Does Chaos Work Better Than Noise?

by Maide Bucolo, Riccardo Caponetto, Luigi Fortuna, Mattia Frasca, Alessandro Rizzo

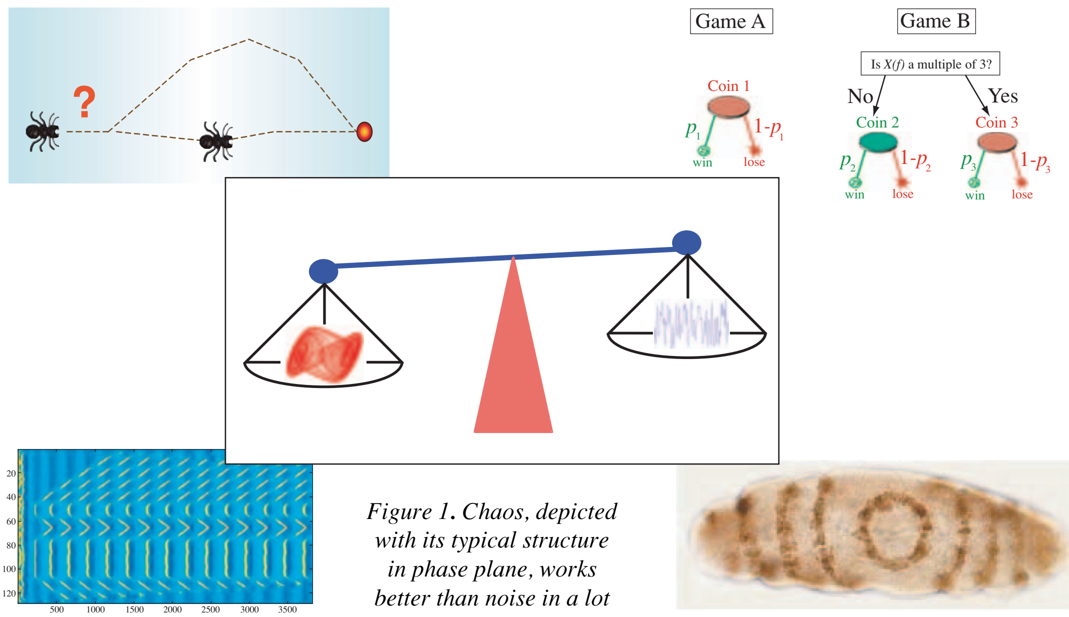

Figure 1. Chaos, depicted with its typical structure in phase plane, works better than noise in a lot of examples presented in this paper.

Abstract—*Chaos and random signals share the property of long term unpredictable irregular behavior and broad band spectrum. The aim of this paper is not to distinguish between random and chaotic dynamics, nor to show the use of chaos, but to focus attention on how chaos and noise help order to arise from disorder. This means to investigate the effect of the introduction of either deterministic chaotic or random sequences in different types of phenomena. In particular the results related to different applications, self-organization in arrays of locally coupled systems in which a chaotic dissymmetry is present, chaos driven optimization strategies, pattern formation in Drosophila embryos and some new topics on game theory are treated with the aim to investigate the subject: "does chaos work better than noise?".*

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Introduction

A classical topic in studying real world phenomena is to distinguish between chaotic and random dynamics. Characterizing the irregular behavior that can be caused either by deterministic chaos or by stochastic processes is not an easy task to perform. Moreover, it is still an open problem to distinguish among these two types of phenomena. Several time series analysis methods have been proposed to investigate the presence of determinism in a set of data [1]. The main difficulty is caused by the surprising similarity that deterministic chaotic and random signals often show, as for example the characteristics of broad band spectra of both of them.

From another point of view, the active use of chaos has been recently widely investigated in the literature. Particularly interesting results have arisen in the area of secure communications [2]. Other topics of great interest are those concerning chaos control and chaotic circuit design. The interest in studying the use of chaotic systems instead of random ones arises when the theme of chaos reaches a high interdisciplinary level involving not only mathematicians, physicians and engineers but also biologists, economists and scientists from different areas. Moreover, several studies showed that order could arise from disorder in various fields (from biological systems to condensed matter, from neuroscience to artificial neural networks [3]). In these cases disorder often indicates both non-organized patterns and irregular behavior, whereas order is the result of self-organization and evolution and often arises from a disorder condition or from the presence of dissymmetries. The origin of self-organization is faced in [4], where starting from evolutionary theory and discussing various key points in biology, the idea that life exists at the edge of chaos has been emphasized. Other examples in which the concept of stochastic driven procedures leads to "ordered" results are Monte Carlo and genetic algorithms for optimization procedures, as well as stochastic resonance in which the presence of noise improves the transmission of the information [5].

The aim of this paper is not to distinguish between random and chaotic dynamics, nor to show the use of chaos, but to focus attention on how chaos and noise help the birth of order from disorder. This means to investigate the effect of the introduction of either deterministic chaotic or random sequences in different types of systems: such as complex systems, optimization procedures, and biological systems. Therefore, the question that this paper tries to discuss is: "does chaos work better than noise?". In other words, which side of the balance of Fig. 1 will prevail?

This paper addresses the question by means of numerous multidisciplinary examples. In the section *Self-Organization in Arrays of Dynamical Systems* the result of the presence of deterministic dissymmetry is treated in two cases: Josephson junctions and Chua's circuits array. In the next section a comparison between the performance of genetic algorithms that run using chaotic signals and that of traditional ones is presented. Then the effect of using chaos in an example of a random-based optimization algorithm,

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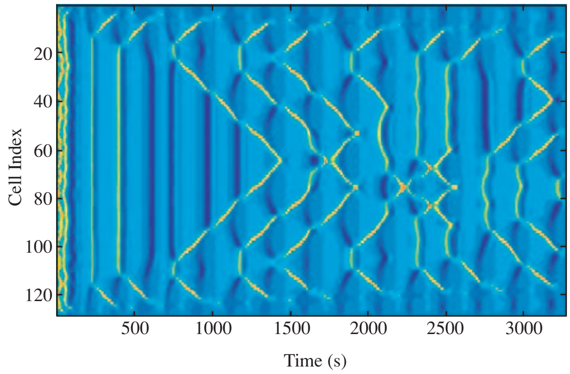

Figure 2a. Chaotic behavior of identical Josephson junctions.

Ant Colony Optimization, is investigated. *Parrondo's Paradox* is considered in the next section in order to further investigate the problem. *Pattern Formation in* Drosophila *Embryos* is perhaps the most fascinating example of self-organization, and this section presents new results showing how chaos can be useful to find in a more effective way the parameters for pattern formation.

Self-Organization in Arrays of Dynamical Systems

In this section topics related to the role of a chaotic induced spatial diversity versus a random one are introduced in regard to the self-organizing behavior of arrays of dynamical systems. In particular, recent results related to some particular nonlinear circuits like the Josephson junctions and the Chua's circuits are reported.

