Modified Getsinger's Model for Accurate Determination of Effective Permittivity Dispersion in Multilayered Microstrip Lines

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Abstract – **Getsinger's Longitudinal Section Electric model describing the propagation characteristics of a single layer microstrip line is modified to cater for 'n' layers. An accurate closed-form expression for two and three layered structures is presented and validated against full-wave numerical results. The computed effective permittivity for a randomly selected two and three layer dielectric microstrip line is presented. In the selected examples, for the modified Getsinger model, the dispersion of effective permittivity is within 2-3% relative error for two-layer and within 6-7% relative error for three-layer dielectric microstrip lines for frequency up to 50 GHz. In contrast, for the same structures, the relative errors in previously reported works are above 10% for two layers and above 15% for three-layer dielectric substrates.**

1 INTRODUCTION

Electromagnetic (EM) simulation software has made very impressive progress over the years. Despite such robust progress, a closed-form expression for dispersion of effective permittivity is still valuable as a starting point for any design problem with microstrip lines on multilayer substrates. This could enable rapid iteration of design variables and reduce overall computation time.

The works of Itoh et al. [1], Jansen et al. [2], Pues et al. [3], Kuester et al. [4] and Getsinger [5] are some of the earlier attempts to describe the dispersion of effective permittivity in microstrip transmission lines. Getsinger [5] proposed a Longitudinal Section Electric (LSE) model, based on the assumption that the quasi-TEM mode on microstrip line is primarily a single LSE mode. Kirschning et al. [6] further improved this model and accurately showed the dispersion in the effective dielectric permittivity up to millimeter wave frequencies for a microstrip transmission line on a homogenous substrate (single layer). In contrast, microstrip lines on multilayered substrate have only been studied extensively for the computation of quasi-static effective dielectric constant. Very little has been reported in the literature on a closed-form expression for the dispersion of effective permittivity in multilayered structures. A computation of dispersion of effective permittivity in microstrip lines on multilayer substrates has been reported by Verma et al. [7]. His method is based on the determination of the quasi-static effective permittivity of multilayer substrate using variational and transmission line approach [8], and computation of filling factor using conformal analysis [9]. An equivalent single layer relative permittivity is then computed for application into Getsinger's LSE model for determination of dispersion of effective permittivity.

In this paper, a direct approach for the determination of equivalent permittivity for lines on a multilayer substrate is presented. Getsinger's LSE model is modified to provide an accurate closed-form expression for the dispersion of effective permittivity of microstrip lines on two- and three-layer dielectric substrates. The results have been validated against a 2D finite-element eigenmode analysis, using the port solution in Ansoft's HFSS™.

2 GETSINGER'S MODEL

Getsinger justifies the validity of the LSE model on the basis that if an analyzable model exhibits similar measurable characteristics to the actual structure, then the model is valid and acceptable. Figure 1 shows the single layered LSE model for a microstrip transmission line.

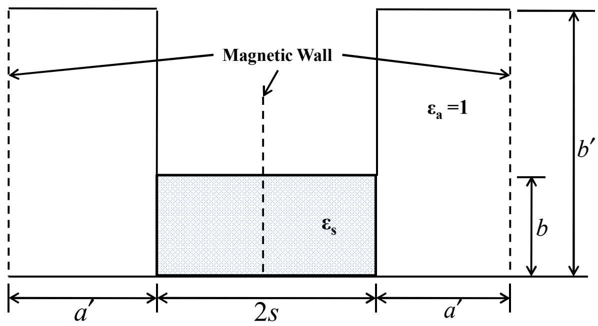

Figure 1: LSE model for single layer dielectric microstrip line.

The solution for the dispersion of effective permittivity ε_{eff} is obtained through the application of this model using the Transverse Resonance (TR) technique [10] and yields

$$\varepsilon_{eff} = \varepsilon_s - \frac{\varepsilon_s - \varepsilon_{e0}}{1 + p(f)} \tag{1}$$

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where $p(f)$ is a polynomial obtained by Kirschning et al. [6], ε_{e0} is the quasi-static effective dielectric constant and ε_s is the permittivity of the single layer dielectric substrate. The model [6] is valid up to 60 GHz under the constraints that (a) $0.1 \le W/b \le 100$ (b) $1 \le \varepsilon_s \le 20$ and (c) $0 \le b/\lambda_0 \le 0.13$, where W is the width of the microstrip line, and b is the thickness of the substrate. The same constraints are applicable to the modified Getsinger model in this paper.

3 MULTILAYER GETSINGER'S MODEL

A multilayered version of Getsinger's LSE model may be developed by recursive application of the transverse resonance method. However, a general closed-form expression for dispersion due to the frequency-dependent effective permittivity rapidly becomes too complex and cumbersome. We therefore restrict to the analysis and development of closed-form expression for microstrip lines on two- and three-layer substrates only. A simpler alternative numerical approach is proposed at the end of the analysis.

