EXAMINATION FOR THE DEGREE OF B.E.

Semester I June 2008

101902 COMMUNICATIONS IV (ELEC ENG 4035)

Official Reading Time: 10 mins
Writing Time: 120 mins
Total Duration: 130 mins

Instructions:
- This is a closed book examination.
- Attempt **ALL FOUR** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

Materials:
- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:
- Fourier Transform Sheet
- Table of the Q Function
- Communications IV Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Total number of pages (including attachments) = 11

# Question 1   **20 marks total**

1a) A message signal $m(t) = \cos(2000\,\pi t) + 2\cos(4000\,\pi t)$ modulates a carrier $c(t) = 100\cos(2\pi f_c t)$ (where $f_c = 20$ kHz) to produce a double sideband suppressed carrier modulated (DSBSC) signal $v(t) = m(t)\,c(t)$.

   (i) Calculate an expression for $v(t)$.   **(1 mark)**

   (ii) Calculate the Fourier transform $M(f)$ of $m(t)$ and sketch it, showing clearly the magnitudes of the various components.   **(2 marks)**

   (iii) Calculate the Fourier transform $V(f)$ of $v(t)$ and sketch it, showing clearly the magnitudes of the various components.   **(3 marks)**

   (iv) Determine an expression for the upper sideband component of $v(t)$. Note that this is a real signal.   **(2 marks)**

   (v) Determine an expression for the spectrum of the upper sideband component of $v(t)$ and sketch it, showing clearly the magnitudes of the various components.   **(2 marks)**

   [Your sketches should show magnitudes of frequency components on the frequency axis].

1b) Consider a stationary random signal $x(t)$ with power spectral density $S_{xx}(f)$, which is passed through the system shown in Figure 1 giving rise to the output signal $y(t)$.
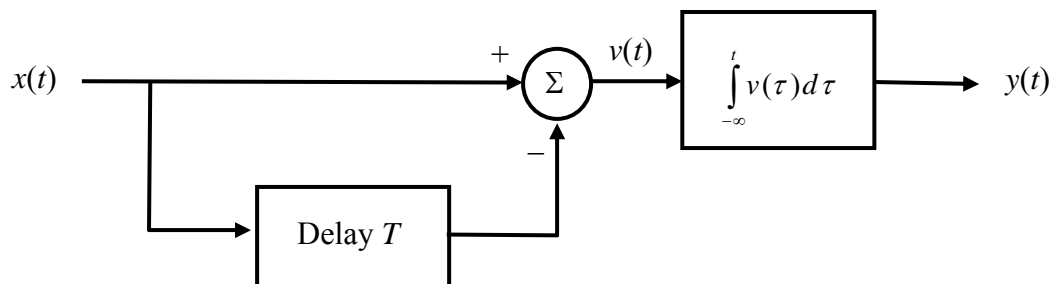
Figure 1

   (i) Express the output $y(t)$ in terms of $x(t)$.   **(2 marks)**

   (ii) Calculate the transfer function $H(f)$ of the system. **Hint:** you will need to recall the identity, $\sin\theta = (e^{j\theta} - e^{-j\theta})/2j$.   **(4 marks)**

   (iii) Calculate the power spectral density $S_{yy}(f)$ of $y(t)$ and sketch it for $S_{xx}(f) = 1$.   **(3 marks)**

   (iv) When $S_{xx}(f) = 1$, what frequency components are absent in the output?   **(1 mark)**

Question 2 follows on Page 3

Question 2
20 marks total

Q2a) In a broadcast communication system the <u>carrier power</u> is 90 kW, the channel attenuation is 80 dB, the noise power spectral density is $S_{nn}(f) = N_o/2$ with $N_o = 1.5 \times 10^{-10}$ W/Hz and the normalised baseband message signal $m(t)$ has a bandwidth of 15 kHz, $|m(t)| \leq 1$ and a mean square value $\langle m^2(t) \rangle = 0.1$.

