

# Superluminal propagation of optical pulses inside diffractive structures

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## Abstract

Inside diffractive structures, the intensity maximum of optical pulses can travel with a velocity exceeding the vacuum speed of light. This effect is due to the occurrence of evanescent waves, and is accompanied by strong attenuation. It is emphasized that, due to the attenuation, causality is not violated. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Superluminal phenomena, i.e. phenomena involving velocities exceeding the speed of light in vacuum, occur in various systems [1,2]. The mechanism behind these situations are quantum mechanical or electromagnetic tunneling processes [1,3–10]. The analogy between the tunneling of electromagnetic waves and the quantum mechanical tunneling of particles has been stressed [11], indicating that electromagnetic tunneling may be considered as a useful tool to study the traversal of a quantum mechanical barrier. Microwave experiments confirm the occurrence of superluminal tunneling velocities [12]. The experimental results are found to be in good agreement with calculations based on Maxwell's equations [13,14]. Recently, it was pointed out that related phenomena may also occur in diffractive optics [15].

Since classical electrodynamics is an inherently causal theory, there is only an apparent violation of causality. Upon tunneling of a wave packet, the intensity is reduced drastically, and at every instant the intensity of a transmitted pulse is less than it would be in the absence of the structure that enforces the tunneling process. A difficulty arises from the fact that customary notions such as phase velocity or group velocity are not usefully applicable to

evanescent waves, because their wave vector is imaginary. The velocity of the maximum of an intensity distribution, as considered in this paper, is not a suitable definition of a velocity to check for an eventual violation of causality. It is widely accepted that the front velocity of a truly temporally limited pulse can be employed for such an endeavor; no superluminal front velocity and, thus, no violation of causality has been observed.

Upon transmission of a pulse through an inverted two-level medium, a superluminal group velocity has been reported, where the transmission is not accompanied by an attenuation [6]. This observation was explained as a fully causal pulse reshaping phenomenon, because transmitted information can be obtained from both the pulse's maximum and its front tail just as well.

## 2. Numerical treatment

In this paper, the temporal behavior upon transmission of an optical pulse through a diffractive structure is examined numerically. A Gaussian pulse incident on a diffractive structure can be represented by a superposition of plane waves of different wavelengths (frequencies). The

spectrum of the pulse is assumed to be a Gaussian with half-width  $\sigma_\omega$  centered around  $\omega_0$ . Then, the field is

$$E(t) = \int_{-\infty}^{\infty} d\omega u_0 \exp\left\{-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}\right\} \exp(i\omega t). \quad (1)$$

The diffraction problem requires the introduction of complex coefficients of the reflection  $R(\omega)$  and the transmission  $T(\omega)$  of each plane wave. This results in a transmitted wave packet

$$E_t(t) = u_0 \mathcal{F}\left(\exp\left\{-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}\right\} |T(\omega)| \exp\{i\Phi(\omega)\}\right), \quad (2)$$

where the coefficient of transmission is  $T(\omega) = |T(\omega)| \exp\{i\Phi(\omega)\}$  and  $\mathcal{F}$  denotes a Fourier transform. Provided that the incident pulse is long enough—or its spectrum is narrow enough—the phase of the coefficient of transmission is approximately linear in  $\omega$  and its modulus is approximately constant, and the transmitted field may be written

$$E_t(t) = |T| \exp\left\{i\left(\Phi_0 - \frac{\partial\Phi}{\partial\omega}\omega_0\right)\right\} E(t) \otimes \delta\left(t + \frac{\partial\Phi}{\partial\omega}\right), \quad (3)$$

where  $\otimes$  indicates a convolution. Apart from a constant factor, this is the incident field temporally shifted by  $\Delta t = -\partial\Phi/\partial\omega$ . This indicates the importance of the phase  $\Phi$  of the diffraction coefficients. If the assumptions mentioned above do not hold, diffraction will in general result in a transmitted wave that deviates from the Gaussian shape.