The basic approach in the modified Getsinger model is: (a) Obtain a transcendental relationship between characteristic impedances and thicknesses of the layer using TR method; (b) Develop relationships between the relative permittivity constants, and propagation constants of different layers with equivalent permittivity constant ε_s. The equivalent permittivity constant is the permittivity constant of a hypothetical equivalent homogenous layer that has the same dispersion of effective permittivity as the actual multilayer structure; (c) In the transcendental equation, substitute the characteristic impedances through the ratio of propagation constants and permittivity of the layers [11] and apply a suitable approximation for the hyperbolic tangents [5]; (d) Solve the resultant equation for the equivalent permittivity constant ε_s; (e) Finally, the dispersion of effective permittivity is obtained by applying the found value of ε_s to (1).

3.1 Two-layer LSE Model

The LSE model for the two-layer dielectric microstrip transmission line is shown in Figure 2. The two layers are characterized by thicknesses b_i and permittivity ε_i, with $i = 1, 2$.

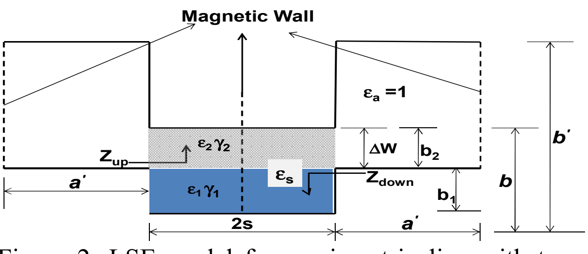

Figure 2: LSE model for a microstrip line with two-layer dielectric substrate.

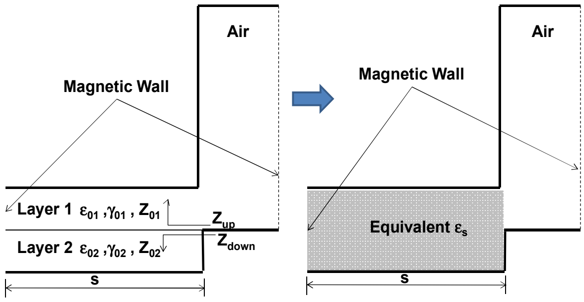

Figure 3: Equivalence of LSE models.

The basic philosophy of the modified Getsinger model is depicted in Figure 3. Using TR [10], the input impedance, as seen looking upwards and downwards, from the junction of the two dielectric interface (Figure) is given by $Z_{up} + Z_{down} = 0$, or

$$Z_{02} \tanh \gamma_2 b_2 + Z_{01} \tanh \gamma_1 b_1 = 0 \qquad (2)$$

Further, in terms of equivalent permittivity of a single homogenous layer $\gamma_s^2 = -k_0^2 \varepsilon_s$ which is related to γ_1, ε_1 through $\gamma_1^2 = k_0^2(\varepsilon_s - \varepsilon_1)$ and γ_2, ε_2 through $\gamma_2^2 = k_0^2(\varepsilon_s - \varepsilon_2)$. Using the approximation of the hyperbolic tangent $\tanh x \approx 3x/(3+x^2)$ [5] in (2), yields after some algebraic manipulation

$$\varepsilon_s = \frac{[3\alpha_2 - \alpha_1(\varepsilon_1+\varepsilon_2)] \pm \{[3\alpha_2 - \alpha_1(\varepsilon_1+\varepsilon_2)]^2 - 4\alpha_1\varepsilon_1\varepsilon_2(\alpha_1 - 3b)\}^{\frac{1}{2}}}{2\alpha_1} \qquad (3)$$

where $\alpha_1 = b_1 b_2 k_0^2(\varepsilon_1 b_1 + \varepsilon_2 b_2)$, $\alpha_2 = \varepsilon_1 b_2 + \varepsilon_2 b_1$ and $b = b_1 + b_2$. As a consistency test, when $b_2 = 0$ (or $b_1 = 0$) then $\gamma_2 = \gamma_1$ and (2) reduces to a single-layer problem. The physically possible solution of ε_s from (3) is applied to (1) for obtaining the dispersion of effective permittivity in microstrip lines with two-layer substrates.