Arrays of Josephson Junctions

As a starting point, this section will deal with a valuable example concerning arrays of Josephson junctions. Research on synchronization of arrays of Josephson junctions has been motivated by the fact that Josephson junctions are effective components capable of generating extremely fast voltage oscillations (typically at terahertz frequency); nevertheless their output power is extremely low (typically 10 nW), making the single junction almost useless for most electronics applications, if it is not part of a long, synchronized array. It has already been shown [6] that spatial diversity helps the tendency for self-synchronization of the junctions: if an array of identical junctions is considered, in-phase periodic orbits in fact exist for a certain range of the parameter set, but they are not asymptotically stable. On the other hand, Braiman *et al*. [7] also showed that a moderate increase in the spatial disorder of the array can lead to significant improvements in the synchronization process. The results obtained parallel those obtained by the same authors for mechanical systems [8], such as arrays of damped, forced pendula, in which a moderate, random, spatial diversity introduced in the length of the pendula enhances the synchronization capability. In [9–11], several experiments show that generating the spatial diversity by using a chaotic law enhances the regularization and the formation of patterns in many spatially extended systems. In particular, the same array of pendula considered in [8] has been synchronized by introducing a moderate amount of diversity generated by chaotic systems.

In this section an array of non-identical, locally connected chaotic Josephson junctions is taken into account. It is known that, under particular conditions, a single junction can exhibit chaotic behavior. In [12] chaos is achieved by introducing a nonlinear effect in the junction, whereas in this work the chaotic behavior of the single junction is achieved by feeding the junction with a periodic driving bias. The global behavior of the array is investigated in the case of identical junctions, where spatio-temporal chaos emerges, and in presence of spatial diversity, generated by either random or chaotic law. In the latter case, it has been confirmed that a certain amount of disorder enhances the self-organization capability. In particular, in agreement with the results presented in [9–11], the introduction of diversity generated by a chaotic law leads to a further improvement in synchronization.

From a mathematical point of view the array of Josephson junctions [6] can be described by the following formula:

$$\ddot{\phi}_j + a_j\dot{\phi}_j + \sin\phi_j = I_0 + k(\phi_{j+1} - 2\phi_j + \phi_{j-1}) + I \cdot \sin(\omega t) \qquad j = 1, \ldots, N \tag{1}$$

where ϕ is the phase difference between the two quantum mechanical wave functions of the two layers of the junction (see Appendix), a_j is a parameter related to the physical properties of the junction, the term $k(\phi_{j+1} - 2\phi_j + \phi_{j-1})$ represents the coupling with the neighboring junctions and $I_0 + I \cdot \sin(\omega t)$ is the current bias.

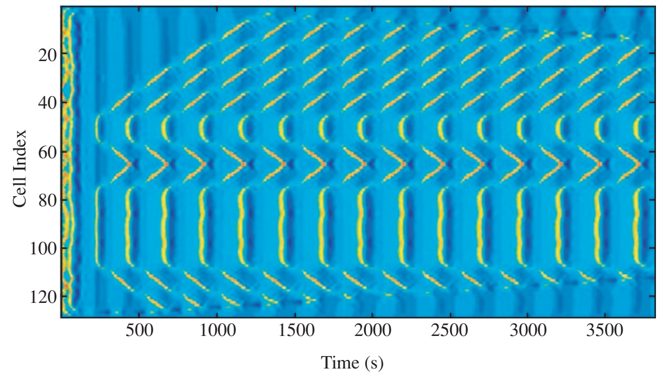

Figure 2c. Self-synchronization by non-organized deterministic dissymmetry.

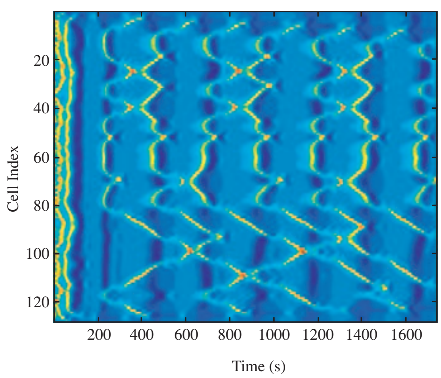

Figure 2b. Self-synchronization by random dissymmetry.

Under these assumptions, when the array consists of identical junctions ($a_j = a\ \forall j$), as time elapses, more and more disordered spatio-temporal patterns emerge, denoting a chaotic behavior. This can be noticed in Fig. 2(a), where a color map codes the variable characterizing the behavior of each junction in terms of the variable ϕ versus time. The prevalence of blue in the color map is due to the d.c. term in the forcing torque. The evolution appears disordered, and is in particular chaotic. By introducing a random, symmetrical disorder in the a_j (in a range of 10–20% of the nominal value), periodic spatio-temporal patterns can be observed (Fig. 2(b)). Our analysis deals now with the effects induced by the introduction of deterministic, non-organized dissymmetry, like the ones generated by a chaotic attractor. In this experiment, a portion of a Chua's attractor is sampled and adequately scaled in order to superimpose a deterministic disorder in the junction parameters. The result of simulation shows (Fig. 2(c)) that a chaotic variation on junction parameters leads the array towards a collective organization. Junctions in the central region of the array are synchronized both in

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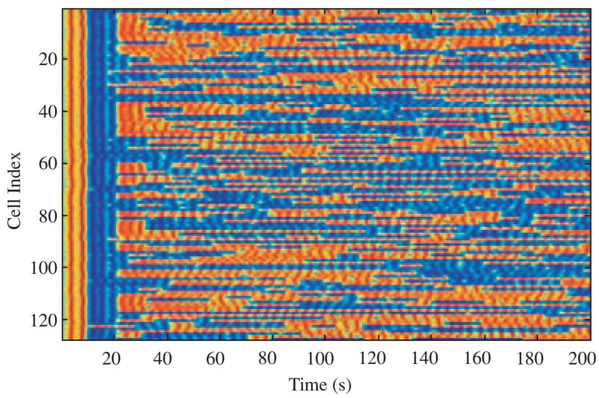
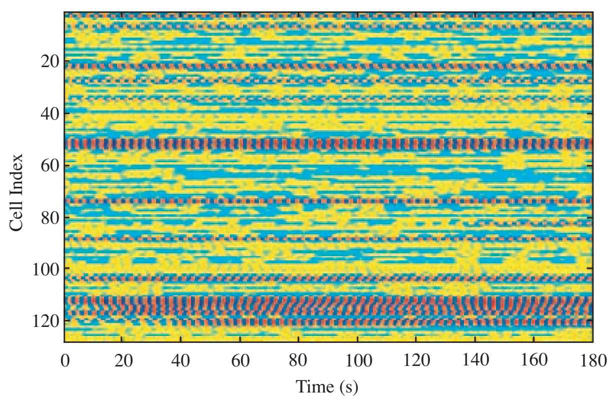
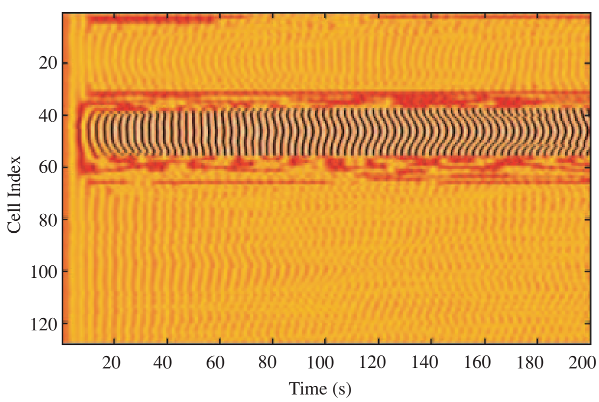


(a) Chaotic behavior of identical circuits.



