Conformal and Multi-Scale Time-Domain Methods: From Tetrahedral Mesh to Meshless Discretisation

Christophe Fumeaux[#1], Thomas Kaufmann[#2], Dirk Baumann[*3], Maciej Klemm[†4]

[#]*School of Electrical and Electronic Engineering, The University of Adelaide*
Adelaide SA 5005, Australia
[1]cfumeaux@eleceng.adelaide.edu.au
[2]thomaska@eleceng.adelaide.edu.au

[*]*Laboratory for Electromagnetic Fields and Microwave Electronics, ETH Zurich*
CH-8092 Zurich, Switzerland
[3]dbaumann@ifh.ee.ethz.ch

[†]*Department of Electrical and Electronic Engineering, University of Bristol*
Bristol BS8 1UB UK
[4]m.klemm@bristol.ac.uk

***Abstract*— Numerical methods applied in Cartesian grids have become workhorses for general purpose time-domain electromagnetic simulations because of their simplicity, efficiency and scalability. Implementations often consider specific treatments for curved and slanted boundaries, as well as sub-cell models and sub-gridding schemes. As alternative, methods based on unstructured discretisation, such as a tetrahedral mesh, have never truly become mainstream techniques despite their remarkable capabilities for accurate multi-scale and conformal modelling. This paper firstly reviews the development of a particular conformal time-domain method applied in tetrahedral meshes, namely the Finite-Volume Time-Domain method, and illustrates its potential for multi-scale problems in a selected example. The second part of the paper points out a novel class of methods which are amenable to conformal time-domain implementation on clouds of points. These so-called "meshless methods" do not require an explicit mesh definition, and open new perspectives towards future applications involving multi-scale multi-physics problems.**

I. Introduction

Full-wave electromagnetic (EM) simulations have become indispensable tools for analysis and design of devices, from radio-frequency equipment to optical nano-structures. In the last four decades, the spectacular evolutions of computing hardware coupled to the steady developments of powerful numerical algorithms have dramatically increased the size and complexity of EM problems that can be solved using a standard computer. The class of time-domain methods based on volume discretisation has become extremely relevant in today's landscape of EM simulators. This can be explained by the natural capabilities of these methods for the treatment of transient and wideband problems involving inhomogeneous dispersive media, as found for example in bio-EM applications. Among the time-domain methods, the most prominent are probably the Finite Integration Technique (FIT) [1], the Finite-Difference Time-Domain (FDTD) method [2] and the Transmission Line Matrix (TLM) method [3]. Those techniques build the core of some of the most prominent commercial EM simulation tools. In their common basic implementation, all these methods are based on space discretisation with a structured hexahedral grid, exemplified by the well-known staggered Yee grid [4] in FDTD. One of the undeniable strengths of structured arrangements is their amenability to efficient parallelisation for computation in clusters and multi-core computers, or with multiple graphical processing units (GPUs).

The basic structured grid arrangement, despite its computational efficiency, is however often challenged by problems including curved and slanted material interfaces. This has motivated the development, starting around the beginning of the 1990's, of local surface treatments [5][6] for improved spatial convergence. Similarly, multi-scale EM problems, i.e. including relevant small details in a larger structure, have motivated the development of techniques that provide higher resolution locally. Among those techniques, graded meshes [7], sub-gridding schemes [8] and sub-cell models can be mentioned. Nevertheless, the complexity of practically relevant EM problems has grown together with the increasing capabilities offered by the software tools and computing hardware. As a consequence, the multi-scale aspect and complexity of geometries still provide extreme challenges for the methods applied in structured grids.

In parallel to these developments aiming at generalising the methods in structured grids, efforts have been started in the early 1990's to implement time-domain methods in unstructured and/or inhomogeneous meshes, either tetrahedral or hexahedral. These efforts have given rise to methods such as the Finite-Volume Time-Domain (FVTD) method [9][10], implementations of TLM in tetrahedral meshes [11], time-domain implementations of the Finite-Element method [12] or the Discontinuous Galerkin Time-Domain (DG-TD) method [13]. Despite their promises for conformal and multi-scale EM modelling, none of these methods are considered as mainstream techniques. This can be arguably attributed to their relative complexity, and comparatively high computational cost for standard problems, i.e. for geometries involving only simple shapes and without tiny details to be resolved.