3.2 Three-layer LSE Model

In a similar manner, the dispersion of effective permittivity in the case of a three-layer microstrip line structure was analyzed (Figure 4). As before, TR is used to obtain the transcendental equation $Z_{up} + Z_{down} = 0$, or

$$Z_{01}Z_{02} \tanh \gamma_1 b_1 + Z_{01}Z_{03} \tanh \gamma_1 b_1 \tanh \gamma_2 b_2 \tanh \gamma_3 b_3$$
$$+ Z_{02}Z_{03} \tanh \gamma_3 b_3 + Z_{02}^2 \tanh \gamma_2 b_2 = 0 \qquad (4)$$

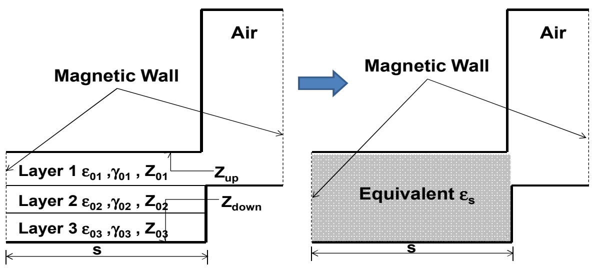

Figure 4: Three-layer Getsinger's LSE model.

Using relationship between ε_s and γ_s with corresponding ones for the substrate layers yields $\gamma_s^2 = -k_0^2 \varepsilon_s$, $\gamma_1^2 = k_0^2(\varepsilon_s - \varepsilon_1)$, $\gamma_2^2 = k_0^2(\varepsilon_s - \varepsilon_2)$ and $\gamma_3^2 = k_0^2(\varepsilon_s - \varepsilon_3)$. Applying the $\tanh x$ approximation for solving (4) results in

$$(\rho_1 + \tau_1)\varepsilon_s^3 + (\alpha_1 + \beta_1 + \rho_2 + \tau_2)\varepsilon_s^2 + (\alpha_2 + \beta_2 + \rho_3 + \tau_3)\varepsilon_s + (\alpha_3 + \beta_3 + \rho_4 + \tau_4) = 0 \quad (5)$$

where

$$\rho_1 = k_0^4 b_1^2 b_2^2 b_3 \varepsilon_1 \varepsilon_2$$
$$\rho_2 = (3k_0^2 b_2^2 + 3k_0^2 b_1^2 - k_0^4 b_1^2 b_2^2 b_3 \varepsilon_1 \varepsilon_2 (\varepsilon_1 + \varepsilon_2 + \varepsilon_3))$$
$$\rho_3 = (9 + k_0^4 b_1^2 b_2^2 \varepsilon_1 \varepsilon_2) b_3 \varepsilon_1 \varepsilon_2 - b_3 \varepsilon_1 \varepsilon_2 \varepsilon_3 (3k_0^2 b_2^2 + 3k_0^2 b_1^2 - k_0^4 b_1^2 b_2^2 (\varepsilon_1 + \varepsilon_2))$$
$$\rho_4 = -b_3 \varepsilon_1 \varepsilon_2 \varepsilon_3 (9 + k_0^4 b_1^2 b_2^2 \varepsilon_1 \varepsilon_2)$$
$$\tau_1 = k_0^4 b_1^2 b_3^2 b_2 \varepsilon_1 \varepsilon_3$$
$$\tau_2 = 3k_0^2 b_1^2 + 3k_0^2 b_3^2 - k_0^4 b_1^2 b_3^2 b_2 \varepsilon_1 \varepsilon_3 (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$
$$\tau_3 = b_2 \varepsilon_1 \varepsilon_3 (9 + k_0^4 b_1^2 b_3^2 \varepsilon_1 \varepsilon_3) - b_2 \varepsilon_1 \varepsilon_2 \varepsilon_3 (3k_0^2 b_1^2 + 3k_0^2 b_3^2 - k_0^4 b_1^2 b_3^2 (\varepsilon_1 + \varepsilon_3))$$
$$\tau_4 = -b_2 \varepsilon_1 \varepsilon_2 \varepsilon_3 (9 + k_0^4 b_1^2 b_3^2 \varepsilon_1 \varepsilon_3)$$
$$\alpha_1 = k_0^2 b_2^2 b_1 \varepsilon_2 \varepsilon_3$$
$$\alpha_2 = 3b_1 \varepsilon_2 \varepsilon_3 - k_0^2 b_2^2 b_1 \varepsilon_2 \varepsilon_3 (\varepsilon_1 + \varepsilon_2)$$
$$\alpha_3 = k_0^2 b_2^2 b_1 \varepsilon_1 \varepsilon_2^2 \varepsilon_3 - 3b_1 \varepsilon_2 \varepsilon_3$$
$$\beta_1 = 9k_0^2 \varepsilon_2^2 b_1 b_2 b_3$$
$$\beta_2 = -9k_0^2 \varepsilon_2^2 b_1 b_2 b_3 (\varepsilon_1 + \varepsilon_3)$$
$$\beta_3 = 9k_0^2 \varepsilon_2^2 \varepsilon_1 \varepsilon_3 b_1 b_2 b_3 \quad (6)$$

When $b_3 = 0$, the model reduces to a two-layer system and $\gamma_3 = \gamma_2$ and (4) reduces to a two-layer problem. The physically possible solution of ε_s from (5) is applied to (1) for obtaining the dispersion of effective permittivity.

