function of the frequency.

Interference does not occur only in light propagation, but also in other wave or oscillation dynamics. Signals in electrical circuits also interfere. In lumped constant electronic circuits, we cannot define the group velocity, because there is no finite length scale. Instead, we can define the group delay, which is the time difference between the input and output signal envelopes. If it is negative, the output pulse precedes the input as shown in Fig. 1. The negative delays are closely connected to the superluminal or negative group velocities in spatially extended systems.

In this paper, we propose a simple electronic circuit that generates negative group delays. The properties of linear circuits are characterized by the transfer function \( H(\omega) = A(\omega)e^{i\phi(\omega)} \). The amplitude \( A(\omega) \) and phase \( \phi(\omega) \) are defined, respectively, as the magnitude ratio and the phase difference between the input and output sinusoids. For light propagation in dispersive media, the phase difference between the input and output is given by \( \phi(\omega) = -k(\omega)L \), where \( L \) is the length of the medium. Mitchell and Chiao\(^5\) demonstrated negative delays in a bandpass filter circuit to which the carrier signal modulated with a pulse is applied. They showed that if the carrier frequency is located outside of the pass band, the envelope of the output pulse precedes the input. In practical implementations, instead of the required large inductor, a simulated inductor with operational amplifiers is used. The use of the carrier signal also increases the complications of the circuit.

The circuit that we discuss generates negative delays for baseband signals or signals with zero carrier frequency. It is easy to produce negative delays as large as a few seconds, and hence we can observe them with the naked eye by watching two LEDs (light emitting diodes), one driven by the input and the other by the output pulse. With this simple electronic circuit, we can clearly demonstrate the mechanism of superluminal light propagation. This experiment can be added to the repertoire of experiments that demonstrate advanced physical concepts with simple electronic circuits.\(^7,8\)

In Sec. II, we explain the relation between the superluminal or negative group velocity in light propagation and the negative group delay in lumped circuits. In Sec. III, we present a circuit that generates negative group delays and the pulse generator used for the input. In Sec. IV, we propose a method for larger negative delays by cascading the circuits. It is shown that the negative delay time can be increased by \( \sqrt{n} \) without introducing additional distortion of the pulse shape, where \( n \) is the number of stages. However for large \( n \), the circuit becomes susceptible to noise.

II. TRANSFER FUNCTION FOR NEGATIVE GROUP DELAY

We discuss negative group delays using a transfer function, or frequency response \( H(\omega) \).\(^9\) The discussion can be
applied to both electric circuits and light propagation in a
dispersive medium. A band-limited input signal $E_{\text{in}}(t)$ can be
expressed as a product of the carrier $\exp(i\omega_0 t)$ and the enve-
lope $E_{\text{en}}(t)$: $E_{\text{in}}(t) = E_{\text{en}}(t)\exp(i\omega_0 t + \text{c.c.)}$, where c.c. represents
the complex conjugate. Our circuit deals with the signal
without the carrier ($\omega_0 = 0$), called the baseband signal, but
we include the carrier for the purpose of comparison with
other cases. The signal is expanded in terms of the Fourier
component $\tilde{E}_{\text{en}}(u)$ of the envelope $E_{\text{en}}(t)$ as

$$E_{\text{in}}(t) = \int_{-\Omega/2}^{\Omega/2} du \tilde{E}_{\text{en}}(u) e^{i(u_{\omega_0} + u)t + \text{c.c.}},$$

where $\Omega$ is the bandwidth and $u$ is the offset frequency from
$\omega_0$.

The transfer function $H(\omega)$ is defined for each frequency $\omega = \omega_0 + u$, and the output $E_{\text{out}}(t)$ can be written as

$$E_{\text{out}}(t) = \int_{-\Omega/2}^{\Omega/2} du \tilde{E}_{\text{en}}(u)H(\omega) e^{i(u_{\omega_0} + u)t + \text{c.c.}}$$

$$= \int_{-\Omega/2}^{\Omega/2} du \tilde{E}_{\text{en}}(u)A(\omega) e^{i(u_{\omega_0} + u)t} e^{i\phi(\omega)} + \text{c.c.}$$

We assume that within the bandwidth ($|u| < \Omega/2$), the
amplitude $A(\omega)$ is nearly unity and the phase $\phi(\omega)$ can be approximated by a linear function, that is,

$$A(\omega) \sim 1, \quad \phi(\omega) \sim \phi(\omega_0) - u t_d.$$  

The group delay $t_d$ is defined as

$$t_d = -\frac{d\phi}{d\omega} \bigg|_{\omega_0}.$$  

The envelope of the output is obtained from Eqs. (2) and (3) as

$$E_{\text{out}}(t) = E_{\text{en}}(t - t_d) e^{i\phi(\omega_0)}.$$  

Equation (5) means that, aside from the phase factor, the
envelope of the output is shifted by the group delay $t_d$, while
maintaining its shape. For $t_d > 0$, the input precedes the output
(normal delay) and for $t_d < 0$, the output precedes the input
(negative delay). We will represent a circuit that satis-
ifies the latter condition in Sec. III.