In order to make the diffraction problem amenable to numerical treatment, the coefficients of transmission are evaluated at a finite number of frequencies  $\omega_n = \omega_0 + n\Delta\omega$ ,  $n = -N, \dots, N$  around the center  $\omega_0$  of the spectrum. The Fourier transform of Eq. (2) is replaced by a discrete Fourier transform. The diffraction coefficients are calculated utilizing a rigorous numerical method [16,17].

The origin is chosen such that the maximum of intensity of the incident wave packet traverses the front plane of the structure at  $t = 0$ . The calculated intensity profile is the time-dependent intensity in the rear plane  $z = h$ . The field in this plane varies with  $x$ . The 0th diffraction order is constant across  $z = h$ . In order to consider the  $x$ -dependence, higher diffraction orders have to be taken into account. According to the Bragg condition  $\mathbf{k}_{\text{inc}} = \mathbf{k}_{\text{out}} + m\mathbf{K}$  and  $|\mathbf{k}_{\text{inc}}| = |\mathbf{k}_{\text{out}}|$ , where  $\mathbf{K}$  is the grating vector and  $m$  the diffraction order, these higher orders do not propagate when the period of the structure is smaller than the wavelength and do not contribute to the energy transport into the far-field. An  $x$ -dependence can no longer be observed at several wavelengths behind the grating.

### 3. Properties of the diffractive structure

A metallic grating period  $d = 0.5 \mu\text{m}$  and refractive index  $n_{\text{metal}} = 1.55 + i7.91$  is deposited on a semi-infinite dielectric substrate (Fig. 1). The assumption of the wavelength-dependent refractive index of aluminium shows that the results are not qualitatively affected by this simplification. The substrate of refractive index  $n = 1.40$  fills the half-space  $z < 0$ . Vacuum exists in region 3 behind the grating as well as in the interstices between the metal stripes. The width of the metallic blocks is half a grating period. Unless stated otherwise, the grating height is  $h = 0.50 \mu\text{m}$ . The grating is illuminated by a pulse of light of a mean wavelength of  $\lambda = 0.85 \mu\text{m}$ . The pulse is normally incident from the substrate side and is TE-polarized, i.e., the electric field is perpendicular to  $x$  and  $z$ .

Since the grating period is smaller than the wavelengths  $\lambda$  in vacuum and  $\lambda_1 = \lambda/n_1$  in region 1, only the 0th diffraction order can propagate in transmission and reflection and higher orders are not considered. As a simple model, the grating can be considered as a periodic arrangement of short planar waveguides in the  $z$ -direction. Since the width  $d/2$  of the waveguides is smaller than the wavelength, the field in the waveguides is evanescent, i.e. exponentially decaying along the  $z$ -axis. The crucial difference compared to an exponentially decaying wave in an absorbing medium is that the wave does not exhibit a periodicity and that its phase is constant in the direction of propagation. Guided modes do not exist for  $\lambda > \lambda_{\text{cutoff}} = d$ .

Fig. 2 shows the diffraction efficiency of the 0th diffraction order of the grating as a function of the wavelength. Above the cutoff at  $0.50 \mu\text{m}$  the diffraction efficiency  $\eta$  in transmission falls to zero, while in reflection it rises to 84.6%. 15.4% of the energy is absorbed in the metal in the limit of long wavelengths. Below cutoff  $\eta$  is influenced by a complicated interaction between wave and matter, which cannot be satisfactorily described in a simple model; energy is diffracted into higher diffraction orders. The result is a curve with numerous extrema.

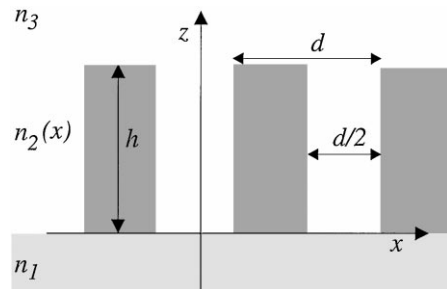


Fig. 1. Geometry of the binary metallic grating and notations.  $n_1 = 1.40$ ,  $n_{\text{metal}} = 1.55 + i7.91$ ,  $n_3 = 1$ ,  $d = 0.5 \mu\text{m}$ ,  $h = 0.5 \mu\text{m}$ .

