EXAMINATION FOR THE DEGREE OF B.E.

Semester I June 2006

101902 COMMUNICATIONS IV (ELEC ENG 4035)

Official Reading Time: 10 mins
Writing Time: 120 mins
Total Duration: 130 mins

Instructions:
- This is a closed book examination.
- Attempt ALL FOUR questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- Explanations are expected where requested, and marks will be given for these.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.

Materials:
- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:
- Fourier Transform Sheet
- Table of the Q Function
- Communications IV Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Total number of pages (including attachments) = 11

Question 1 follows on page 2.
Question 1

Q1a) A band pass signal \( x(t) = 20 \text{sinc}(200t) \cos(2\pi 1000t) \) is applied to an ideal band pass filter centred at 1000 Hz and of bandwidth 100 Hz. The passband gain is unity.

(i) Calculate the fourier transform \( X(f) \) of \( x(t) \) and sketch \( X(f) \) showing clearly the amplitude and frequency scales. \(3 \text{ marks}\)

(ii) Sketch the frequency response of the filter and find an expression for its transfer function \( H(f) \). \(2 \text{ marks}\)

(iii) Calculate the impulse response \( h(t) \) of the bandpass filter. \(2 \text{ marks}\)

(iv) Calculate the output \( y(t) \) of the bandpass filter. \(3 \text{ marks}\)

Q1b) A binary PAM system generates a signal

\[
s(t) = \sum_{k=-\infty}^{\infty} A a_k p(t - kT)
\]

where \( A = 10, a_k = \pm 1 \) is random uncorrelated binary data, the pulse shape is \( p(t) = \Delta(t/T) \) and \( T = 1 \text{ ms} \) is the bit period.

(i) The signal \( s(t) \) is cyclostationary. Explain clearly what this means in relation to the mean and autocorrelation function of \( s(t) \). \(2 \text{ marks}\)

(ii) Sketch the signal \( s(t) \) for \( 0 \leq t \leq 4T \) if the data is \( a_k = [-1 \quad +1 \quad +1 \quad -1 \quad +1] \) for \( 0 \leq k \leq 4 \). \(3 \text{ marks}\)

(iii) Calculate the power spectrum \( S_{ss}(f) \) of \( s(t) \) and sketch it showing clearly the amplitude and frequency scales (use the result on data sheet, derivation is not required). \(3 \text{ marks}\)

(iv) Calculate the autocorrelation function \( R_{ss}(\tau) \). \(2 \text{ marks}\)

Question 2 follows on Page 3
Question 2

Q2a) In a broadcast communication system the carrier power is 90 kW, the channel attenuation is 80 dB, the noise power spectral density is $S_m(f) = N_0/2$ with $N_0 = 1.5 \times 10^{-10}$ W/Hz and the normalised baseband message signal $m(t)$ has a bandwidth of 15 kHz, $|m(t)| \leq 1$ and a mean square value $<m^2(t)> = 0.1$.

(i) If the modulation used is amplitude modulation (AM) with a modulation index $a = 0.90$, calculate the following for a receiver with bandwidth equal to that of the signal:

- The bandwidth of the signal
- The predetection signal to noise ratio (SNR$_p$) in decibels
- The output signal to noise ratio (SNR$_o$) in decibels  

(ii) If the modulation used is frequency modulation (FM) with peak frequency deviation 75 kHz, calculate the following for a receiver with a bandwidth given by Carson’s rule:

- The (approximate) bandwidth of the signal
- The predetection signal to noise ratio (SNR$_p$) in decibels
- The output signal to noise ratio (SNR$_o$) in decibels  

(iii) What is the maximum channel attenuation (in decibels) allowed if the FM system in (ii) is to be above threshold?