To translate the above discussion into light propagation
through a dispersive medium of length $L$, we let $E_{\text{in}}$ and $E_{\text{out}}$
represent the input and output field of the medium, respec-
tively. When monochromatic light with frequency $\omega$ is
propagated in the medium, the phase of the field is shifted by
$\phi(\omega) = -k(\omega)L$, where $k(\omega)$ is the wave number in the
medium. If $k(\omega)$ is linear in the bandwidth, we have with the help of Eq. (4)

$$t_d = \frac{1}{v_g} \frac{dk}{d\omega} \bigg|_{\omega_0},$$

where $d\omega dk|_{\omega_0}$ is the group velocity $v_g$. The envelope of
the light is delayed by $L/v_g$.

If the difference between the propagation time of the enve-
lope in the dispersive medium and that in a vacuum with
the same length $L$ is negative, that is,

$$\Delta t = t_d - L/c = L(v_g^{-1} - c^{-1}) < 0,$$

then the light propagation in the medium is superluminal.
There are two cases that satisfy this condition; $v_g > c$ and
$v_g < 0$. In the former case, the output envelope precedes the
output for the vacuum case but does not precede the input. In
the latter case, the output precedes the input, or the medium
produces the negative group delay $t_d < 0$.

### III. Circuits and Experiment

#### A. Negative delay circuit for baseband signals

In Fig. 2, we show a negative delay circuit for baseband ($\omega_0 = 0$) signals. This circuit is basically a noninverting (im-
perfect) differentiator. Its transfer function is obtained as

$$H(\omega) = A(\omega) e^{i\phi(\omega)} = 1 + i\omega T,$$

where $T = RC$.

We note that the corresponding operation in the time do-
main is $1 + T(d/dt)$. For the rising edge, which has a posi-
tive slope, the two terms interfere constructively, while for
the falling edge, which has a negative slope, they interfere
destructively. Thus the pulse peak is advanced.

In the low-frequency region ($|\omega| \ll 1/T$), $H(\omega)$ is approxi-
mated by

$$A(\omega) = 1 + O(\omega^2 T^2),$$

$$\phi(\omega) = \omega T + O(\omega^3 T^3),$$

which mean that the amplitude is nearly constant and the
phase increases linearly with frequency. Then the group de-
lay becomes negative,

$$t_d = -\frac{d\phi}{d\omega} \bigg|_{\omega=0} = -T < 0.$$  

As seen in Fig. 3, the amplitude $A(\omega)$ and the phase $\phi(\omega)$
of the transfer function are not linear except when $|\omega|T \ll 1$
due to the higher order terms in Eqs. (9) and (10). These
terms induce distortion of the wave form of the output.
To keep the distortion as small as possible, the spectrum of the
input signal must be restricted to the frequency region $|\omega| \ll 1/T$. For this purpose low-pass filters are needed.

**B. Low-pass filter**

To prepare a band-limited pulse we introduce low-pass filters. The initial source is a rectangular pulse from a timer IC. Because the rectangular pulse has high-frequency components, the negative delay circuit does not work properly. We must eliminate the high-frequency components with low-pass filters. As shown in Fig. 4, we introduce a two-stage low-pass filter. The transfer function of each filter is

$$H_{LP}(\omega) = \frac{\alpha}{1 + i\omega T_{LP}(3 - \alpha) + (i\omega T_{LP})^2},$$

where $T_{LP} = R_1 C_1$ and $\alpha = (1 + R_3 / R_2)$. The order of the filter is given by the order of the denominator and the total order of the two-stage low-pass filters is given by $m = 4$. By changing $\alpha$, we can tailor the characteristics of the filter. In our experiment we choose $\alpha = 1.268$, which corresponds to a Bessel filter. The Bessel filter is designed so that the overshoot for rectangular waves is small. The cutoff frequency is defined as $\omega_c = 0.7861 / T_{LP}$.