978-1-4577-1559-4/12/$26.00 ©2012 IEEE

This paper reviews some of the developments that enhance the FVTD method's capability for conformal and multi-scale problems. Despite being presented for FVTD, the featured techniques can be easily adapted to other conformal time-domain methods. The descriptions are build around a challenging example – the simulation of the conformal 31-antenna array for breast cancer imaging developed at the University of Bristol [14][15]. The second part of the paper points at new developments in meshless time-domain numerical techniques. The class of meshless methods is based on interpolation of fields at arbitrary node locations and bypasses the need of creating a costly geometrically-defined mesh. Despite being still in their infancy, meshless methods provide a new perspective for future developments in computational electromagnetics.

II. FINITE VOLUME TIME-DOMAIN METHOD

The FVTD method is based on a conservative formulation of Maxwell's equations obtained through integration over a volume V

$$\begin{aligned} \frac{\partial}{\partial t}\int_V \vec{D}\cdot dv &= \int_{\partial V} \vec{n}\times\vec{H}\ da \\ -\frac{\partial}{\partial t}\int_V \vec{B}\cdot dv &= \int_{\partial V} \vec{n}\times\vec{E}\ da \end{aligned} \qquad (1)$$

The terms on the right-hand side are numerical fluxes through the boundary ∂V of the volume, and $\vec{n}$ is the normal vector to the surface. Among the different possible numerical implementations of these fluxes, the most commonly used algorithm is described in detail in [16], together with an explicit predictor-corrector time iteration. The discretised version of the coupled equation system (1) does not impose restriction on the type of volumetric mesh, as long as the applied time step satisfies the Courant–Friedrichs–Lewy (CFL) condition for stability, as elaborated in [16] for FVTD. Most FVTD implementations are based on a tetrahedral discretisation. Tetrahedral meshes can be created with strong inhomogeneity in cell size, i.e. using small cells to resolve geometrical details [17]. The tetrahedral mesh for the example presented next was generated with Altair HyperMesh.

In the following, the modelling of a conformal array of ultra-wideband (UWB) antennas is described to showcase three algorithmic developments towards efficient application of conformal time-domain methods. The conformal array has been developed as part of a near-field radar imaging system for early detection of breast cancer [15]. The third generation of the system comprises 31 UWB wide-slot antennas, which are described in [18]. The full-wave simulation of the array around a breast phantom is extremely challenging by today's standards. The FVTD modelling of this arrangement (shown in Fig. 1) has been presented in [19] for a scenario involving detection of a small tumour. The phantom consists of homogeneous dispersive breast tissue with Debye parameters ($\varepsilon_s = 10$, $\varepsilon_\infty = 10$, $\tau = 7$ ps), modelled as described in [20]. It is covered by a 2 mm-thick dispersive skin layer ($\varepsilon_s = 37$, $\varepsilon_\infty = 4$, $\tau = 7.2$ ps) and contains a non-dispersive

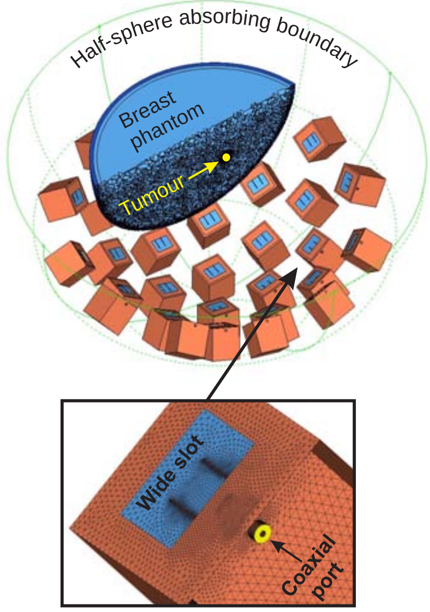

Fig. 1 Geometrical arrangement of the 31-antennna conformal array for breast cancer imaging. Only a part of the discretised breast phantom is shown in cut view. The remaining part of the tetrahedral mesh is not shown for the sake of visibility. The outer absorbing boundary is shown as wire frame. The bottom inset is a zoomed image onto one of the antennas' surface mesh, illustrating the tilted port and the inhomogeneous discretisation of the fork feeding the wide radiating slot.

spherical tumour ($\varepsilon_r = 50$) with 6 mm diameter. The results with pulses covering the 3-10 GHz band have been validated through comparison with phantom measurements [19].