  (i) If amplitude modulation (AM) is used, with a modulation index $a = 0.90$, calculate the following for a receiver with bandwidth equal to that of the signal:

  The bandwidth of the signal, B
  The predetection signal to noise ratio (SNR_p) in decibels
  The output signal to noise ratio (SNR_o) in decibels   **(6 marks)**

  (ii) If the modulation used is frequency modulation (FM) with peak frequency deviation 75 kHz, calculate the following for a receiver with a bandwidth given by Carson's rule:

  The (approximate) bandwidth of the signal
  The predetection signal to noise ratio (SNR_p) in decibels
  The output signal to noise ratio (SNR_o) in decibels
  Comment on how this FM case compares with the AM case in part (i).

  (4 marks)

  (iii) What is the maximum channel attenuation (in decibels) allowed if the FM system in (ii) is to be above threshold?   **(3 marks)**

Q2b) An AM system has a received signal of the form:

$$v(t) = A\big[1 + am(t)\big]\cos(2\pi f_o t) + n(t)$$

where $A = 100$ μV, $m(t)$ has a bandwidth of 10 kHz with $|m(t)| \leq 1$ and $<m^2(t)> = 0.1$, the modulation index is $a = 0.9$, the carrier frequency is $f_o = 1.6$ MHz and $n(t)$ is white noise of power spectral density $S_{nn}(f) = 2.5 \times 10^{-15}$ V²/Hz. The receiver has the form shown in the figure below.
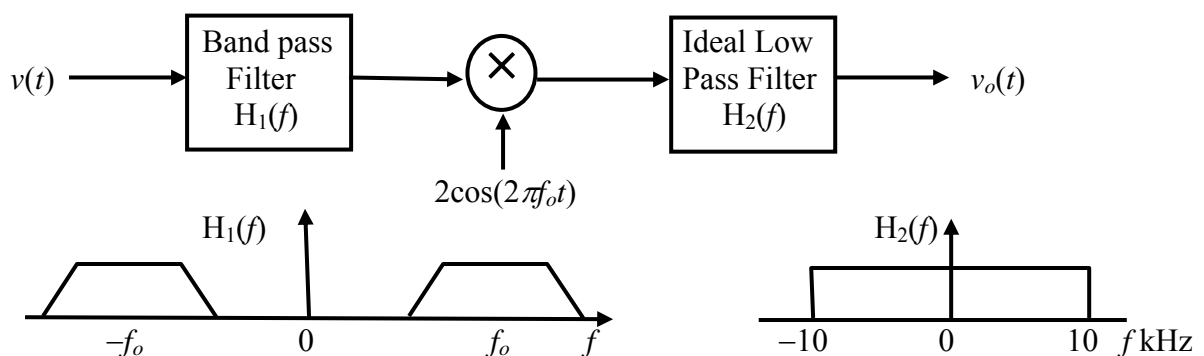
The a pass band gains are 1, the flat parts of $H_1(f)$ are 20 kHz wide centred at $\pm f_o$ with transitions at each end of 5 kHz width (ie. the total bandwidth is 30 kHz, noise bandwidth, $B_n = 23333$ Hz).

  (i)   Calculate the predetection signal to noise ratio (ie. at the output of H_1) in decibels.
  (4 marks)

 **(ii)**   Calculate the output signal to noise ratio in decibels (ie. at the output of H_2).
  (3 marks)

 Question 3 follows on Page 4

Question 3   **20 marks total**

Q3a) A 16QAM communications system uses a square 4×4 constellation, as shown below, such that the symbols are equally spaced by d both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is $R = 50\ \Omega$.
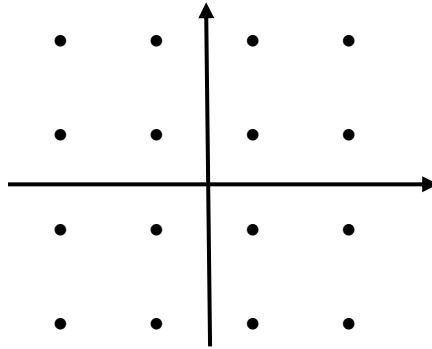
    (i) Determine the *minimum* bandwidth B required for transmission.   **(2 marks)**