**C. Experiment**

We show the overall circuit diagram for the negative delay experiment in Fig. 4 and the parameters in Table I. The pulse generator in the upper section of Fig. 4 is subdivided into the generator of the single-shot rectangular wave and the low-pass filters. In the first part, when triggered by the switch, the timer IC generates a single pulse, whose width is determined by the time constant $T_{rec} = R_0 C_0 = 1.5$ s. The rectangular pulse is shaped by the two-stage low-pass filters. We take the cutoff frequency of the low-pass filter to be $\omega_c = 0.35 / T$, so that $A(\omega)$ and $\phi(\omega)$ can be considered to be constant and linear, respectively, below the cutoff frequency (see Fig. 3). Finally, the band-limited single pulse becomes the input of the negative delay circuit in the lower section of Fig. 4.

Two delay circuits shown in Fig. 2 are cascaded for larger advance times. The input and output terminals are monitored by LEDs. Their turn-on voltage is about 1.1 V. The variable resistor at the input is adjusted so that the input and the output have the same height.

The experimental result is shown in Fig. 5. The input and output wave forms are recorded with an oscilloscope. The origin of the time ($t = 0$) is the moment when the switch in Fig. 4 is turned on. We see that the output precedes the input.

![Fig. 3. The amplitude of the transfer function $A(\omega) = |H(\omega)|$ (upper) and the phase $\phi(\omega) = \arg H(\omega)$ (lower).](image)

![Fig. 4. Overall circuits. Upper section, pulse generator. Lower section, negative delay circuit.](image)

![Fig. 5. Experimental results. The oscilloscope traces show the input and output pulses. The output precedes the input due to the negative delay.](image)
considerably (more than 20% of the pulse width). The slight distortion of the output wave form is caused by the nonideal frequency dependence of $A(\omega)$ and $\phi(\omega)$, as mentioned in Sec. III A.

The expected negative delay derived from Eq. (11) is $2T = 0.44$ s; we have connected two circuits (each with time constant $T' = 0.22$ s) in series for a larger effect. This delay quantitatively agrees with the experimental result, where the time difference between the output and the input peaks is about 0.5 s. The time scale is chosen so that we can directly observe the negative delay with two LEDs connected at the input and the output terminals. We also could use two voltmeters (or circuit testers) to monitor the wave forms, and we could dispense with an oscilloscope.

The actual negative delay circuit in Fig. 4 differs from that shown in Fig. 2. The resistor $R'$ and the capacitor $C'$ are added to suppress high-frequency noises. As shown in Fig. 3, the gain at $|\omega|T > 1$ is large. Although the high-frequency components of the input signal are suppressed by the low-pass filters, internal and external noise with high frequency are unavoidable. With $R'$ and $C'$, the high-frequency gain is limited. The parameters are chosen to satisfy $R'C<C'R \ll T$, so that the phase $\phi(\omega)$ for $|\omega|T < 1$ is not affected.

IV. REALIZATION OF LARGER NEGATIVE DELAY

The negative delay in the above experiment is about 20% of the pulse width. This delay is larger than the values obtained in other superluminal-velocity or negative-delay experiments. We now consider how to make even larger negative delays. We assume a noise-free environment for simplicity.

By cascading the negative delay circuits, the time advance can be increased. One might expect that, by increasing the number of stages $n$, the total time advance can be increased linearly with $n$. But, unfortunately, this is not the case. It is obvious from Eqs. (9) and (10) that the distortion of the wave form of the output is also increased. When $n$ circuits are connected in cascade, the total transfer function can be written as $H^n(\omega)$. Correspondingly, the amplitude and the phase are given as

$$A^n(\omega) \approx 1 + \frac{n(\omega T)^2}{2},$$

$$n \phi(\omega) \approx n \omega T .$$

To keep the wave distortion below a certain level, we have to limit the excess gain $A^n(\omega) - 1$ within the bandwidth by some value $\gamma$;

$$\frac{n(\omega T)^2}{2} \leq \frac{n(\omega_c T)^2}{2} = \gamma .$$

Then the advance time per circuit should satisfy

$$T = \sqrt{\frac{2 \gamma}{n \omega_c}} = \sqrt{\frac{2 \gamma}{n T_w}} .$$

where $T_w$ is the pulse width determined by the cutoff frequency of the low-pass filter.

