Time reversal and object reconstruction with single-cycle pulses

A. Boh Ruffin, Joan Decker, Laurent Sanchez-Palencia, Lénaic Le Hors, John F. Whitaker, and Theodore B. Norris

Center for Ultrafast Optical Science, University of Michigan, 1006 IST Building, 2200 Bonisteel Boulevard, Ann Arbor, Michigan 48109-2099

J. V. Rudd

Picometrix, Inc., P.O. Box 130243, Ann Arbor, Michigan 48113-0243

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We demonstrate the reconstruction of one- and two-dimensional objects by numerically backpropagating measured scattered terahertz transients. The spatial resolution determined by the Sparrow criterion is found to correspond to approximately 30% of the peak wavelength and 85% of the mean wavelength of the power spectrum of the single-cycle waveform. © 2001 Optical Society of America

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An important class of inverse problem in optics is imaging, or object reconstruction. It is well known that object reconstruction can be accomplished if the phase as well as the amplitude distribution of the field scattered by the object is measured. In diffraction or imaging problems, one typically considers the scattering of a narrow-band field; in this Letter we consider the scattering of extremely broadband, single-cycle fields and demonstrate a new method for object reconstruction based on time reversal. This study has been facilitated by recent advances in single-cycle terahertz (THz) pulse generation and measurement and builds on recent investigations of the scattering of THz pulses1–3 and on developments in direct THz imaging.4 In this spirit, it is also our expectation that this study can serve as a scale model for time-reversal imaging with single-cycle pulses in other systems.

Our approach is based on the time-reversal symmetry of Maxwell’s equations and the Huygens–Fresnel diffraction theory.5 The diffraction of broadband electromagnetic pulses can be treated with the time-domain Huygens–Fresnel diffraction formula6

\[
u(P_0, t) = \int_X \frac{\cos \theta}{2\pi c r_0} \frac{\partial}{\partial t} u(P_1, t - \frac{r_{01}}{c}) \, d\sigma,
\]

where \( \theta \) is the zenith angle made with respect to the optic axis, \( c \) is the speed of light, and \( r_{01} \) is the distance from object point \( P_1 \) to far-field point \( P_0 \). This integral calculates a diffracted field \( u(P_0, t) \) from any given input field \( u(P_1, t - r_{01}/c) \), where \( d\sigma \) is an area element in the aperture \( E \). Because of the time-reversal symmetry of Maxwell’s equations, the diffracted field at far-field position \( P_0 \) can be mathematically time reversed and then used as an input field in the above integral equation to reconstruct the field at the object’s position \( P_1 \). That is, it is possible to determine the (spatial) transmission function at object \( P_1 \) by taking a THz waveform measured at several off-axis positions \( P_0 \) and applying a backpropagation algorithm to these fields. Because the time-domain THz measurement yields directly the electric field amplitude, these scattered waveforms contain all the phase (time delay) information that one needs to obtain the spatial distribution of the electric field across the object. Intuitively, each temporal feature of the scattered waveform corresponds to the arrival of an impulse from a different region of the object. The backpropagation algorithm simply numerically differentiates the measured diffracted transient, computes the time-reversed field \( u(P_0, t + r_{01}/c) \), and uses the result as an input to the time-reversed Kirchhoff diffraction integral. In our experiments the scattered fields are measured at several angles on a sphere (referred to as the intermediate screen) centered on the object. It should be noted that the boundary conditions that yield Eq. (1) are for a planar aperture; for a curved surface (at the intermediate screen), proper inclusion of the boundary condition leads to a modification of the obliquity factor. Thus the field in the object plane is given by

\[
u(P_1, t) = -\frac{1}{4\pi c} \int_X (1 + \cos \theta) \times \frac{\partial}{\partial t} u(P_0, t + \frac{r_{01}}{c}) \, d\sigma',
\]

where \( u(P_0, t + r_{01}/c) \) is the time-reversed scattered THz field in the intermediate screen and \( d\sigma' \) is an area element in the intermediate screen.

In our first experiments we demonstrated the concept of time-reversal imaging by using a one-dimensional object. A collimated, single-cycle THz pulse with a beam diameter of approximately 3.0 cm was incident on a 14-slit grating formed by aluminum-foil strips on a low-density polyethylene substrate with a periodicity, albeit irregular, of approximately 2 mm. The grating diffracted the pulse in a plane perpendicular to the grating axis, as illustrated schematically in Fig. 1. The diffracted THz fields were measured at 62 zenith positions \( (0^\circ \leq \theta \leq 45^\circ) \) at a radial distance of 25 cm centered on the object. To maintain the time-zero reference for all off-axis positions, we used a commercial THz measurement system in which femtosecond optical pulses were delivered to the emitter and detector photoconductive antennas via single-mode optical fiber.7 With the
Fig. 1. One-dimensional (1-D) experimental setup. A collimated input field diffracts in all directions from an object. (The input field shown represents a field measured in the near field of the THz emitter.) These scattered transients are then recorded at different azimuths along the intermediate (detection) screen. For the numerical backpropagation, the positions at the intermediate screen are locations for virtual sources, \( u(P_0, t + r_{01}/c) \), that will propagate in the reverse direction to the object plane.

exception of the fiber coupling of the antennas, the system operates in the same manner as a conventional time-domain THz system. Both emitter and receiver used a 90° bow-tie antenna fabricated upon low-temperature-grown GaAs and were coupled to silicon aplanatic hyperhemispherical substrate lenses with a radius of 4 mm. This photoconductive THz system had a mean frequency of 288 GHz with a FWHM of 350 GHz and usable frequency components up to 1.0 THz.