    (ii) Calculate E_b, the average energy per bit (in V^2s), for the constellation in terms of the separation distance d (the horizontal and vertical separation of the symbols). **Hint:** the energy of each symbol in the constellation is given by the square of its distance from the origin.
    (3 marks)

    (iii) If the average received power $P_r = 1.2 \times 10^{-6}$ W, determine the symbol spacing d.
    (3 marks)

    (iv) If outermost corner symbols have an energy of 9×10^{-10} J, write down an expression for the corner symbol energy d^2 in units of V^2s, as a function of the receiver input impedance R. If the noise power spectral density, $N_o = 1 \times 10^{-11}$ W/Hz, also write down an expression for the noise energy in units of V^2s, as a function of the receiver input impedance R. Then determine the probability of selecting an adjacent symbol in error during demodulation, given that a matched filter receiver is used.   **(4   marks)**

Q3b) A BPSK (binary phase shift keyed) system is used to transmit the same <u>bit rate</u> as in part a), but this time the receiver input impedance, R, is unknown.

    (i) Determine the minimum bandwidth B required for transmission.   **(2 marks)**

    (ii) For an average received power $P_r = 1.2 \times 10^{-6}$ W and noise spectral density $N_o = 1 \times 10^{-11}$ W/Hz, determine the probability of error for a matched filter receiver.   **(4 marks)**

    (iii) Comment on the differences between BPSK and 16QAM as revealed by your answers.
    (2 marks)

Question 4 follows on Page 5

Question 4   20 marks total

4a) A source *X* has an alphabet {A, B, C, D, E} with corresponding probabilities {0.20, 0.15, 0.05, 0.10, 0.50}.

   (i) Calculate the source entropy $H(x)$ in bits and explain what this means.   **(3 marks)**

   (ii) Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of five symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i).   **(2 marks)**

   (ii) Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency*.   **(4 marks)**

   (iii) Explain why it is possible to uniquely decipher a Huffman code.   **(2 marks)**

4b) A BPSK (binary phase shift keyed) digital transmission system transmits data at 10^6 symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to $P_e = Q\left\{\sqrt{2E_c/N_o}\right\} = 10^{-4}$, where E_c is the energy per transmitted channel bit and $N_o/2$ is the spectral density of the accompanying additive white Gaussian noise (AWGN). Error correction is achieved by using a (15,11) Hamming block code.

   (i) Calculate the value of E_c/N_o.   **(1 mark)**

   (ii) How many errors can the Hamming code *correct* in each block of 15?   **(1 mark)**

   (iii) Up to how many errors can the Hamming code *detect* in each block of 15?

   (iv) What is the minimum bandwidth required to transmit the signal?   **(1 mark)**

   (v) Calculate the bit error probability after error correction.   **(3 marks)**

   (vi) If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to $(15/11) \times 10^6$ symbols/sec so that the <u>message bit rate</u> is 10^6 bits/sec, calculate the corrected probability of a bit error. (**Hint:** Calculate the reduced value of E_c/N_o, and hence the new value of *p*, the uncorrected probability of a bit error).   **(3 marks)**

End of Examination Questions

Data Sheets follow on Pages 6 – 11

Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\ dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\ df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\lvert T \rvert X(fT)$
$x(t-T)$	$X(f)e^{-j2\pi f T}$
$x(t)e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\dfrac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\displaystyle\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(f)}{j2\pi f} + \tfrac{1}{2} X(0)\delta(f)$
$t\, x(t)$	$-\dfrac{1}{j2\pi}\dfrac{dX(f)}{df}$
$X(t)$	$x(-f)$
$\mathrm{rep}_T\{x(t)\}$	$\lvert F \rvert \mathrm{comb}_F(f)\, X(f)$
$\lvert T \rvert \mathrm{comb}_T(t)\, x(t)$	$\mathrm{rep}_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)\, Y(f)$
$x^*(t)$	$X^*(-f)$