(b) Self-synchronization by random dissymmetry.



(c) Self-synchronization by non-organized deterministic dissymmetry.

Figure 3. Chua's circuits array experiment.

space and time, oscillating with the same frequency as the forcing current, while in the external bands a regular spatial wave propagates. Therefore, spatio-temporal chaos disappeared, leading to a periodic behavior by simply perturbing the symmetry of the system.

Array of Chua's Circuits

The second example reported in this work concerns arrays of Chua's circuits. Interest arises on arrays of Chua's circuits to investigate spatio-temporal chaos, [13, 14], propagation of impulsive information [15], and formation of spiral waves in a two-dimensional circuit matrix [16].

All the works performed agree on the fact that, for arrays constituted by identical circuits, global behavior is strongly affected by changes in the connection coefficient, denoting in a spatio-temporal context all the phenomena that can be observed in the single circuit: equilibrium states, limit cycles, and, obviously, spatio-temporal chaos. Taking inspiration from the previous example, an array of Chua's circuits [17] has been considered. The following equation describes the k-th cell of the array in the well known dimensionless form:

$$\begin{aligned}
\dot{x}_k &= \alpha_k[y_k - m_1 x_k \\
&\qquad - 0.5(m_0 - m_1)[\,|x_k + 1| - |x_k - 1|\,] \\
&\qquad + D(x_k - y_{k+1})] \\
\dot{y}_k &= x_k - y_k + z_k + D(y_k - x_{k+1}) \qquad (2) \\
\dot{z}_k &= -\beta y_k
\end{aligned}$$

A mono-dimensional array of 128 adjacent units coupled through linear resistors (each circuit is characterized by parameters chosen according to [17] to generate the *Double Scroll Chua Attractor*) is taken into account, and both a random and a deterministic dissymmetry on a circuit parameter (the α_k parameter is allowed to vary in a range of 10–20% of its nominal value) are introduced. The parameters k and D represent the introduced diffusive coupling. As can be observed in Fig. 3, very regular spatio-temporal patterns emerge only when the imposed perturbation is chaotic, confirming the conjecture that chaos can help systems to achieve order and synchronization.

A General Remark

Even if only some specific cases of induced spatial disorder leading to regular patterns have been presented, various experiments have been carried out by the authors where the same scenario has been observed. Two-dimensional arrays like Cellular Nonlinear Networks [10], arrays of Hindmarsh-Rose neurons [18] and distributed networks of fuzzy dynamical systems [19] have been also considered. In each case a weak spatial dissymmetry leads to self-organization, but a strong improvement has been emphasized when chaos is used instead of noise.

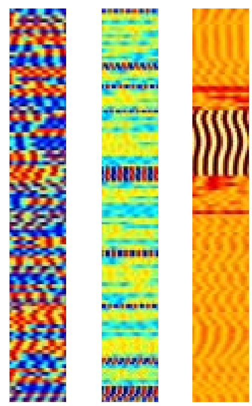
Improving Performance of Genetic Algorithms

In this section and in the next one, the role played by nonlinear chaotic dynamics versus random processes in optimization algorithms is examined.

The random sequence generation algorithms, on which most used GA tools rely, usually satisfy on their own some statistical tests like chi-square or normality. However, there are no analytical results that guarantee an improvement of the performance indexes of GA algorithms depending on the choice of a particular random number generator.

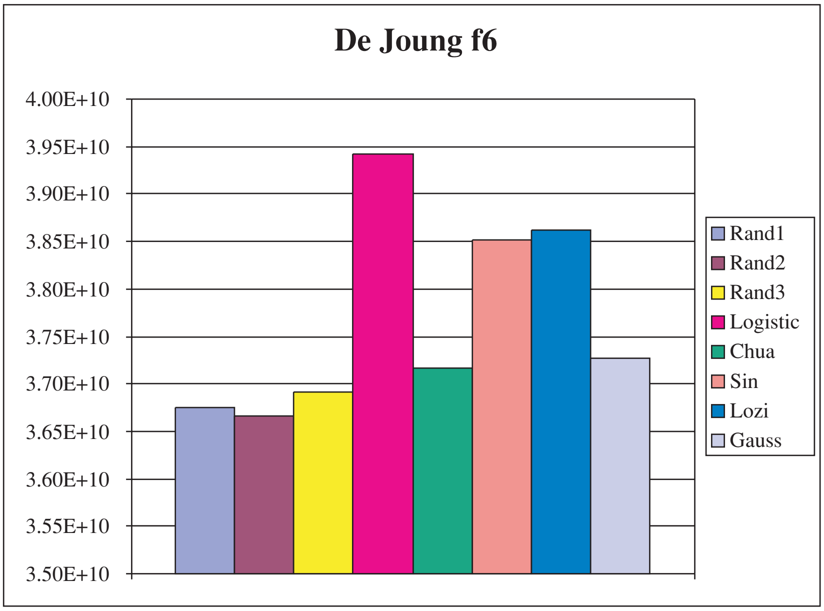
The convergence properties of Genetic Algorithms (GAs) [20] are closely connected to the random sequence applied on genetic operators during a run. In particular, when starting some genetic optimizations with different random sequences, experience shows that the final results may be very close but not equal, and require also different numbers of generations to reach the same optimal value. The random sequence generation algorithms, on which most used GA tools rely, usually satisfy on their own some statistical tests like chi-square or normality. However, there are no analytical results that guarantee an improvement of the performance indexes of GA algorithms depending on the choice of a particular random number generator.

Chaotic systems have already been exploited to define new operators to be applied during genetic optimization, in order to improve the performance of GAs. In particular, in [21] a special mutation operator, applied during gene recombination and based on the logistic function, is introduced showing interesting results in exploration and exploitation of GA capabilities. Also in [22] chaotic time series are used in DNA computing procedures. More recently, in [23] chaotic sequences have



been used to increase population size dynamically in order to avoid GAs' premature convergence.

