

# A Comparative Study of Volumetric vs. Subcell Modeling of Thin-Wire Structures in FVTD

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**Abstract**—Unstructured time-domain solvers such as the finite-volume time-domain method have the capability to directly model very fine geometric features. Nevertheless, subcell thin-wire models are beneficial for reducing the number of volumetric cells. In this work, we compare broadband network parameter extraction when wires are modeled by volumetric discretization and fed by appropriately selected port models versus wires modeled using a mesh-independent thin-wire model. The results of these methods are compared to measurements for broadband S-parameter extraction of a simple monopole antenna.

## I. INTRODUCTION

Time-domain solvers such as the finite-volume time-domain method (FVTD) make use of an unstructured partition of the computational domain for flexible geometric modeling [1]. FVTD, combined with local time-stepping schemes permits volumetric discretizations of very fine geometric features including thin-wire structures [2]. For broadband system characterization, a port model can be used to both excite and capture the electrical response of the system [3].

In contrast to directly modeling thin-wires, a subcell model can be used. The model in [4] allows for arbitrary wire placement within unstructured meshes, making it possible to reposition the wires without regenerating the volumetric mesh. Recently, this model has been adapted to FVTD and coupled with a subcell circuit model for driving and/or interconnecting wires and circuits [5]. Voltages and currents from these circuits can be used for broadband system characterization.

In this work, we compare simulations of a simple monopole antenna by both a direct volumetric discretization and a circuit-driven subcell thin-wire model. The comparison is performed over a large frequency band for a very thin wire. We also outline the steps required to extract the correct wire voltage from the subcell model in the absence of PEC symmetry.

## II. THE COMPUTATIONAL ENGINES

Herein, two computational engines are used: a fully volumetric FVTD engine augmented with both local-time-stepping capabilities and port models [2], [3], and a fully parallel FVTD engine augmented with a coupled subcell circuit and thin-wire model [5]. The volumetric update algorithm for both solvers is a cell-centered, field-collocated, flux-split upwind scheme that uses second-order time-stepping [1].

The port model of the first engine is based on a fully discretized transmission line profile. The excitation and S-parameter extraction are implemented by exploiting flux-splitting to separate incident and reflected waves [3].

In the second engine, the system of equations introduced to subcell model a lossless,  $\hat{\zeta}$ -directed thin-wire is

$$\frac{\partial}{\partial t} \begin{bmatrix} C\mathcal{V}(t, \zeta) \\ LI(t, \zeta) \end{bmatrix} + \frac{\partial}{\partial \zeta} \begin{bmatrix} \mathcal{I}(t, \zeta) \\ \mathcal{V}(t, \zeta) \end{bmatrix} = \begin{bmatrix} 0 \\ \langle \mathcal{E}_{\zeta} \rangle \end{bmatrix}, \quad (1)$$

where  $\mathcal{V}(t, \zeta)$  is the wire voltage,  $\mathcal{I}(t, \zeta)$  is the wire current,  $C$  and  $L$  are the per-unit-length capacitance and inductance and  $\langle \mathcal{E}_{\zeta} \rangle$  is a weighted projection of the electric field at the wire coordinate  $\zeta$  [4], [5]. The quantities  $L$ ,  $C$  and  $\mathcal{V}(t, \zeta)$  are dependent on the mechanism by which the weighted field is computed – the reference for the voltage lies at an empirical coupling distance denoted  $\rho_0$  in the literature. As a result, the thin-wire voltage cannot be *directly* used (without modification) to compute system parameters such as input impedance except in special cases [5]. This is addressed for a coax-fed subcell monopole in the following section. Limitations of the subcell model are assumed homogeneity surrounding the wire to  $\rho_0$  and large element sizes compared to the wire radius [4]. The port model is more general but is limited by the computational cost associated with the resolution of very thin wires.

## III. NUMERICAL RESULTS

As a comparative example, we consider a 5 cm monopole antenna over an infinite PEC ground plane. The antenna diameter is 0.51 mm and we assume that the bandwidth of interest extends up to 8.5 GHz. The diameter of the wire is much smaller than even the shortest wavelength of interest (3.5 cm) so a subcell model is applicable. The wire diameter is roughly  $\lambda/70$  at the maximum frequency and  $\lambda/400$  at the first resonance. The volumetric region used for the subcell model is shown in Figure 1a and consists of roughly 500,000 tetrahedral cells. A circuit is introduced at the wire/PEC junction as shown in Figure 1b and from the circuit the voltage  $\mathcal{V}_1(t)$  and the current  $\mathcal{I}_1(t)$  are extracted during the simulation. These values are used to compute the input impedance, which is then converted to  $S_{11}$ . Due to PEC symmetry, the circuit voltage can be used directly in this