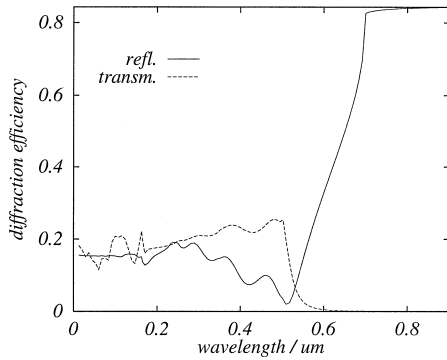


Fig. 2. Diffraction efficiency of the grating versus wavelength.

There is no sharp cutoff at  $\lambda = 0.5 \mu\text{m}$ , because the cutoff phenomenon is a result of the assumption that the waveguide is infinitely extended. However, at  $\lambda = 0.85 \mu\text{m}$  these interactions do not have to be taken into account.

Upon transmission through the grating the wave is exponentially damped. To compare the temporal behavior of the transmitted wave with a wave that propagates the same distance  $h$  through vacuum, it is expedient to plot the intensity normalized in such a way that the area under each curve is unity. The intensity is measured at  $z = h$ . The origin is chosen such that at  $t = 0$  the maximum intensity occurs in the plane  $z = 0$ .

#### 4. Numerical results and discussion

In Fig. 3 the normalized intensity of a 5 femtosecond pulse transmitted through the grating is compared with the same pulse traveling in vacuum. Over the complete depicted time interval, the pulse transmitted through the grating advances the pulse propagated in vacuum. Particularly, the maximum of intensity travels with a velocity

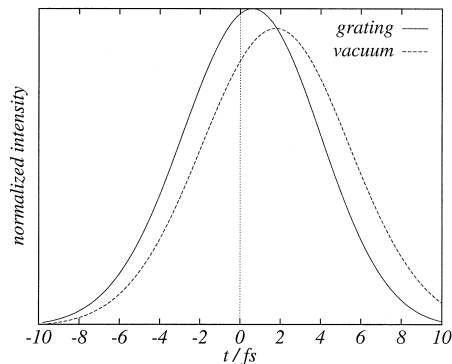


Fig. 3. Intensity versus time of a 5.0 femtosecond pulse transmitted through the grating in comparison with propagation in vacuum. Pulse intensities are normalized to unit area; the grating-transmitted pulse is attenuated by a factor  $1.8 \times 10^{-4}$ .

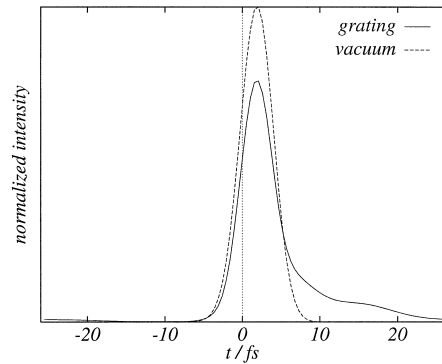


Fig. 4. Normalized intensity versus time of a 2.8 femtosecond pulse transmitted through the grating in comparison with propagation in vacuum. The attenuation of the grating-transmitted pulse amounts to  $3.3 \times 10^{-4}$ .

exceeding the speed of light in vacuum. Noteworthy is that in transmission the pulse is shortened by 0.34 fs. Although the pulse reshaping causes a decrease of the pulse duration, the temporal intensity profile shows no visible deviation from a Gaussian.

With decreasing pulse duration, the superluminal effect vanishes. The maximum of a pulse of 2.80 fs traverses the plane  $z = h$  after 1.88 fs, corresponding to a velocity of  $0.89 c$ . Remarkably, the temporal intensity profile exhibits a distinct deviation from the Gaussian shape of the incident pulse (cf. Fig. 4). The pulse shows an intensity tail, suggesting that a fraction of its energy dwells inside the grating for a time of the order of 10 fs.