According to the conjecture introduced in this work, all the random sequence generators in a GA are replaced by chaotic generators, without affecting the original operator definitions. Therefore, chaotic sequences influence the behavior of all genetic operators. In particular, a GA uses random sequences for the following purposes:



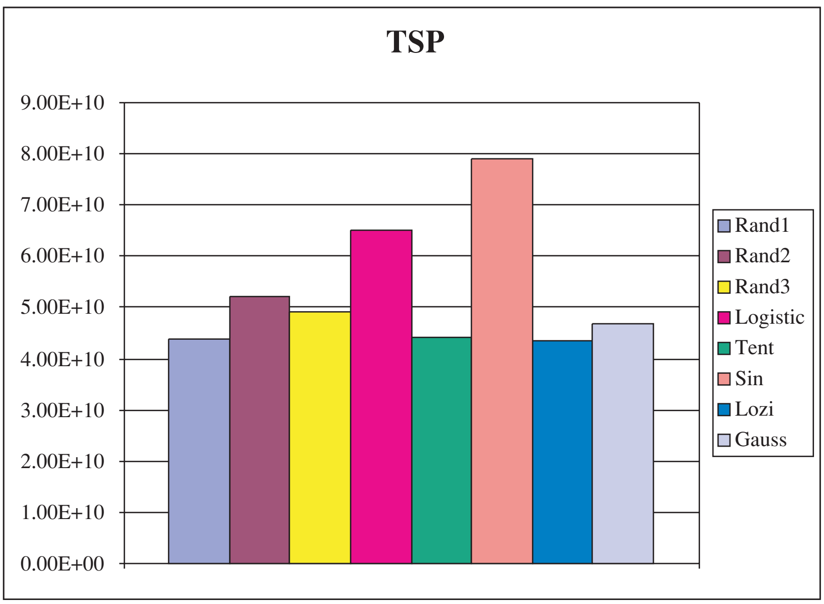


Figure 4. Comparison of performance for different random and chaotic generators, made on (a) De Joung function f6, (b) TSP.

- during the creation of the initial population it is necessary to generate the required number of individuals using a random number generator;
- the selection algorithm is based on the probabilistic choice of individuals according to their fitness; random generators are used also in this operation;
- crossover algorithms are based on the random choice of points inside the chromosomes or on the random generation of bit masks;
- the mutation operator is based on the random change of bits in chromosomes.

Four different types of test problems have been considered: De Joung functions [24], an eigenvalues Linear Matrix Inequalities problem [25], the Iterated Prisoner Dilemma (IPD), and the Traveling Salesman Problem (TSP) [26].

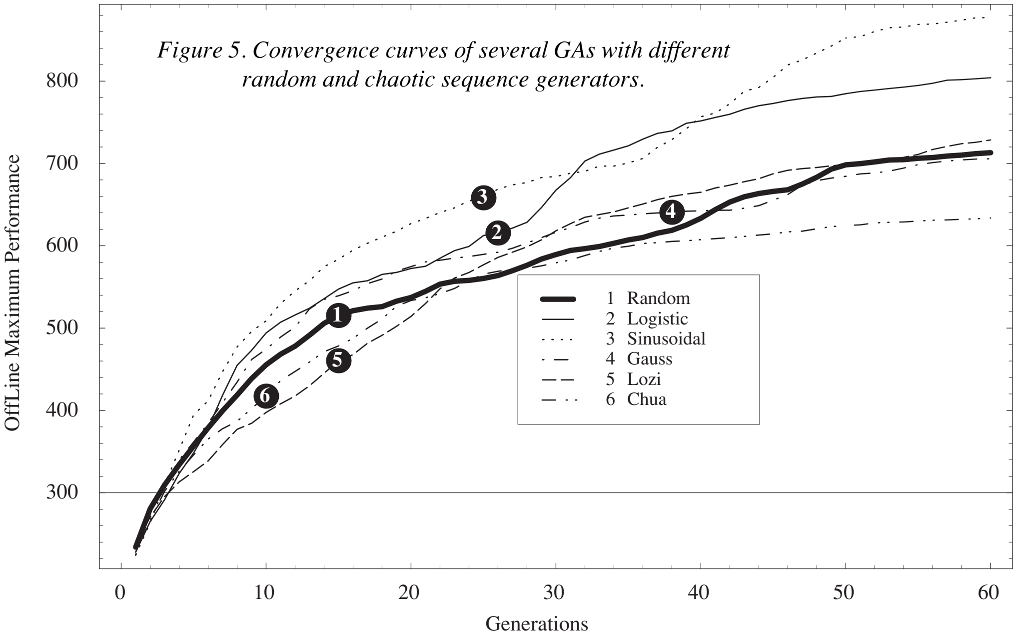
Figure 4 shows comparisons made on a specific performance index named *off-line performance* [24], evaluated by using different random or chaotic sequence generators on the De Joung function and TSP test problems. As it can be clearly seen, best performance is always obtained by using chaotic generators.

An improvement has also been obtained in terms of speed of convergence of the algorithm, as illustrated in Fig. 5.

Ant Colony Optimization Algorithm

Recent research on ethological systems has emphasized self-organization in animal colonies as a crucial point for the accomplishment of those tasks which require a high degree of co-ordination among workers. Ant colonies, for example, can build nests, feed broods, forage for food, and so on [27]. Beyond biological interest, the

Figure 5. Convergence curves of several GAs with different random and chaotic sequence generators.



computer science community has envisioned in these studies a powerful source of inspiration [28] to develop techniques to solve complex problems, exploiting a branch of Artificial Intelligence called *Swarm Intelligence* [29]. In this context, classical optimization problems like the *Traveling Salesman Problem* (TSP) have been faced by taking as an underlying intelligence model the collective intelligence of social insect colonies like ants. In nature, ant colonies find shortest routes from nest to food and vice-versa by laying and following pheromone trails. TSP, which consists of finding the shortest tour between n cities, visiting each one only once and ending at the starting point, is solved by an algorithm which parallels the collective ant behavior by exploiting an *artificial pheromone*. The artificial pheromone is a paradigm to take into account the most tracked paths; its strength is enforced by further visits of the route and weakened as time elapses through an *evaporation rate*. The features of the algorithm offer the possibility of implementation in an agent-based, distributed environment [30].