Transforms

$u(t)e^{-at}$	$\dfrac{1}{a+j2\pi f}\quad ; a > 0$
$e^{-a\lvert t \rvert}$	$\dfrac{2a}{a^2 + (2\pi f)^2}\quad ; a > 0$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a} e^{-\lvert 2\pi f a \rvert}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\dfrac{1}{j2\pi f} + \tfrac{1}{2}\delta(f)$
$\mathrm{sgn}(t)$	$\dfrac{1}{j\pi f}$
$\dfrac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$\mathrm{rect}(t/T)$	$\lvert T \rvert \mathrm{sinc}(fT)$
$\mathrm{sinc}(t/T)$	$\lvert T \rvert \mathrm{rect}(fT)$
$\Delta(t/T)$	$\lvert T \rvert \mathrm{sinc}^2(fT)$
$\mathrm{comb}_T(t)$	$\lvert F \rvert \mathrm{comb}_F(f)$
$e^{-t^2/2T^2}$	$\lvert T \rvert \sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$\mathrm{sgn}(t)\,\mathrm{rect}(t/T)$	$\dfrac{1-\cos(\pi f T)}{j\pi f}$

Note that F and T are real constants, with $FT = 1$.

Note that a is a real positive constant.

Definitions

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\text{rep}_P \{f(x)\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\text{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area = 1)}$$

$$\text{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\text{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda) \, g(x - \lambda) \, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f) \, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t) \, dt = \text{area of } x(t)$$

$$X(-f) = X^*(f) \text{ if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) = \int_{-\infty}^{\infty} X(f) Y^*(f) \, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.676E-04	9.354E-04	9.043E-04	8.740E-04	8.447E-04	8.164E-04	7.888E-04	7.622E-04	7.364E-04	7.114E-04
3.2	6.871E-04	6.637E-04	6.410E-04	6.190E-04	5.976E-04	5.770E-04	5.571E-04	5.377E-04	5.190E-04	5.009E-04
3.3	4.834E-04	4.665E-04	4.501E-04	4.342E-04	4.189E-04	4.041E-04	3.897E-04	3.758E-04	3.624E-04	3.495E-04
3.4	3.369E-04	3.248E-04	3.131E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.602E-04	2.507E-04	2.415E-04
3.5	2.326E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.926E-04	1.854E-04	1.785E-04	1.718E-04	1.653E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.363E-04	1.311E-04	1.261E-04	1.213E-04	1.166E-04	1.121E-04
3.7	1.078E-04	1.036E-04	9.961E-05	9.574E-05	9.201E-05	8.842E-05	8.496E-05	8.162E-05	7.841E-05	7.532E-05
3.8	7.235E-05	6.948E-05	6.673E-05	6.407E-05	6.152E-05	5.906E-05	5.669E-05	5.442E-05	5.223E-05	5.012E-05
3.9	4.810E-05	4.615E-05	4.427E-05	4.247E-05	4.074E-05	3.908E-05	3.747E-05	3.594E-05	3.446E-05	3.304E-05
4.0	3.167E-05	3.036E-05	2.910E-05	2.789E-05	2.673E-05	2.561E-05	2.454E-05	2.351E-05	2.252E-05	2.157E-05
4.1	2.066E-05	1.978E-05	1.894E-05	1.814E-05	1.737E-05	1.662E-05	1.591E-05	1.523E-05	1.458E-05	1.395E-05
4.2	1.335E-05	1.277E-05	1.222E-05	1.168E-05	1.118E-05	1.069E-05	1.022E-05	9.774E-06	9.345E-06	8.934E-06
4.3	8.540E-06	8.163E-06	7.801E-06	7.455E-06	7.124E-06	6.807E-06	6.503E-06	6.212E-06	5.934E-06	5.668E-06
4.4	5.413E-06	5.169E-06	4.935E-06	4.712E-06	4.498E-06	4.294E-06	4.098E-06	3.911E-06	3.732E-06	3.561E-06
4.5	3.398E-06	3.241E-06	3.092E-06	2.949E-06	2.813E-06	2.682E-06	2.558E-06	2.439E-06	2.325E-06	2.216E-06
4.6	2.112E-06	2.013E-06	1.919E-06	1.828E-06	1.742E-06	1.660E-06	1.581E-06	1.506E-06	1.434E-06	1.366E-06
4.7	1.301E-06	1.239E-06	1.179E-06	1.123E-06	1.069E-06	1.017E-06	9.680E-07	9.211E-07	8.765E-07	8.339E-07
4.8	7.933E-07	7.547E-07	7.178E-07	6.827E-07	6.492E-07	6.173E-07	5.869E-07	5.580E-07	5.304E-07	5.042E-07
4.9	4.792E-07	4.554E-07	4.327E-07	4.111E-07	3.906E-07	3.711E-07	3.525E-07	3.348E-07	3.179E-07	3.019E-07

