THE UNIVERSITY OF ADELAIDE

EXAMINATION FOR THE DEGREE OF B.E.

JUNE 1998

COMMUNICATION SYSTEMS (1312)

FIRST PAPER

Time : ONE and a HALF hours

(In addition, candidates are allowed ten minutes before the examination begins, to read the paper.)

The only calculators authorised for this examination are the Sharp model EL-556G or the EL-556L.

Attempt ALL three questions.

All questions carry equal marks; part question marks are given in brackets where appropriate.

Attached is a set of Fourier Transforms.

ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.

Question 1 follows on page 2.
1. (a) \( X(t) \) is a wide-sense stationary process with power-spectral density \( S_{xx}(f) \).

Determine (in terms of \( S_{xx}(f) \)) the power spectral density of

\[
\begin{align*}
(i) & \quad y_1(t) = X(t) - X(t - T) \\ (ii) & \quad y_2(t) = \frac{d}{dt} X(t)
\end{align*}
\]

(2 marks)

(b) A white-noise process \( n(t) \), with power spectral density \( S_{nn}(f) = \gamma \), is applied at the input of a low-pass filter as shown in Figure 1.

\[
\begin{array}{c}
\text{n(t)} \\
\uparrow \\
\text{h_1(t)} \\
\downarrow \\
\text{y(t)}
\end{array}
\]

Figure 1: Low Pass Filter.

The impulse response of the filter is

\[ h_1(t) = e^{-\alpha t} u(t), \alpha > 0. \]

Find \( S_{yy}(f) \), the power spectral density of the output process \( y(t) \). (4 marks)

(c) Determine \( R_{yy}(\tau) \), the autocorrelation function of the process \( y(t) \) in (b) above. (3 marks)

(d) The impulse response \( h(t) \) of a linear time-invariant system is

\[ h(t) = u(t+1) - u(t-1) \]

where \( u(t) \) is the unit step function. Comment on the causality of the system. (1 mark)

Using time-domain convolution only, find and sketch the system output \( z(t) \) when the input is

\[ x(t) = u(t) - u(t-1) \]

Explain your method with the aid of appropriate sketches. (8 marks)

Question 2 follows on page 3.
2. (a) A message signal $m(t)$ modulates a carrier $v_c(t)$ given by

$$v_c(t) = A_c \cos 2\pi f_c t$$

Write down an equation for the modulated signal if

(i) Double sideband amplitude modulation (DSB-AM or AM) is employed (with modulation index $\mu$) (1 mark)

(ii) Double sideband suppressed carrier (DSB-SC) amplitude modulation is used. (1 mark)

(b) Sketch time domain and frequency domain waveforms for an AM modulated signal when

$$m(t) = A_m \cos 2\pi f_m t.$$ 

Assume $f_c \gg f_m$ and $\mu = 0.8$. (3 marks)

(c) Repeat (b) for DSB-SC modulation. (3 marks)

(d) Briefly discuss the demodulation techniques available respectively for AM and DSB-SC modulated waveforms, pointing out the relative advantages and disadvantages. (4 marks)

(e) A modulated signal of bandwidth 10 kHz is transmitted over a channel with 50dB attenuation and additive white noise of power spectral density

$$S_{nm(t)} = \frac{N_0}{2} = 10^{-9} \text{ W/Hz}.$$ 

Determine the required transmitter power to achieve a SNR of at least 50dB at the receiver output if

(i) DSB-SC modulation is used (6 marks)

(ii) AM is used with $\mu = 0.9$. Assume that the message signal $m(t)$ is a random process with $|m(t)| \leq 1$ and $<m^2(t)> = 0.2$. (2 marks)

Question 3 follows on page 4.
3. Frequency modulation (FM) and Phase modulation (PM) are examples of Angle Modulation. A frequency modulated signal $v_f(t)$ can be described by

$$v_f(t) = A_c \cos 2\pi \left( f_c t + K_f \int_{-\infty}^{t} m(\tau) d\tau \right)$$

where $A_c, f_c$ are the carrier amplitude and frequency

$m(t) = \text{message signal}$

$K_f = \text{frequency deviation constant}$

$= \text{maximum frequency deviation if } |m(t)| \leq 1.$

(a) Write down an equation for a phase modulated signal $v_p(t)$. Define the instantaneous frequency $f_i$ for FM and PM respectively. (3 marks)

(b) For the waveform shown in Figure 2(a) sketch the FM modulated signal. For the waveform in Figure 2(b) sketch the PM modulated signal. Compare and comment (the time scales of the two sketches should be in the range $0 \leq t \leq 4$). (3 marks)

Figure 2: Message signals.

Question 3 continues on page 5.
3. (continued)
(c) The Armstrong method is used to generate a wideband angle-modulated signal as shown in Figure 3.

\[
g_1(t) = A \cos \pi (f_c t + k \phi m(t))
\]

where \( f_c = 105 \text{ Hz}, k = 0.05, |m(t)_{\text{min}}| = |m(t)_{\text{max}}| = 200, \) and bandwidth \( B \) of \( m(t) = 100 \text{ Hz}. \)

Determine the maximum frequency deviation and bandwidth of
(i) \( g_2(t) \)  
(ii) \( g_3(t) \)

(d) For a FM receiver the output signal-to-noise ratio \( \text{SNR}_o \) is
\[
\text{SNR}_o = \frac{3A^2 f_d^2 <m^2>}{4\alpha W^3}
\]

where \( f_d = \) maximum frequency deviation  
\( W = \) bandwidth of message signal  
\( <m^2> = \) average power of normalised message signal  
\( A = \) amplitude of carrier  
\( 2\alpha = \) noise power spectral density level.

The predetection signal-to-noise ratio (\( \text{SNR}_p \)) is given by
\[
\text{SNR}_p = \frac{1}{2}\frac{A^2}{2\alpha B}
\]

where \( B = \) bandwidth of FM signal.

At the FM threshold \( \text{SNR}_p = 10 \). Assuming sinusoidal modulation, show that the output signal-to-noise ratio at the threshold is given by
\[
\text{SNR}_{o, \text{th}} = 30\beta^2 (\beta + 1)
\]

where \( \beta = \) the modulation index.