All the implementations of Ant Colony Optimization (ACO) algorithms rely on a guided random search procedure. Instead of considering random variables to make decisions, a chaotic law is adopted. Performance has been evaluated on different TSP benchmarks by adopting several chaotic laws and different, well known versions of the algorithm (Ant System, Ant System with Elitist Strategy, Rank-Based Ant System, Max-Min Ant System, Ant Colony System). If the average length of the best solution over several trials is considered as a comparison term, chaos and random perform quite similarly. However, if for each algorithm run, the best path lengths obtained with both the algorithms (chaos based and random

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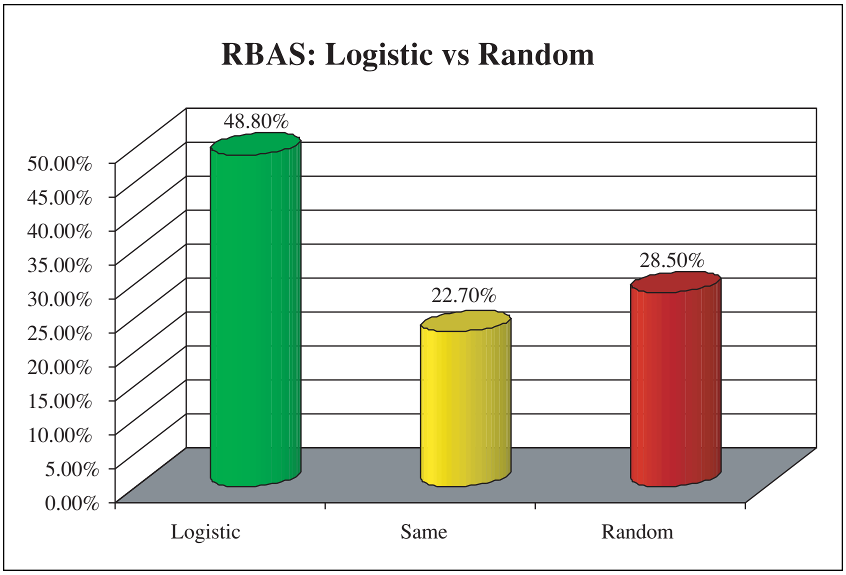


Figure 6. Comparison of performance on the TSP by Ant Colony System Algorithm.

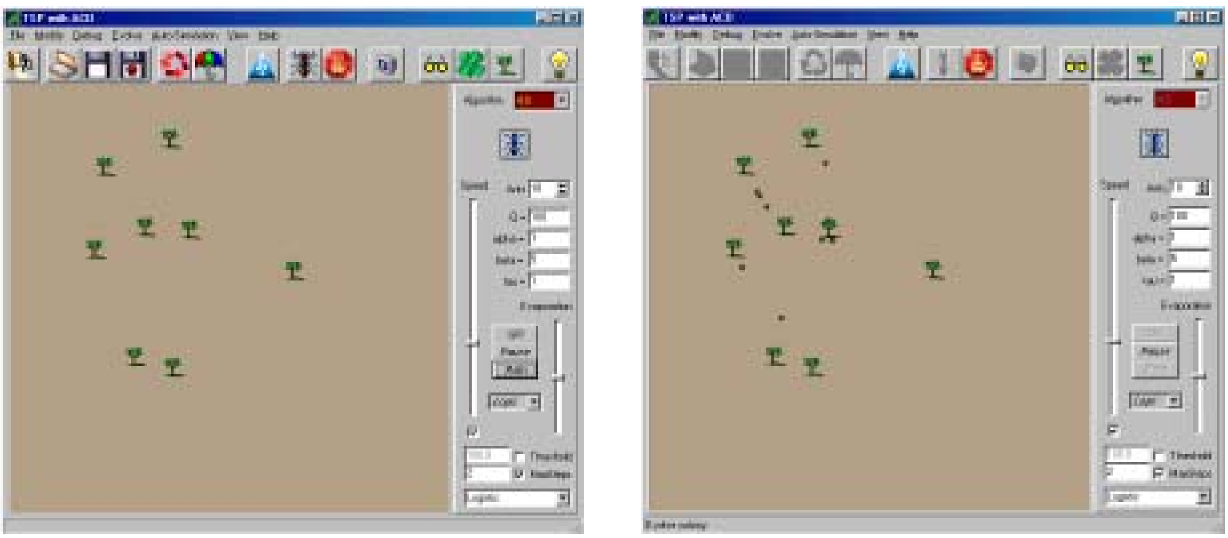
based) are compared, the results, shown in Fig. 6, outline that in 48.8% of the runs the logistic map works better than the random based algorithm, while in 22.7% of the runs the results obtained with the two algorithms are the same and only in 28.5% of the runs the random based algorithm works better than the logistic map. This result obtained on the standard benchmark *EIL51*, taken from TSPLIB [26] and the Rank-Based Ant System, is still valid also when other chaotic maps are applied.

Figure 6 illustrates the performance obtained by a dedicated visual software developed, whose interface is illustrated in Fig. 7.

Parrondo's Paradox

Should chaos play a central role also in game theory? This question is investigated by considering Parrondo's Paradox. This paradox has been introduced by Parrondo as a pedagogical illustration of the Brownian ratchet. It states that a resulting winning game can be obtained by playing in a random or periodic fashion two games which are separately losing [31, 32].

The two losing games are called *Game A* and *Game B*. *Game A* consists of a coin having p as winning probability ($p < 0.5$ is chosen to obtain a losing game). *Game B* consists of two alternatively played coins: if the present played capital is multiple of a given integer M then the coin to be played will have a winning probability p_1, otherwise (the present capital is not a multiple of M) the coin to be played has a winning probability p_2. The two



(a) Map of the cities. *(b) Evolution of the colony.*

Figure 7. Visual software tool for the chaos-based ant optimization algorithm.

games, when played independently from each other, are losing games if the following conditions hold:

$$\frac{1-p}{p} > 1 \qquad (3)$$

for Game A, and

$$\frac{(1-p_1)(1-p_2)^{M-1}}{p_1 p_2^{M-1}} > 1 \qquad (4)$$

for Game B. The paradox consists in constructing a game in which *Game A* and *Game B* are alternatively played with a probability γ: this composite game is winning.

Parrondo showed that the paradox occurs if the following condition holds:

$$\frac{(1-q_1)(1-q_2)^{M-1}}{q_1 q_2^{M-1}} > 1 \qquad (5)$$

where $q_1 = \gamma p + (1-\gamma)p_1$ and $q_2 = \gamma p + (1-\gamma)p_2$.

While in the original Parrondo's Paradox the strategy is based on a random choice of one of the two games with the probability γ, in this work this choice is based on the value given by a chaotic sequence. Also in this case, many chaotic sequence generators have been considered. Figure 8 illustrates the trend of the capital gained by playing the two games separately, alternated by a random law, alternated by chaotic laws. It is evident that performance is much increased when a chaotic law is adopted.