Q2b) An AM system has a received signal of the form:

$$v(t) = A\left[1 + am(t)\right]\cos(2\pi f_0 t) + n(t)$$

where $A = 100$ $\mu$V, $m(t)$ has a bandwidth of 10 kHz with $|m(t)| \leq 1$ and $<m^2(t)> = 0.1$, the modulation index is $a = 0.9$, the carrier frequency is $f_0 = 1.6$ MHz and $n(t)$ is white noise of power spectral density $S_m(f) = 2.5 \times 10^{-15}$ V$^2$/Hz. The receiver has the form shown in the figure below.

![Diagram of AM receiver](image)

The pass band gains are 1, the flat parts of $H_1(f)$ are 20 kHz wide centred at $\pm f_0$ with transitions at each end of 5 kHz width (i.e. the total bandwidth is 30 kHz, noise bandwidth = 23333 Hz).

(i) Calculate the predetection signal to noise ratio (i.e. at the output of $H_1$) in decibels.  

(ii) Calculate the output signal to noise ratio in decibels (i.e. at the output of $H_2$).  

Question 3 follows on Page 4
Question 3

Q3a) A 16QAM system uses the constellation shown below such that the symbols are equally spaced by \( d \) both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is \( R = 50 \) ohm.

\[ \text{Constellation diagram} \]

(i) Determine the minimum bandwidth required for transmission. (2 marks)

(ii) Calculate the average energy per bit (in \( V^2s \)) for the constellation in terms of the separation distance \( d \) (the horizontal and vertical separation of the symbols). (Note that the energy of each symbol in the constellation is given by the square of its distance from the origin). (3 marks)

(iii) If the average received power \( P = 1.200 \times 10^{-6} \) W, determine the symbol spacing \( d \). (3 marks)

(iv) If outermost corner symbols have an energy of \( 9.000 \times 10^{-10} \) J and \( N_o = 1.000 \times 10^{-11} \) W/Hz, determine the probability of selecting an adjacent symbol in error during demodulation. (Assume a matched filter receiver). (4 marks)

[Hint: Convert watts to \( V^2s \) and joules to \( V^2s \) sec by multiplying by the resistance \( R \)].

Q3b) A BPSK (binary phase shift keyed) system is used to transmit the same bit rate as in part a):

(i) Determine the minimum bandwidth required for transmission. (2 marks)

(ii) For an average received power \( P = 1.200 \times 10^{-6} \) W and noise spectral density \( N_o = 1.000 \times 10^{-11} \) W/Hz, determine the probability of error for a matched filter receiver. (4 marks)

(iii) Comment on the differences between BPSK and 16 QAM as revealed by your answers. (2 marks)

Question 4 follows on Page 5
Question 4

4a) A source X has an alphabet \{A, B, C, D, E\} with corresponding probabilities \{0.10, 0.15, 0.20, 0.20, 0.35\}.

(i) Calculate the source entropy \(H(x)\) in bits and explain what this means. \(3\) marks

(ii) Design a binary Huffman code for these symbols, and calculate its efficiency. \(5\) marks

(iii) Explain why it is possible to uniquely decipher a Huffman code. \(2\) marks

4b) A BPSK (binary phase shift keyed) digital transmission system transmits data at \(10^6\) symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to \(p = Q\left(\sqrt{2E_c/N_0}\right) = 10^{-4}\), where \(E_c\) is the energy per transmitted channel bit and \(N_0/2\) is the spectral density of the accompanying additive white gaussian noise. Error correction is achieved by using a (15,11) Hamming block code.

(i) Calculate the value of \(E_c/N_0\). \(1\) mark

(ii) How many errors can the code correct in each block of 15? \(1\) marks

(iii) What is the minimum bandwidth required to transmit the signal? \(2\) marks

(iv) Calculate the bit error probability after error correction. \(3\) marks

(v) If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to \((15/11) \times 10^6\) symbols/sec so that the message bit rate is \(10^6\) bits/sec, calculate the corrected probability of a bit error. (Hint: Calculate the reduced value of \(E_c/N_0\), and hence the new value of \(p\), the uncorrected probability of a bit error). \(3\) marks

End of Questions

Data Sheets follow on Pages 6 – 11