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	2.867E-07	2.722E-07	2.584E-07	2.452E-07	2.328E-07	2.209E-07	2.096E-07	1.989E-07	1.887E-07	1.790E-07
5.1	1.698E-07	1.611E-07	1.528E-07	1.449E-07	1.374E-07	1.302E-07	1.235E-07	1.170E-07	1.109E-07	1.051E-07
5.2	9.964E-08	9.442E-08	8.946E-08	8.476E-08	8.029E-08	7.605E-08	7.203E-08	6.821E-08	6.459E-08	6.116E-08
5.3	5.790E-08	5.481E-08	5.188E-08	4.911E-08	4.647E-08	4.398E-08	4.161E-08	3.937E-08	3.724E-08	3.523E-08
5.4	3.332E-08	3.151E-08	2.980E-08	2.818E-08	2.664E-08	2.518E-08	2.381E-08	2.250E-08	2.127E-08	2.010E-08
5.5	1.899E-08	1.794E-08	1.695E-08	1.601E-08	1.512E-08	1.428E-08	1.349E-08	1.274E-08	1.203E-08	1.135E-08
5.6	1.072E-08	1.012E-08	9.548E-09	9.010E-09	8.503E-09	8.022E-09	7.569E-09	7.140E-09	6.735E-09	6.352E-09
5.7	5.990E-09	5.649E-09	5.326E-09	5.022E-09	4.734E-09	4.462E-09	4.206E-09	3.964E-09	3.735E-09	3.519E-09
5.8	3.316E-09	3.124E-09	2.942E-09	2.771E-09	2.610E-09	2.458E-09	2.314E-09	2.179E-09	2.051E-09	1.931E-09
5.9	1.818E-09	1.711E-09	1.610E-09	1.515E-09	1.425E-09	1.341E-09	1.261E-09	1.186E-09	1.116E-09	1.049E-09
6.0	9.866E-10	9.276E-10	8.721E-10	8.198E-10	7.706E-10	7.242E-10	6.806E-10	6.396E-10	6.009E-10	5.646E-10
6.1	5.303E-10	4.982E-10	4.679E-10	4.394E-10	4.126E-10	3.874E-10	3.637E-10	3.414E-10	3.205E-10	3.008E-10
6.2	2.823E-10	2.649E-10	2.486E-10	2.332E-10	2.188E-10	2.052E-10	1.925E-10	1.805E-10	1.693E-10	1.587E-10
6.3	1.488E-10	1.395E-10	1.308E-10	1.226E-10	1.149E-10	1.077E-10	1.009E-10	9.451E-11	8.854E-11	8.294E-11
6.4	7.769E-11	7.276E-11	6.814E-11	6.380E-11	5.974E-11	5.593E-11	5.235E-11	4.900E-11	4.586E-11	4.292E-11
6.5	4.016E-11	3.758E-11	3.515E-11	3.288E-11	3.076E-11	2.877E-11	2.690E-11	2.516E-11	2.352E-11	2.199E-11
6.6	2.056E-11	1.922E-11	1.796E-11	1.678E-11	1.568E-11	1.465E-11	1.369E-11	1.279E-11	1.195E-11	1.116E-11
6.7	1.042E-11	9.731E-12	9.086E-12	8.483E-12	7.919E-12	7.392E-12	6.900E-12	6.439E-12	6.009E-12	5.607E-12
6.8	5.231E-12	4.880E-12	4.552E-12	4.246E-12	3.960E-12	3.692E-12	3.443E-12	3.210E-12	2.993E-12	2.790E-12
6.9	2.600E-12	2.423E-12	2.258E-12	2.104E-12	1.960E-12	1.826E-12	1.701E-12	1.585E-12	1.476E-12	1.374E-12
7.0	1.280E-12	1.192E-12	1.109E-12	1.033E-12	9.612E-13	8.946E-13	8.325E-13	7.747E-13	7.208E-13	6.706E-13
7.1	6.238E-13	5.802E-13	5.396E-13	5.018E-13	4.667E-13	4.339E-13	4.034E-13	3.750E-13	3.486E-13	3.240E-13
7.2	3.011E-13	2.798E-13	2.599E-13	2.415E-13	2.243E-13	2.084E-13	1.935E-13	1.797E-13	1.669E-13	1.550E-13
7.3	1.439E-13	1.336E-13	1.240E-13	1.151E-13	1.068E-13	9.910E-14	9.196E-14	8.531E-14	7.914E-14	7.341E-14
7.4	6.809E-14	6.315E-14	5.856E-14	5.430E-14	5.034E-14	4.667E-14	4.326E-14	4.010E-14	3.716E-14	3.444E-14