A. *Local Time Steps*

The full discretisation of the considered problem includes 22 million cells. This is only a fraction of the number of cells that would be required for a FDTD simulation (estimated at 800 million cells). The relatively small number of cells in the FVTD model is achieved by generating a strongly inhomogeneous mesh, which provides a spatial refinement only where needed. For example, the wide slot of the UWB antennas is fed by a fork made out of 0.2 mm wide metal strips (Fig. 1, inset). A very fine mesh is required to properly resolve the current variations on those strips – a crucial point for accurate simulations. This requires at least 3 cells along each strip width, which translates into cell dimensions below $\lambda_\varepsilon / 150$, where λ_ε denotes the wavelength in the medium. Over a short distance, the typical cell dimension can grow to a standard discretisation of $\lambda_\varepsilon / 15$ to $\lambda_\varepsilon / 10$.

As mentioned, the CFL criterion set the maximal time step granting a stable explicit time iteration. In an unstructured mesh, the smallest cell determines this time step. Therefore the resolution of small details is bound to dramatically decrease the time step and therefore significantly increase the number of iteration steps. Fortunately, this can be alleviated by introducing local time steps, matched to the geometrical inhomogeneity. An implementation of local time steps has been proposed in [21]. It introduced nested sub-domains where time steps can be incremented by a factor of 2 while satisfying locally the stability criterion. Only the smallest cells are updated every time step, the other ones being updated every 2^n-th step according to their size. In the present example, the application of local time stepping algorithm translates into having only 2% of the cells updated every time step, a few percents updated every 2^{nd}, 4^{th} and 8^{th} time step, and more than 80 % of the cells updated only every 16^{th} or 32^{nd} time step. This relaxes the limitation associated with the explicit stability criterion in inhomogeneous meshes. Similar implementations have been described for the DG-TD method, e.g. in [22].

B. *Arbitrarily Oriented Ports*

The conformal arrangement of the antennas in the array can be problematic for structured discretisations. In particular, the definition of ports with arbitrary orientations is still an open problem in FDTD. Port orientation can cause consistency problems in the simulated performance of individual antennas of the array. In the FVTD unstructured mesh, all ports are defined according to [23] and automatically rotated in space using standard matrix operations. All antennas in the array behave then identically, in the limit of the digital accuracy.

C. *Locally Conformal PML-like Absorbers*

For radiation problems, the absorbing boundary in an unstructured mesh does not need to take the form of a rectangular box. Shaping the outer boundary can be beneficial to reduce the size of the computational domain and/or to adapt absorbing surfaces to radiated wave fronts. In particular, antennas are often simulated in FVTD using Silver-Mueller absorbing boundary conditions (SM-ABC) [16]. A spherical outer boundary is beneficial since SM-ABC are quite efficient when operating close to normal incidence, despite their first-order accuracy. Similarly, an approximate implementation of spherical perfectly matched layers (PML) has been introduced for FVTD in [24] and extended to conformal configurations in [25]. For the problem at hand, conformal PML are applied to a hemispherical truncating boundary, as shown in Fig. 1.

D. *Other Developments*

Numerous other developments of the past decade have aimed at increasing the efficiency of conformal time-domain methods. Among these developments, special source treatments [26] can be mentioned, or the hybridisation with standard techniques, e.g. with FDTD [16] or integral equations [27]. Finally, it is worth mentioning that several approaches exist for parallelisation in unstructured mesh (e.g. [28]). Parallelisation is clearly desired and possible despite being slightly less efficient and natural than for FDTD.

