

THE UNIVERSITY OF ADELAIDE
EXAMINATION FOR THE DEGREE OF B.E.
NOVEMBER 1999
ADVANCED COMMUNICATION THEORY (9334)

Time : ONE and a HALF hours

(In addition, candidates are allowed ten minutes before the examination begins to read the paper.)

The use of calculators is permitted; this equipment is to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links will be barred from the examination room.

Attempt **ALL 3** questions.

All questions carry equal marks; part marks are given in brackets where appropriate.

ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.

Question 1 follows on page 2

1. Consider the random process $x(t) = A \cos(\omega_c t + \theta)$, where A and ω_c are constants, and θ is a random variable with probability density function (pdf)

$$f(\theta) = 4/\pi, \quad -\pi/8 \leq \theta \leq \pi/8.$$

- Find the statistical mean of $x(t)$ by computing $E\{x(t)\}$.
- Find the Autocorrelation Function of $x(t)$ by computing $R(t, \tau) = E\{x(t) x(t + \tau)\}$
- From your answer in part (b) what is the variance of $x(t)$?
- What is the time average mean and time average variance of $x(t)$?
- Is $x(t)$ a stationary process? Why or why not? (20 marks, 4 marks per part)

2. Consider the pulse signal $s(t) = e^{-1} [u(t) - u(t - T)]$, where T is the pulse width.

- Sketch the impulse response, $h(t)$, of the matched filter for detecting the presence of $s(t)$ in additive noise.
- Find the shape of the output pulse when $s(t)$ is passed through filter $h(t)$.
- If white noise with Power Spectral Density $S_n(f) = N_0/2$ Watts/Hz is added to $s(t)$, then passed through the matched filter, $h(t)$, find the maximum Signal-to-Noise Ratio (SNR).
- Does the matched filter you found in this problem preserve the shape of $s(t)$? (20 marks, 5 marks per part)

3. Let a random signal, $s(t)$, be added to noise to form the random process $x(t) = s(t) + n(t)$. Given $x(t)$, we desire a Minimum Mean Squared Error (MMSE) estimate of $s(t)$ by passing $x(t)$ through a linear filter with impulse response, $h(t)$. Assume $s(t)$ and $n(t)$ are uncorrelated. The filter defined by $h(t)$ may be non-causal.

- a) Given that $s(t)$ is characterised by a Power Spectral Density (PSD) of

$$S_s(f) = 3 / (1 + 2\omega^2 + \omega^4)$$

and that the noise is characterised by its PSD

$$S_n(f) = 3 / (1 + \omega^2)$$

find the transfer function, $H(f)$, for the MMSE linear filter to estimate $s(t)$.

(15 marks)

- b) Find the Minimum Squared Error for this filter.

(5 marks)