If we want to increase the time advance while conserving the pulse width and the shape of the signal, we must reduce the time advance $T$ per circuit by the factor $1/\sqrt{n}$. Therefore, the total time advance $T_{\text{total}}$ scales as

$$T_{\text{total}} = n T = \sqrt{2n \gamma} T_w ,$$

which is a slowly increasing function of $n$.

In addition, there is another factor to be considered. It is impossible to advance the signal beyond the time when the switch is turned on in the rectangular pulse generator in Fig. 4. A causal transfer function cannot generate negative delays unconditionally. The reason for the advance is that the slow rising part of the pulse, which has been suppressed by the low-pass filter, is amplified by the negative delay circuit. The slowness of the rising part of the pulse is determined by the order $m$ of the low-pass filter. Figure 6 represents the responses of various order Bessel filters with the same cutoff frequency ($\omega_c$). To a rectangular wave with a unit height and a pulse width $\omega_c^{-1}$. The pulse shapes, especially the widths of the pulses, are similar to each other due to the same cutoff frequency, but the higher the order of the filter, the more the rising part is delayed. Hence we need a high order filter to attain a large time advance. In other words, the pulse must be delayed appropriately in advance to obtain a large negative delay.

Moreover, the short-time behavior of the total circuit including the low-pass filters and the negative delay circuits is determined by the composite transfer function at high frequency ($\omega \to \infty$). The order of the low-pass filter $m$ should not be smaller than the number of the stages $n$ of the negative delay circuits. Otherwise, the total transfer function would diverge at $\omega \to \infty$, and the derivative of the rectangular pulse would appear at the output.

Figure 7 represents a result of a simulation for the response of 10 negative delay circuits with the input of a tenth-order Bessel filter. The parameters used are $T_w = 1$ s and $\gamma = 0.2$. The total time advance is estimated to be 2 s from

![Fig. 6. Responses of the Bessel filter to a rectangular wave. The orders of the filters are $m=2$, 4, 6, 8, and 10.](image)

![Fig. 7. A simulation result for a multistage negative delay circuit ($n=10$).](image)
Eq. (17). This estimate is consistent with the result of the simulation. The advance time is comparable to the pulse width. It is so large that the input starts to rise when the output begins to fall.

V. DISCUSSION

Let us consider a positive delay circuit. A circuit known as a all-pass filter has the transfer function

$$H_p(\omega) = \frac{1 - i\omega T}{1 + i\omega T}. \tag{18}$$

The all-pass filter can be constructed with an operational amplifier, three resistors, and a capacitor. It is very convenient for generating positive delays for baseband signals because the amplitude and phase of the transfer functions are $A(\omega) = 1$ and $\phi(\omega) = -2\tan^{-1}\omega T \sim -2\omega T$, respectively. The flat amplitude response makes it possible to cascade the $n$ circuits without introducing pulse distortion and results in the delay $t_d = 2nT$. Unfortunately, the negative version ($T \rightarrow -T$) of an all-pass filter

$$H_n(\omega) = \frac{1 + i\omega T}{1 - i\omega T} = \frac{H(\omega)}{1 - i\omega T}, \tag{19}$$

will not work because it is not causal. The pole of $H_n(-is)$, $s = 1/T$, is located in the right-half plane. If one makes this circuit, it will be unstable due to the time response function $\exp(it/T)$. This example reminds us of the asymmetry between negative and positive delays. The former is much more difficult to achieve than the latter.

The group velocity has no direct connection with relativistic causality, and therefore it can exceed the speed of light $c$ in a vacuum. But the front velocity $v_f$ (or the wave front velocity) is constrained by causality and is equal to $c$, namely, $|v_f| = L/t_f = c$. In lumped systems ($L = 0$), the wave front delay $t_f$ must vanish. Actually all of the pulses in our system have their wave fronts at $t = 0$, the moment when the original rectangular pulse rises or the switch is turned on.

To conclude, we have demonstrated negative group delays in a simple electronic circuit. A considerably large negative delay (0.44 s) can easily be achieved. It is slow enough to be seen with the naked eye using LEDs or voltmeters. The negative delay amounts to 20% of the pulse width. In the light experiment, $t_d - L/c$ is 62 ns and is only a few percent of the pulse width, 4 $\mu$s.

Including the pulse generator for the input of the negative delay circuit, the apparatus consists of common parts available in any laboratory. It is so simple that a beginner can build it in one hour. The setup can be operated stand-alone and no expensive instruments such as oscilloscopes and function generators are required. It is useful for understanding the physics of superluminal propagation as well as negative group delays.

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