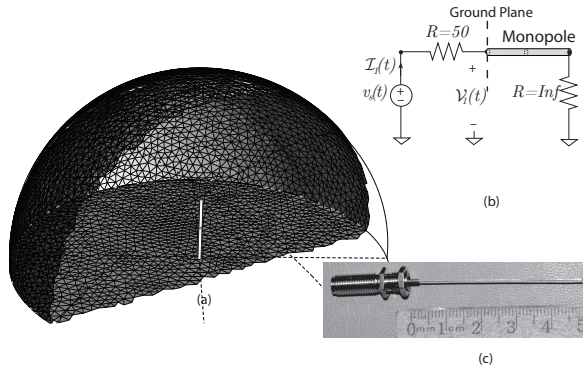


Fig. 1. a) Hemispherical meshed domain used for the subcell simulation of a 5 cm thin-wire monopole - the PEC groundplane extends to the edges of the domain. b) Circuit configuration for driving the thin-wire - circuits are associated with the end points of the monopole and have zero dimension. c) Physical wire and SMA connector used for measurements.

calculation as it is effectively dereferenced from  $\rho_0$  [5]. Note that the groundplane is continuous as no volumetric feeding device is connected to the monopole. The simulation was run to 5 ns.

For the port model, a coaxial cable is used to drive the volumetrically discretized monopole as shown in Figure 2a. The coax was designed to have a  $50 \Omega$  impedance by selecting an outer diameter of 1.75 mm and a relative permittivity of 2.18 between the conductors. The model uses roughly 1.5 millions tetrahedral cells, in a strongly inhomogeneous mesh.

We can include the coax as part of the subcell simulations by replacing the centre coax conductor with the subcell model as shown in Figure 2b. A circuit is used to join the coax to the base of the monopole and the voltage and current are extracted. To ensure large cell sizes relative to the wire radius, a 10 mm diameter coax with a permittivity of 12 was chosen. The voltage at the coax/monopole junction cannot be used directly because the coax removes PEC symmetry. Instead, we compute the per-unit-length charge  $Q(t) = C\mathcal{V}(t)$  at the monopole base. The electric field supported by the model can be approximated from  $Q(t)$  and integrated to the coax radius to determine the correct voltage [5].

A comparison of the computed reflection coefficient is shown in Figure 3 and is compared to measurements taken using the SMA feed of Figure 1c [5]. The results show good agreement - the small discontinuity in the ground plane introduced by the coax is negligible. The benefit of the subcell model is illustrated by a high computational cost of the port model for this very thin wire. The anomaly at 7.5 GHz in the coax-fed subcell model is likely due to the coax/PEC aperture which is a quarter-wavelength at 7.5 GHz. A voltage computed by directly integrating the volumetric electric field values may remove this effect and will be investigated in future work.

#### IV. CONCLUSIONS AND OUTLOOK

An FVTD subcell thin-wire model and a volumetric port model have been compared for a thin monopole. The subcell model requires significantly fewer volumetric cells to resolve

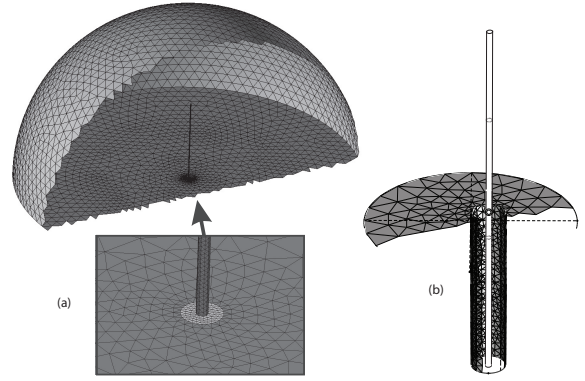


Fig. 2. a) Volumetric discretization of the port-fed monopole. b) For the coax-fed subcell model, the center conductor is replaced with the wire model.

this fine geometry. When feeding the monopole with a coax, the thin-wire voltage must be converted properly for accurate results. If the feed is small but appreciable, a thin-wire model may not accurately capture its effects. The port model does not suffer from these or other subcell limitations but requires resolution of the wire which becomes very costly. Further comparison of these two techniques will be the focus of future work.

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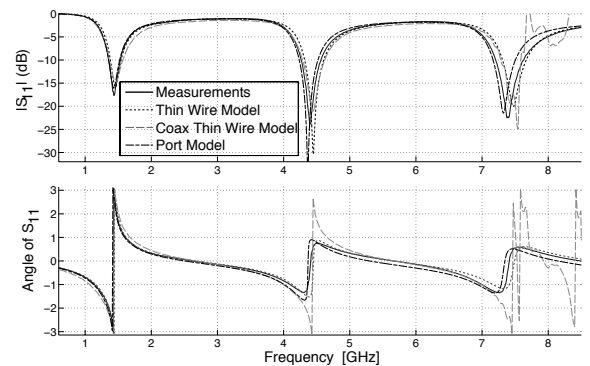


Fig. 3. Reflection coefficient of the monopole antenna computed using the subcell and port models. At 7.5 GHz the 10 mm coax feed is 1/4 wavelength.