One of the difficulties associated with unstructured discretisation is undoubtedly the mesh generation. Based on a digitised geometry, the creation of a mesh with acceptable quality for time-domain simulations often requires labour-intensive operations from the user. In order to bypass this costly pre-processing stage in the modelling process, while retaining the advantages of an unstructured discretisation, the concept of meshless methods appears very attractive. An implementation of time-domain meshless method for computational electromagnetics is described in the following.

III. Meshless Methods in Time Domain

Meshless methods have been investigated in computational sciences since more than two decades [29]. The general idea is to solve the differential equations governing a physical effect on a cloud of arbitrarily located points (nodes). As particular flavour of meshless method, the Radial Point Interpolation Method (RPIM) has been introduced in computational electromagnetics in [30], with extension to 3D in [31]. The principle of RPIM is briefly summarised in the following, and the interested reader is directed to [32] for more details.

A field quantity $u(x)$ is approximated in RPIM using radial and polynomial basis functions $r_n(x)$ and $p_m(x)$

$$u(x) \approx \langle u(x) \rangle = \sum_{n=1}^{N} a_n r_n(x) + \sum_{m=1}^{M} b_m p_m(x) \quad . \qquad (2)$$

The radial basis functions $r_n(x)$ (e.g. Gaussian) are evaluated on *N* nodes in the direct vicinity of the point of interest. The polynomials $p_m(x)$ are usually of low order. The coefficients a_n and b_m are determined in a point-matching procedure at the discretisation nodes. Once the coefficients are known, low-order derivatives can be directly estimated by considering the derivatives of the basis functions. These derivatives are used to solve the governing differential equations.

The time-domain implementation of RPIM for Maxwell's equations considers a staggered arrangement of E- and H-nodes, where electric and magnetic fields are sampled, respectively. In this sense, RPIM can be considered as a generalised FDTD, and inversely, it can be shown that FDTD constitutes a special case of RPIM.

RPIM shares the capability to resolve complex and multi-scale geometries with more conventional conformal time-domain techniques. However, it potentially brings additional advantages arising from the simplicity of the arbitrary node discretisation. These advantages concern the possibility of dynamic node placement/adaptation, the simplicity of geometry modification for optimisation purpose, and the ease of combination of various differential equations in multi-physics problems.

As example, the simulation of a substrate-integrated waveguide (SIW) is shown in Fig. 2 to conceptualise multi-scale modelling in RPIM. The typical node distance is around 20 times smaller around the metallic posts of the SIW compared to other places in the computational domain.

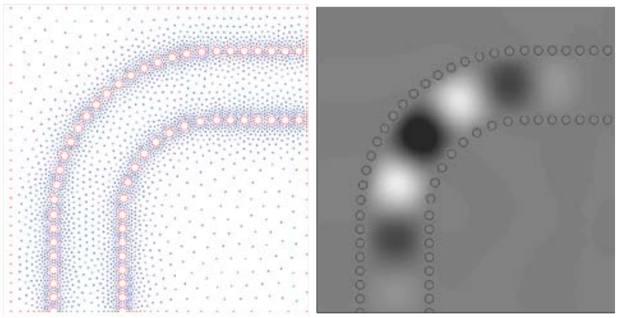

Fig. 2 Two-dimensional simulation of a SIW bend using RPIM. The graph on the left-hand side shows the distribution of E-field nodes, with boundary nodes shown in red (including metallic vias and absorbing boundary), and source nodes in green. The graph on the right-hand side is a snapshot of the field distribution in the bend for a sine-modulated Gaussian pulse excitation.

IV. Conclusions

This paper has reviewed some developments of numerical techniques for time-domain EM simulations in unstructured meshes. The potential for efficient multi-scale and conformal simulations has been illustrated through a FVTD example. Despite the advantages for some high-end problems, it is still questionable if this class of methods can be viable for general purpose application, because of their relatively high computational cost for standard problems. However, they surely can present an alternative for problems involving multi-scale geometrical features.

The second part of the paper has considered recent advances in meshless methods applied to EM modelling. Further developments will be necessary for this class of methods to find its place among more established techniques and bring new perspectives in terms of dynamic, multi-scale and multi-physics modelling.