7.5	3.191E-14	2.956E-14	2.739E-14	2.537E-14	2.350E-14	2.176E-14	2.015E-14	1.866E-14	1.728E-14	1.600E-14
7.6	1.481E-14	1.370E-14	1.268E-14	1.174E-14	1.086E-14	1.005E-14	9.297E-15	8.600E-15	7.954E-15	7.357E-15
7.7	6.803E-15	6.291E-15	5.816E-15	5.377E-15	4.971E-15	4.595E-15	4.246E-15	3.924E-15	3.626E-15	3.350E-15
7.8	3.095E-15	2.859E-15	2.641E-15	2.439E-15	2.253E-15	2.080E-15	1.921E-15	1.773E-15	1.637E-15	1.511E-15
7.9	1.395E-15	1.287E-15	1.188E-15	1.096E-15	1.011E-15	9.326E-16	8.602E-16	7.934E-16	7.317E-16	6.747E-16
8.0	6.221E-16	5.735E-16	5.287E-16	4.874E-16	4.492E-16	4.140E-16	3.815E-16	3.515E-16	3.238E-16	2.983E-16
8.1	2.748E-16	2.531E-16	2.331E-16	2.146E-16	1.976E-16	1.820E-16	1.675E-16	1.542E-16	1.419E-16	1.306E-16
8.2	1.202E-16	1.106E-16	1.018E-16	9.361E-17	8.611E-17	7.920E-17	7.284E-17	6.698E-17	6.159E-17	5.662E-17
8.3	5.206E-17	4.785E-17	4.398E-17	4.042E-17	3.715E-17	3.413E-17	3.136E-17	2.881E-17	2.646E-17	2.431E-17
8.4	2.232E-17	2.050E-17	1.882E-17	1.728E-17	1.587E-17	1.457E-17	1.337E-17	1.227E-17	1.126E-17	1.033E-17
8.5	9.480E-18	8.697E-18	7.978E-18	7.317E-18	6.711E-18	6.154E-18	5.643E-18	5.174E-18	4.744E-18	4.348E-18
8.6	3.986E-18	3.653E-18	3.348E-18	3.068E-18	2.811E-18	2.575E-18	2.359E-18	2.161E-18	1.979E-18	1.812E-18
8.7	1.659E-18	1.519E-18	1.391E-18	1.273E-18	1.166E-18	1.067E-18	9.763E-19	8.933E-19	8.174E-19	7.478E-19
8.8	6.841E-19	6.257E-19	5.723E-19	5.234E-19	4.786E-19	4.376E-19	4.001E-19	3.657E-19	3.343E-19	3.055E-19
8.9	2.792E-19	2.552E-19	2.331E-19	2.130E-19	1.946E-19	1.777E-19	1.623E-19	1.483E-19	1.354E-19	1.236E-19
9.0	1.129E-19	1.030E-19	9.404E-20	8.584E-20	7.834E-20	7.148E-20	6.523E-20	5.951E-20	5.429E-20	4.952E-20
9.1	4.517E-20	4.119E-20	3.756E-20	3.425E-20	3.123E-20	2.847E-20	2.595E-20	2.365E-20	2.155E-20	1.964E-20
9.2	1.790E-20	1.631E-20	1.486E-20	1.353E-20	1.232E-20	1.122E-20	1.022E-20	9.307E-21	8.474E-21	7.714E-21
9.3	7.022E-21	6.392E-21	5.817E-21	5.294E-21	4.817E-21	4.382E-21	3.987E-21	3.627E-21	3.299E-21	3.000E-21
9.4	2.728E-21	2.481E-21	2.255E-21	2.050E-21	1.864E-21	1.694E-21	1.540E-21	1.399E-21	1.271E-21	1.155E-21
9.5	1.049E-21	9.533E-22	8.659E-22	7.864E-22	7.142E-22	6.485E-22	5.888E-22	5.345E-22	4.852E-22	4.404E-22
9.6	3.997E-22	3.627E-22	3.292E-22	2.986E-22	2.709E-22	2.458E-22	2.229E-22	2.022E-22	1.834E-22	1.663E-22
9.7	1.507E-22	1.367E-22	1.239E-22	1.123E-22	1.018E-22	9.223E-23	8.358E-23	7.573E-23	6.861E-23	6.215E-23
9.8	5.629E-23	5.098E-23	4.617E-23	4.181E-23	3.786E-23	3.427E-23	3.102E-23	2.808E-23	2.542E-23	2.300E-23
9.9	2.081E-23	1.883E-23	1.704E-23	1.541E-23	1.394E-23	1.261E-23	1.140E-23	1.031E-23	9.323E-24	8.429E-24