However, one of the limitations of applying the original paradox in real world problems, such as genetics, evolution, and economics [32], is that it involves only two games and one of them is a capital dependent game. The idea underlying the Parrondo's Para-

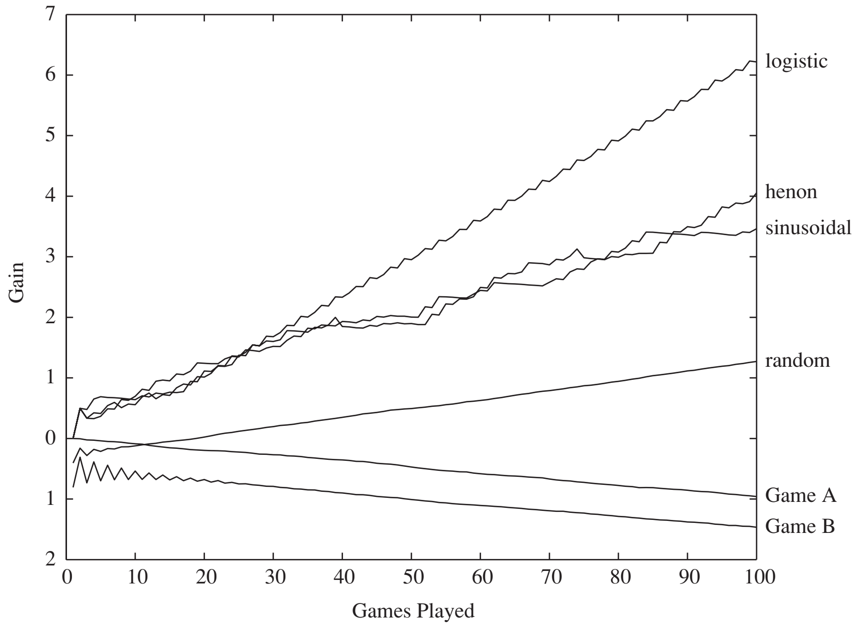
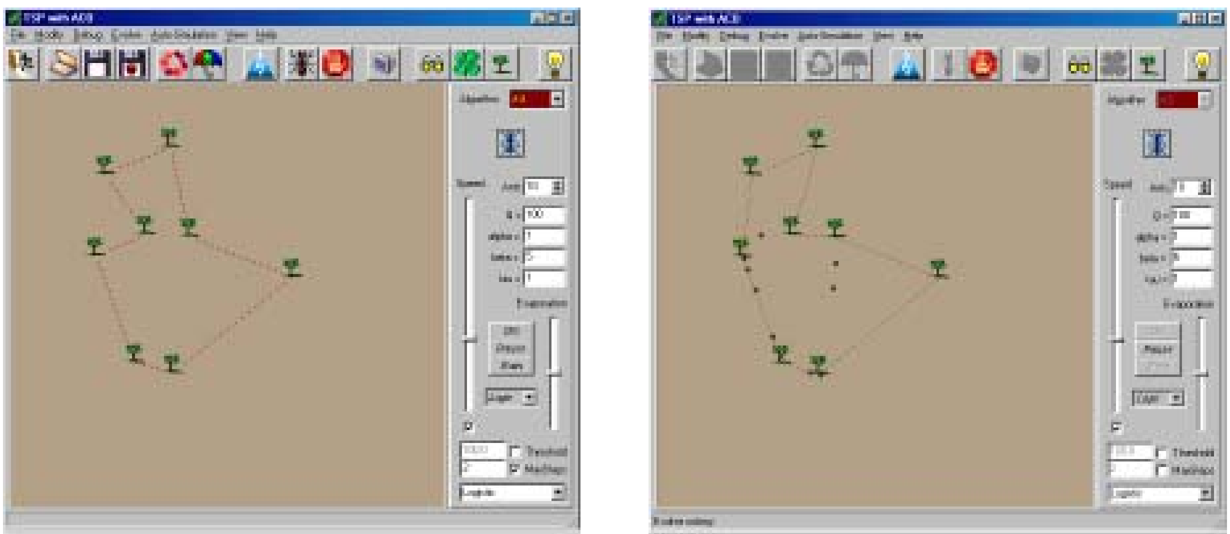


Figure 8. Trend of capital gain in Parrondo's Paradox (the game parameters are: M=3; p=1/2–0.005; p_1=1/10–0.005; p_2=3/4–0.005).



(c) Best path found after 2 iterations. *(d) Optimal path (found after 4 iterations).*

Figure 7. Visual software tool for the chaos-based ant optimization algorithm.

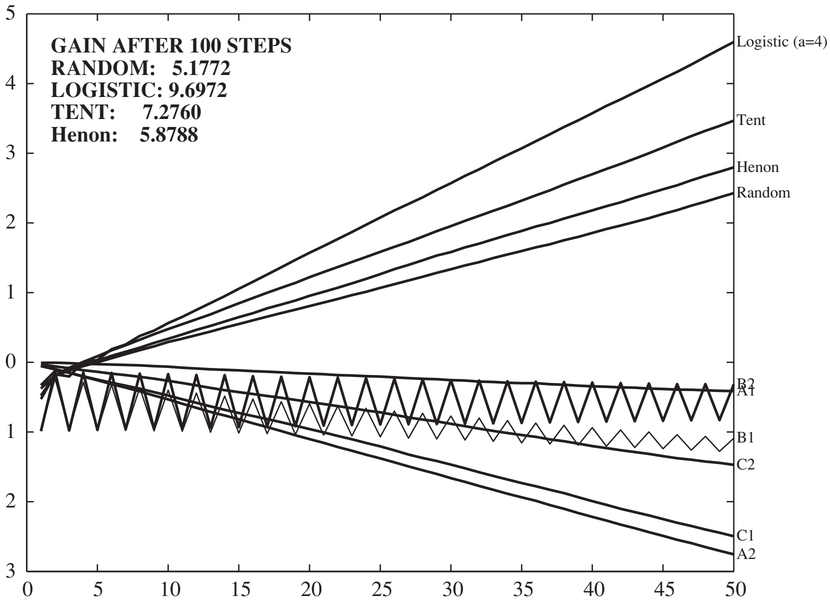


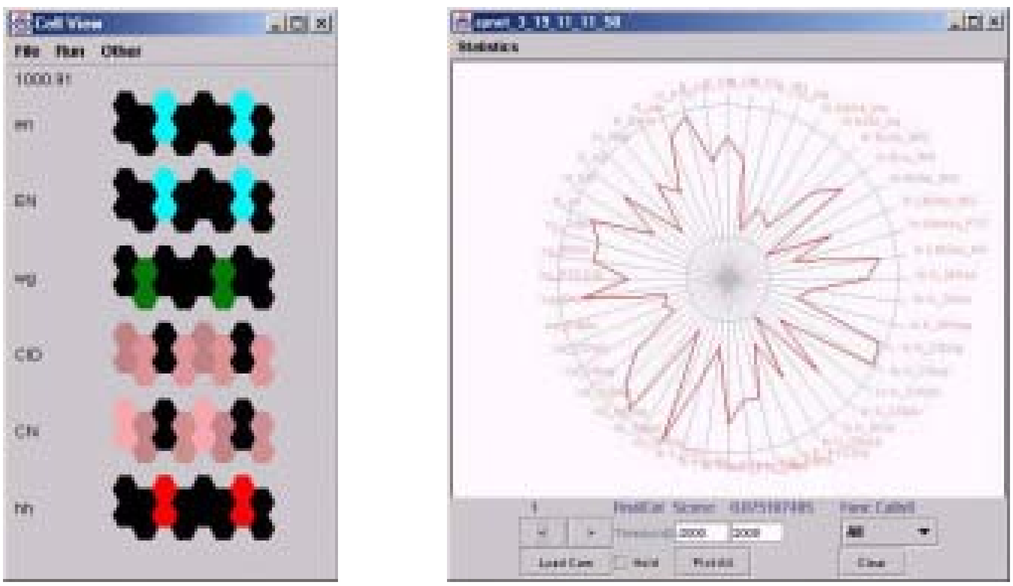
Figure 9. Trend of capital gain in a six games Parrondo's Paradox when different strategies are adopted to choose the game to be played. The six games are labeled as A1, A2, B1, B2, C1, C2. The capital reductions for each game when independently played are also shown.

dox may be extended to n different games. While for $n = 3$ it is still possible to find analytically the parameters characterizing the various games (as for example the losing probabilities of each game when independently played) and leading to the paradox, for more than three games, an optimization strategy should be used. Even in the generalized Parrondo's Paradox the games are played according to a random choice. When a chaotic choice is performed instead of a random one, the gain in the capital is increased. The comparison of the results obtained with a random choice and those obtained with chaotic maps for a six games paradox is shown in Fig. 9.