Acknowledgment

C. Fumeaux acknowledges the support of the Australian Research Council (ARC) under the Future Fellowship funding scheme (FT100100585). M. Klemm acknowledges the support of the UK's Engineering and Physical Sciences Research Council (EPSRC).

References

[1] T. Weiland, "A Discretization Method for the Solution of Maxwell's Equations for Six-Component Fields," *Electronics and Comm. AEUE*, vol. 31, no. 3, pp. 116–120, 1977.

[2] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite- Difference Time-Domain Method*, 3rd ed. Norwood, MA: Artech House, 2005.

[3] W.J.R. Hoefer, "The transmission-line matrix method – Theory and Applications," *IEEE Trans. Microwave Theory Tech.*, vol. 33, no. 10, pp. 882-893, October 1985.

[4] K. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. 14, no. 3, pp. 302–307, 1966.

[5] T. G. Jurgens, A. Taflove, K. R. Umashankar, and T. G. Moore, "Finite-difference time-domain modeling of curved surfaces," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 4, pp. 357–366, 1992.

[6] H. Du, P.P.M. So, and W.J.R. Hoefer, "Cells with tensor properties for conformal TLM boundary modeling," in *2006 IEEE MTT-S International Microwave Symposium Dig.*, pp. 157-160, San Francisco, CA, Jun. 11-16, 2006.

[7] D. Choi, and W.J.R. Hoefer, "A graded mesh FD-TD algorithm for eigenvalue problems," in *Proc. 17th European Microwave Conf.*, pp. 413-417, 1987.

[8] I.S. Kim, and W.J.R. Hoefer, "A Local mesh refinement algorithm for the time domain-finite difference method using Maxwell's curl equations," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 6, pp. 812-815, June 1990.

[9] N.K. Madsen, and R.W. Ziolkowski, "A three dimensional modified finite volume technique for Maxwell's equations," *Electromagnetics*, vol. 10, pp. 147-161, 1990.

[10] V. Shankar, A.H. Mohammadian, and W.F. Hall, "A time-domain, finite-volume treatment for the Maxwell equations," *Electromagnetics*, vol. 10, pp. 127- 145, 1990.

[11] P. Sewell, T.M. Benson, C. Christopoulos, D.W.P. Thomas, A. Vukovic, and J.G. Wykes, "Transmission-line modeling (TLM) based upon unstructured tetrahedral meshes," *IEEE Trans. Microwave Theory Tech.*, vol. 53, no. 6, pp. 1919-1928, 2005.

[12] J. F. Lee, R. Lee, and A. C. Cangellaris, "Time-domain finite element methods," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 430-442, 1997.

[13] J. S. Hesthaven and T. Warburton, *Nodal Discontinuous Galerkin Methods: Analysis, Algorithms, and Applications*, Springer-Verlag, Berlin, Germany, 2008.

[14] M. Klemm, J.A. Leendertz, D. Gibbins, I.J. Craddock, A. Preece, and R. Benjamin, "Microwave radar-based breast cancer detection: Imaging in inhomogeneous breast phantoms," *IEEE Antennas Wireless Propagat. Lett.*, vol.8, pp.1349-1352, 2009.

[15] M. Klemm, I. J. Craddock, A. Preece, J. Leendertz, and R. Benjamin, "Microwave radar-based differential breast cancer imaging: Part 1 Imaging in homogeneous breast phantoms and low contrast scenarios," *IEEE Trans. Antennas Propagat.*, vol. 58, no.7, pp. 2337-2344, June 2010.

[16] P. Bonnet, X. Ferrieres, B. Michielsen, P. Klotz, and J. Roumiguieres, Chap. 9., in *Time Domain Electromagnetics*, Academic Press, 1999.

[17] C. Fumeaux, D. Baumann, and R. Vahldieck, "Finite-Volume Time-Domain analysis of a cavity-backed Archimedean spiral antenna," *IEEE Trans. Antennas and Propagat.*, vol. 54, no. 3, pp. 844-851, March 2006.