Communications IV Data Sheet

1. Correlation and Power Spectrum

$$R_{xy}(t_1, t_2) = E\{y(t_1)y^*(t_2)\}$$

$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$

$$R_{xx}(-\tau) = R_{xx}(\tau)$$

$$R_{yx}(\tau) = R_{xy}^*(-\tau)$$

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$

$$S_{yx}(f) = S_{xy}^*(f)$$

2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda = h(t) \otimes x(t)$$

$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$

$$S_{xy}(f) = H^*(f)S_{xx}(f)$$

$$S_{yx}(f) = H(f)S_{xx}(f)$$

3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\,\hat{x}(t) \qquad (analytic\ signal)$$

$$X^+(f) = 2u(f)X(f)$$

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t-\lambda}\,d\lambda \qquad (hilbert\ transform)$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty} |H(f)|^2\,df$$

5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$

$$S_{nn}^{(+)}(f) = u(f)S_{nn}(f)$$

$$S_{nn}^{(-)}(f) = u(-f)S_{nn}(f)$$

$$S_{n_c n_c}(f) = S_{n_s n_s}(f) = S_{nn}^{(+)}(f+f_o) + S_{nn}^{(-)}(f-f_o)$$

$$S_{n_c n_s}(f) = j\{S_{nn}^{(+)}(f+f_o) - S_{nn}^{(-)}(f-f_o)\}$$

6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \leq 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