Pattern Formation in *Drosophila* Embryos

In this section our study focuses on a parameter identification procedure where a complex nonlinear model is involved. The traditional procedure is based on a random sequence, while in this section the effect of using chaos instead of noise is examined.

The phenomenon of pattern formation in *Drosophila* embryos has been recently studied by carrying out an analysis of a mathematical model realized by Von Dassow and colleagues [33].



(a) (b)

Figure 10. (a) Pattern of gene expressions in 8 x 2 network of cells. The brightness of the color, arbitrarily chosen, reflects the concentration of the gene product in the cell. (b) Graphic representation of a solution. Each parameter value is reported in a spoke representing the logarithm scale range of the parameter.

The model takes into account the biological elements involved in the pattern formation, these elements are products of genes (both mRNAs and proteins). For all cells a regular hexagonal form is assumed. The model consists of a set of nonlinear ordinary differential equations describing the interactions among the products of genes of the *Drosophila* in terms of concentrations of the components in an indexed cell or cell face. In all 136 variables and 50 free parameters are taken into account for each segment, that consists of four cells. The parameters involved in the model have unknown values, that can vary in a large range (several orders of magnitude) of biologically plausible values.

In order to deal with the high number of unknown parameters of the model, an iterative procedure is usually set-up by randomly choosing a candidate set of possible parameters and performing a numerical simulation of the model through the Java tool *Ingenue* [34]. This tool is built specifically to model networks of genes interacting in a field of cells and provides a user-friendly interface. Ingenue provides a procedure, called *Iterator*, to explore the space of possible solutions by comparing the patterns obtained with those actually observed in developing *Drosophila* embryos.

Figure 10(a) shows the pattern of gene expression in the cells as obtained with the model, while Fig. 10(b) is a graphic representation of the solutions. This pattern closely matches the pattern of the *Drosophila* embryo.

Usually the iterative procedure of *Ingenue* makes use of random routines. A chaos guided generation of the parameters of the model is here proposed. This new approach reveals better results than a random based choice of the possible solutions. In a representative case after 1000 iterations the traditional random based procedure found 3 feasible solutions, while after the same number of iterations the chaos guided algorithm, which exploits a tent map, found 15 feasible solutions. Moreover, the first solution occurs after 84 iterations with the random based procedure, while it occurs after 51 iterations when chaos guides the choice of parameters.

The results obtained with different chaotic maps (Tent map, Lozi map, Logistic map, Lorenz peak-to-peak dynamics) are compared as shown in Fig. 11 on the basis of several iterations starting from different initial conditions. Figure 11 reports the average number of feasible solutions found by

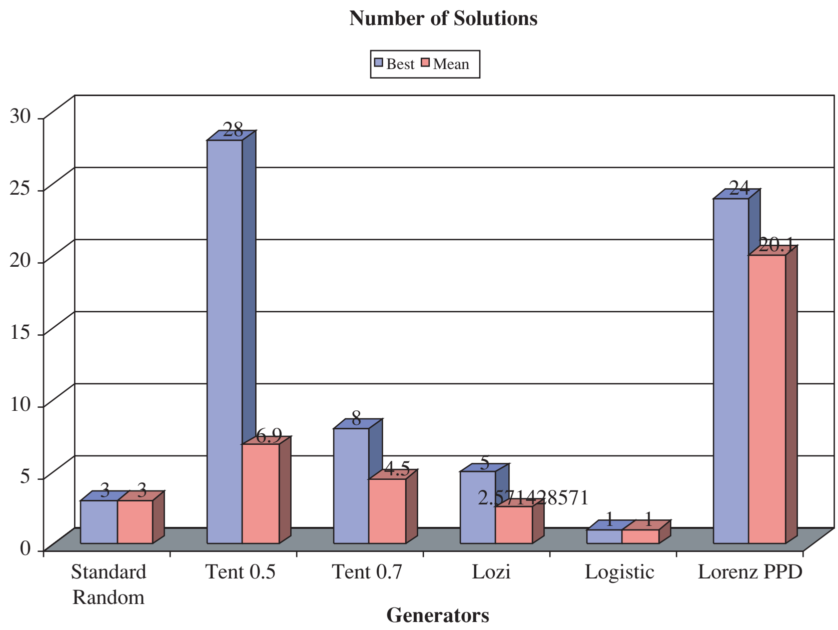
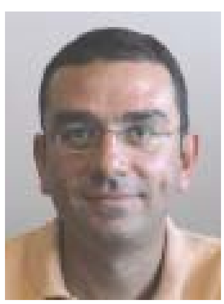


Figure 11. The average number of feasible solutions and the number of solutions found in the best case by the algorithm based either on the random generator, traditionally used in Ingenue, or on a chaotic map. The results obtained with several chaotic maps (Tent map, Lozi map, Logistic map, Lorenz peak-to-peak dynamics) are compared.



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the algorithm as well as the number of solutions found in the best case.

Conclusions

In this work the topic of using deterministic chaotic signals instead of random signals has been explored. Often the question "does chaos work better than noise?" arises. In our experiments we are encouraged to assert that the benefits of chaos are often evident even if a general answer cannot be formulated.

Several studies have been performed and only a few results have been summarized in the paper. Our attention has been devoted to two classes of problems: the first one regards the role of spatial diversity to control complex systems in order to improve self-organization and synchronization in circuits and systems organization; the second one regards in general the class of random based optimization algorithms, where random number generators are usually introduced.