[18] D. Gibbins, M. Klemm, I.J. Craddock, J.A. Leendertz, A. Preece, and R. Benjamin, "A comparison of a wide-slot and a stacked patch antenna for the purpose of breast cancer detection," *IEEE Trans. Antennas Propagat.*, vol.58, no.3, pp. 665-674, March 2010.

[19] C. Fumeaux, M. Klemm, and D. Baumann, "Large-scale transient simulations of breast cancer tumour's reflections using a 31-element conformal array," in *Asia-Pacific Microwave Conference 2011 (APMC 2011)*, Melbourne, Australia, 5-8 December 2011.

[20] D. Baumann, C. Fumeaux, C. Hafner, and E.P. Li, "A modular implementation of dispersive materials for time-domain simulations with application to gold nanospheres at optical frequencies," *Optics Express*, vol. 17, no. 17, pp. 15186-15200, August 2009.

[21] C. Fumeaux, D. Baumann, P. Leuchtmann, and R. Vahldieck, "A generalized local time-step scheme for efficient FVTD simulations in strongly inhomogeneous meshes," *IEEE Trans. Microwave Theory Tech*., vol. 52, no. 3, pp. 1067-1076, March 2004.

[22] H. Songoro, M. Vogel, and Z. Cendes, "Keeping time with Maxwell's equations," *IEEE Microwave Mag.*, vol. 11, no. 2, pp. 42-49, April 2010.

[23] D. Baumann, C. Fumeaux, and R. Vahldieck, "Field-based scattering-matrix extraction scheme for the FVTD method exploiting a flux-splitting algorithm," *IEEE Trans. Microwave Theory Tech.*, vol. 53, no. 11, pp. 3595-3605, November 2005.

[24] C. Fumeaux, K. Sankaran, and R. Vahldieck, "Spherical perfectly matched absorber for finite-volume 3-D domain truncation," *IEEE Trans. Microwave Theory Tech.*, vol. 55, no. 12, pp. 2773-2781, December 2007.

[25] D. Baumann, C. Fumeaux, R. Vahldieck, and E. Li, "Conformal perfectly matched absorber for finite-volume time-domain simulations," in *2008 Asia-Pacific Symposium on Electromagnetic Compatibility (APEMC)*, Singapore, May 19-23, pp. 188-191, 2008.

[26] D. Isleifson, I. Jeffrey, L. Shafai, J. LoVetri, and D.G. Barber, "An efficient scattered-field formulation for objects in layered media using the FVTD method," *IEEE Trans. Antennas Propagat.*, vol. 59, no. 11, pp. 4162-4170, November 2011.

[27] D.K. Firsov and J. LoVetri, "FVTD - Integral Equation Hybrid for Maxwell's Equations," *Int. J. Numer. Model.*, vol. 21, pp. 29-42, 2007.

[28] J.S. Shang, M. Wagner, Y. Pan, and D. Blake, "Strategies for adopting FVTD on multicomputers [finite-volume time-domain analysis]," *Computing in Science & Engineering*, vol. 2, no. 1, pp. 10-21, 2002.

[29] V.P. Nguyen, T. Rabczuk, S. Bordas, and M. Duflot, "Meshless methods: A review and computer implementation aspects," *Math. and Computers in Sim.*, vol. 79, no. 3, pp.763–813, 2008.

[30] T. Kaufmann, C. Fumeaux, and R. Vahldieck, "The meshless radial point interpolation method for time-domain electromagnetics," *in IEEE MTT-S International Microwave Symposium Digest*, pp. 61–65, Atlanta, June 2008.

[31] Y. Yu and Z. D. Chen, "A 3-D radial point interpolation method for meshless time-domain modeling," *IEEE Trans. Microwave Theory and Techn.*, vol. 57, no. 8, pp. 2015–2020, Aug. 2009.

[32] T. Kaufmann, Y. Yu, C. Engström, Z. Chen, and C. Fumeaux, "Recent developments of the meshless radial point interpolation method for time-domain electromagnetics," *Int. J. Num. Modelling: Electronic Networks, Devices and Fields*, doi: 10.1002/jnm.1830, 2012.