3.3 Alternative numerical approach

Present day computational power of computers allows us to solve transcendental equations of type (2) and (4) very efficiently using software such as Maple™ and Mathematica™. The advantage is that a Taylor approximation (with limited validity) of the hyperbolic functions is not required.

4 SIMULATIONS AND RESULTS

The simulation of microstrip lines on two- and three-layer substrates was done using Ansoft HFSS™ to provide a reference. As the microstrip line is an open line, the port has to be bounded. The simulations were run by considering the port boundaries sufficiently far away from the microstrip line so that they do not influence the microstrip line eigenmode. A sufficient port size is achieved when the eigenmode solution becomes independent of the outer boundary condition, i.e. identical for radiation, Perfect Magnetic Conductor (PMC), Perfect Electric Conductor (PEC), or Perfectly Matched Layer (PML). This consistency of results across the various port outer boundary conditions guarantees accuracy of the results obtained by the model. This numerical reference solution was used to validate the closed-form dispersion expressions obtained from the proposed multilayered Getsinger LSE model. Additionally, the results were compared against various other methods using Single Layer Reduction (SLR) [7] in conjunction with other methods for determining the quasi-static effective dielectric constant ε_{e0} [12, 13], such as Variational and TL, conformal mapping and the series capacitance method. The Unified Dispersion Model [14] was applied to obtain the dispersion of effective permittivity. Examples of computed two- and three-layer cases are shown below for illustration.

4.1 Two-layer microstrip line

A microstrip line of width W = 1.41 mm was simulated on a substrate with ε_1 = 4.4, b_1 = 0.8 mm and a top layer of ε_2 = 2.33, b_2 = 0.15 mm. It is evident from Figure 5 that the modified Getsinger model follows the actual dispersion of effective permittivity very closely. The relative error in the results is within 2-3% as against other approaches where it is within 9-10%.

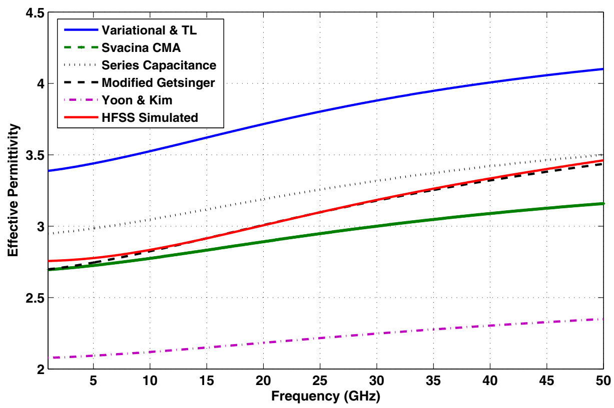

Figure 5: Dispersion of effective permittivity for a two-layer system.

4.2 Three-layer microstrip line

A three layered dielectric microstrip line was simulated of width W =1.41 mm with bottom layer: ε_1 = 4.5, b_1 = 0.8mm, middle layer, ε_2 = 3.27, b_2 = 1.0 mm and top layer: ε_3 = 2.33, b_3 = 1.6 mm. Figure 6 clearly indicates that the results of the modified Getsinger model are within 6-7% as against other approaches which are within 15-20% of the actual dispersion.

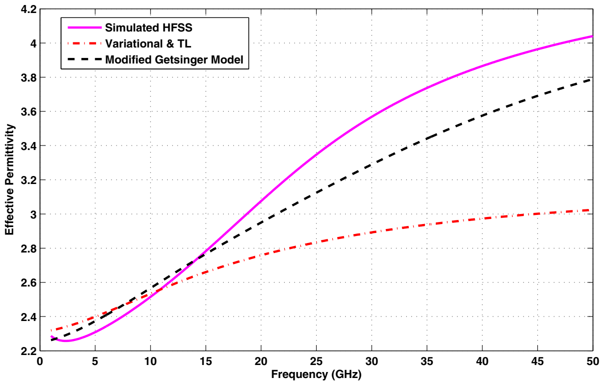

Figure 6: Dispersion of effective permittivity for a three-layer system.

4 CONCLUSIONS

From the results presented, it may be concluded that the results from the proposed modified Getsinger model are within 2-3% error for two-layered and within 6-7% error for three layered microstrip line structure, while other approaches can provide above 10% errors in two layered and above 15% error for three layered microstrip line structures. The modified Getsinger's model therefore provides a good starting point for solving multilayered microstrip line problems.

Acknowledgments

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