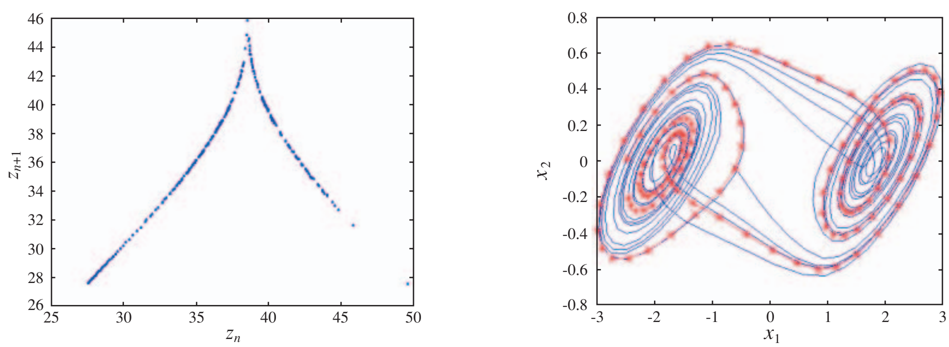
In both cases an improvement has generally been noticed when chaos instead of random number generators has been introduced both to generate spatial diversities and to introduce non-organized patterns into the implementation of numerical procedures. Moreover, the same encouraging results have been obtained when Parrondo Games have been considered. A further optimization example refers to classical pattern formation in biology, where the introduction of a chaotic driven search algorithm of parameter identification has been performed.

The question introduced in this paper is still open and we hope that this contribution will stimulate more examples either to reinforce our feeling or to deny it.

In any case we hope to encourage the debate in new areas of research where, so far, only experiments and simulations are useful to understand complex phenomena.

Appendix

In the appendix both the equations of the chaotic dynamical systems reported in the experiments and the model



(a) (b)

Figure 12. (a) The Lorenz peak-to-peak dynamics. (b) The Double Scroll Chua Attractor (marks indicate the samples of the chaotic sequence generated from the chaotic attractor).

> In this work the topic of using deterministic chaotic signals instead of random signals has been explored. Often the question "does chaos work better than noise?" arises. In our experiments we are encouraged to assert that the benefits of chaos are often evident even if a general answer cannot be formulated.

of Josephson junctions are illustrated.

Chaotic Maps

Determinism, long term unpredictability and high sensitivity to initial conditions are the peculiarity of chaos. Systems showing chaos can be both continuous-time systems and discrete-time maps. The chaotic sequences used in most of our experiments have been generated by using different well-known chaotic maps [35], reported in the following:

- Logistic Map

$$x_{n+1} = 4x_n(1 - x_n) \tag{6}$$

- Tent Map

$$x_{n+1} = \begin{cases} \dfrac{x_n}{0.7} & x_n \leq 0.7 \\ \dfrac{1 - x_n}{0.3} & \text{otherwise} \end{cases} \tag{7}$$

- Sinusoidal Map

$$x_{n+1} = a x_n^2 \sin x_n \tag{8}$$

with $a = 2.3$

- Gaussian Map

$$x_{n+1} = \begin{cases} 0 & x_n = 0 \\ \dfrac{1}{x_n} \bmod 1 & x_n \neq 0 \end{cases} \tag{9}$$

- Lozi Map

$$\begin{aligned} x_{n+1} &= y_n + 1 - a|x_k| \\ y_{n+1} &= bx_k \end{aligned} \tag{10}$$

with $a = 1.7$; $b = 0.5$.

Moreover, two other maps have been used to generate chaotic sequences; these maps are obtained starting from continuous-time chaotic systems such as the Lorenz system and Chua's circuit. The equations of both chaotic oscillators are:

Lorenz system

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{aligned} \tag{11}$$

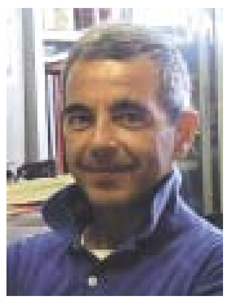
Chua's circuit

$$\begin{aligned} \dot{x} &= \alpha(y - m_1 x \\ &\quad - 0.5(m_0 - m_1)[|x + 1| - |x - 1|]) \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{12}$$

with the following parameters:

$\sigma = 10$; $\rho = 28$; $b = 8/3$; $\alpha = 9$; $\beta = 14.286$; $m_0 = -1/7$; $m_1 = -2/7$.

The peak-to-peak dynamics of the Lorenz system has been taken into account [35], while as regards Chua's circuit a chaotic sequence is obtained by the sampling of a variable of the Double Scroll Chua Attractor [17]. Figure 12 illustrates the Lorenz peak-to-peak dynamics and the phase plane projection of the Double Scroll Chua Attractor (marks indicate samples of the chaotic sequence).



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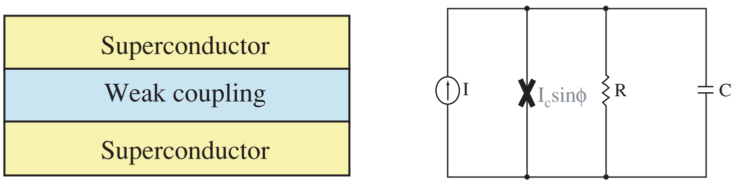


(a) (b)

Figure 13. (a) The Josephson junction and (b) the equivalent electrical circuit.

The Josephson Junction Model

The physics underlying the Josephson effect is regulated by quantum mechanics. Nevertheless, the dynamics of a Josephson junction, constituted by two closely spaced semiconductors separated by a weak connection, is usually described in classical terms. The current flowing in a Josephson junction consists of three main contributors: the supercurrent, due to the actual Josephson effect, the displacement current, which can be modeled by the contribution of a capacitor, and the ordinary current, modeled by the contribution of a resistor. Based on this consideration, the junction model adopted in this work is illustrated in Fig. 13.

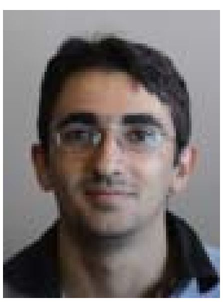
Applying Kirchoff's current and voltage laws to the circuit in Fig. 13, and exploiting the Josephson current-phase and voltage-phase relations [35], the following dynamical model can be written:

$$\frac{hC}{4\pi e}\frac{d^2\phi}{dt^2} + \frac{h}{4\pi eR}\frac{d\phi}{dt} + I_c \sin\phi = I_b \qquad (13)$$

Equation (13) describes the dynamics of the single junction only in terms of variable ϕ, which is the phase difference between the two quantum mechanical wave functions of the two layers of the junction. h represents the Planck constant, e the charge of the electron, and I_c is a critical current, typical of the junction considered. Moreover, following the normalization reported in [35], Eq. 13 can be rewritten in dimensionless form as follows:

$$\frac{d^2\phi}{dt^2} + a\frac{d\phi}{dt} + \sin\phi = I \qquad (14)$$

The forcing signal has been chosen to be a signal consisting of a sine current plus a constant bias (i.e. $I_b = I_0 + I\sin\omega t$), and the following parameter values have been assumed: $I_0 = 0.7155$, $I = 0.4$, $\omega = 0.25$, $a = 0.75$.



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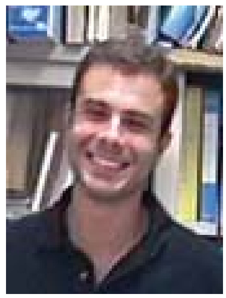

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