In Fig. 2(a) we show the scattered THz transients measured at each zenith position on the intermediate screen. For increasing nonzero angles one can observe the linear increase in the wave-form period that corresponds to the later arrival times of the fields from slits at an increased separation from the detector. We reconstructed the object by using the time-reversed data of Fig. 2(a) as the input field \( u(P_0, t + r_{01}/c) \) in Eq. (2) and then plotting the electric field versus lateral position in the object plane. The result is shown in Fig. 2(b); the grating periodicity is clearly resolved, although the edge resolution is limited by the system bandwidth. From the edge response, determined by convolution of the grating dimensions with a Gaussian point-spread function, we found the spatial resolution of the system to be approximately 350 μm.

We next demonstrate the principle of time-reversal imaging with a two-dimensional (2-D) object. Ideally, in the 2-D case the scattered fields should be measured at points on a spherical grid that extend over the intermediate screen in both zenith (\( \theta \)) and azimuthal (\( \phi \)) directions, but doing so is experimentally difficult. Instead, we fixed the detector at a single angle \( \theta = \theta_d \neq 0 \) and rotated the object about the \( z \) axis from 0 to \( 2\pi \), as shown in Fig. 3. Rotation in \( \phi \) of the object is entirely equivalent to rotation of the detector on the sphere in a circle \( \theta = \theta_d, 0 \leq \phi \leq 2\pi \). Intuitively, it is clear that it is sufficient to acquire data for only one zenith angle \( \theta \) because, as the object is rotated through \( 0 \leq \phi \leq 2\pi \), there is access to all spatial frequencies that correspond to \( -\theta_d \leq \theta \leq \theta_d \) for both the \( x \) and \( y \) directions in the object plane. Acquiring data for only one zenith angle permitted the use of a conventional photoconductive free-space THz system. For the 2-D experiments this system had a peak and a

![Fig. 2](image-url)

Fig. 2. One-dimensional experimental results: (a) the diffracted fields at the intermediate screen. These waveforms were used as input sources in Eq. (2) to reproduce the transmission function of the object (Fig. 1, inset); the result is shown in (b).

![Fig. 3](image-url)

Fig. 3. 2-D experimental setup. Diffracted electric fields are measured at one off-axis position \( \theta_d = 12^\circ \) as the object (spiral) is rotated about the \( z \) axis from 0 to \( 2\pi \). This figure also demonstrates how measuring at one off-axis position while rotating the object is equivalent to measuring the fields on a spherical screen.
mean frequency of 136 and 377 GHz, respectively, with a FWHM of 351 GHz. For the results shown here, the diffracted fields were measured at one zenith position, \( \theta_d = 12^\circ \), and 72 azimuthal (\( \phi \)) positions from 0 to 2\( \pi \).

To test the principle of 2-D time-reversal imaging by using the scheme described above, we first carried out a simulation, numerically propagating a single-cycle THz plane wave that best approximated the spatiotemporal profile of the actual beam onto a binary aperture screen for the \( \phi = 0^\circ \) and \( \phi = 15^\circ \) orientations. From delay between the waveforms and Eq. (3), the spatial resolution was determined to be 674 \( \mu \)m.

Fig. 4 (a) Time-reversal imaging simulation. Here the scattered field distribution of the electric field incident upon the object was calculated with Eq. (1). The field was then time reversed and backpropagated by use of Eq. (2) under the same conditions as in the experiment. (b) Image obtained from the experimental data after the backpropagation algorithm described in the text has been performed.

In our 2-D experiment, by this criterion the temporal resolution was 467 fs; thus we found that waveforms taken for orientations separated by \( \Delta \phi \approx 15^\circ \) are resolved, as illustrated in Fig. 5. For a detector position \( \theta_d = 12^\circ \) this value of \( \Delta x \) corresponds to a spatial resolution of 674 \( \mu \)m. Intuitively, one might expect the spatial resolution for imaging with single-cycle pulses to be approximated between the wavelengths at which the power spectrum peaks and its centroid; for our system those wavelengths are \( \lambda_{\text{peak}} = 2.2 \) mm and \( \lambda_{\text{mean}} = 796 \) \( \mu \)m, respectively. Thus the actual spatial resolution obtained by the time-reversal imaging technique is approximately 30% of the peak wavelength and 85% of the mean wavelength of the single-cycle pulses used in this experiment.

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References