With an increase in the grating thickness the intensity behind the grating decreases exponentially, just like in an absorbing medium due to the imaginary part of the wave vector. The maximum of intensity shows a temporal behavior that is significantly different from the propagation in vacuum (Fig. 5). For a grating thickness less than 80 nm it reaches the rear plane with an additional delay compared

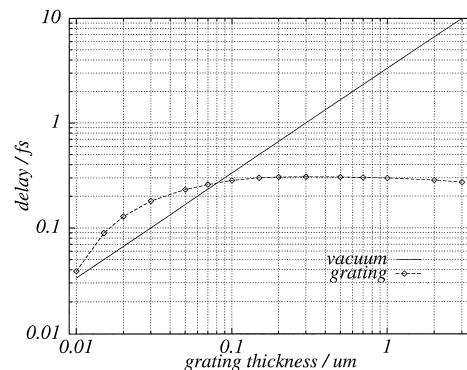


Fig. 5. Delay of the grating-transmitted wave packet versus grating thickness compared to propagation in vacuum over the same distance. Pulse duration is 246 fs.

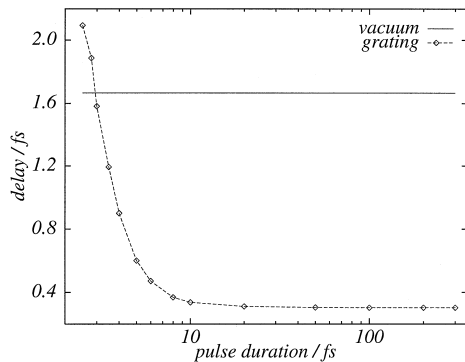


Fig. 6. Pulse duration versus its delay. Grating thickness is 0.5  $\mu\text{m}$ .

to the vacuum case. When the grating thickness exceeds 80 nm the tunneling time becomes independent of the grating thickness. The velocity of the maximum of intensity associated with the tunneling process can reach  $36.7c$  at a grating thickness of 3.0  $\mu\text{m}$ ; however, in this case the intensity is attenuated by a factor of  $10^{-22}$ . With an increase of the grating thickness, the independence of the transit time suggests that inside the vacuum regions of the grating the wave vector is not just complex but purely imaginary.

A similar phenomenon is observed in the quantum mechanical tunneling of a particle through a potential barrier and is known as the Hartmann effect [18].

Provided that the incident pulse is long enough, i.e., it has a narrow spectrum, its spectrum lies in a wavelength interval where the phase of the coefficient of diffraction is linear in  $\lambda$ . Therefore, the pulse delay becomes independent of the pulse duration for a pulse  $> 50$  fs (Fig. 6). For short pulses the superluminal tunneling velocity vanishes.

For a pulse traversing a solid layer of metal no superluminal propagation has been observed. Interpreting the grating as a stack of waveguides, the  $z$ -component of the wave vector  $k_z$  in the interstices of vacuum is purely imaginary, i.e., it is no longer periodic in the  $z$ -direction. On the other hand, inside a metallic layer,  $k_z$  is complex with a non-vanishing real part which gives rise to an exponentially decreasing wave with a periodicity in the  $z$ -direction. This property of the field is vital to its temporal transmission behavior, because the periodicity of the field establishes a phase relation between the field in front of and behind the grating. The lack of a periodicity in the grating case, i.e., the phase of the coefficient of diffraction does not change along  $z$ , gives rise to the phenomenon that an increase of the grating thickness does not necessar-

ily lead to an increase of the transmission time, as shown in Fig. 5.

## 5. Conclusions

The above results show that the propagation of the maximum of intensity of a wave packet with a velocity exceeding the speed of light in vacuum is possible. In this context the question arises, whether the principle of causality is violated, which demands that “information” cannot be transmitted with superluminal velocities. The results presented are obtained by the numerical solution of Maxwell’s equations. Since Maxwell’s equations are inherently Lorentz-invariant, there exists an explanation of the superluminal effect conforming to causality: The numerical data give evidence that, due to the attenuation, at every instant the intensity of the transmitted pulse does not exceed the intensity in absence of the grating. Similar observations have been reported for tunneling through photonic band-gap materials [1,2].

Thus, the effect described above is to be understood as a pulse reshaping phenomenon, where the rear part of the incident pulse is attenuated more than the front part.

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