_ _

$$s_{base}(t) = Am(t)$$

$$B = W$$

$$SNR_{base} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

_ _

$$s_{am}(t) = A\{1 + am(t)\}\cos(2\pi f_o t)$$

$$B = 2W$$

$$SNR_{am} = \frac{A^2 a^2 <m^2>}{2N_o W} = \frac{a^2 <m^2>}{1+a^2 <m^2>}\frac{P_r}{N_o W}$$

_ _

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$

$$B = 2W$$

$$SNR_{dsbsc} = \frac{A^2 <m^2>}{2N_o W} = \frac{P_r}{N_o W}$$

_ _

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$

$$B = W$$

$$SNR_{ssbsc} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

_ _

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$

$$B = 2(f_d + W)$$

$$SNR_{fm} = \frac{3A^2 f_d^2 <m^2>}{2N_o W^3} = 3 <m^2> \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_o W}$$

Threshold at $\frac{P_r}{N_o B} = 10$ (10 dB)

_ _

7. Information Theory

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$$H(x) = -\sum_i P(x_i)\log_2 P(x_i) \; bits \,/\, symbol$$

$$H(x,y) = -\sum_i \sum_j P(x_i, y_j)\log_2 P(x_i, y_j)$$

$$H(y \mid x) = -\sum_i \sum_j P(x_i, y_j)\log_2 P(y_j \mid x_i)$$

$$= H(x,y) - H(x)$$

$$I(x,y) = H(x) - H(x \mid y) = H(y) - H(y \mid x)$$

$$C = r \max I(x,y) \; bits \,/\, \sec \; (discrete \; channel)$$

$$C = B\log_2(1 + P_s / P_n) \, bits/\sec \, (continuous \, channel)$$

8. Digital Modulation

$$s_o(t_o) = \pm V, \; \Gamma = \frac{V^2}{<n_o^2>} \; (binary \; antipodal \; signals)$$

$$P_b = Q\{\sqrt{\Gamma}\} \; (bit \; error, \; binary \; antipodal \; system)$$

$$h(t) = c\, s(t_o - t) \; (matched \; filter)$$

$$\Gamma = \frac{2E_s}{N_o} \; (matched \; filter, baseband \; \& \; BPSK)$$

$$P_{sym} \approx Q\{\sqrt{d^2/2N_o}\} \; (matched \; filter, spacing \, d)$$

$$Constellation \; radius = \sqrt{(symbol \; energy)}$$

$$S_{xx}(f) = \frac{E\{a_k^2\}}{T}|P(f)|^2 \quad ; \; x(t) = \sum_{k=-\infty}^{\infty} a_k \, p(t - kT)$$

$$s_{QAM}(t) = \sum_{k=-\infty}^{\infty} A\, p(t - kT)\big[a_k \cos\omega_o t - b_k \sin\omega_o t\big]$$

$$s_{PSK}(t) = \sum_{k=-\infty}^{\infty} A\, p(t - kT)\cos(\omega_o t + \theta_k)$$

9. Nyquist Pulse

$$W = \frac{1}{2T}$$

$$P(f) = \begin{cases} T & ;| f | < (1-\rho)W \\ 0.5T - 0.5T\sin\left(\frac{\pi(|f|-W)}{2\rho W}\right) & ;\| f | - W | < \rho W \\ 0 & ;elsewhere \end{cases}$$

$$p(t) = \frac{\pi}{4}\mathrm{sinc}\left(\frac{t}{T}\right)\left\{\mathrm{sinc}\left(\frac{\rho t}{T}+\frac{1}{2}\right)+\mathrm{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)\right\}$$

10. Coding

$$P(i) = \binom{n}{i}P_b^i Q_b^{n-i} \; (binomial \; distribution)$$

$$P_b = Q\{\sqrt{RE_b/\alpha}\} \; (BER \; before \; correction)$$

$$P_{cbe} \approx \frac{2t+1}{n}\binom{n}{t+1}P_b^{t+1} \; (BER, correcting \; t \; errors)$$

$$\tilde{x} = \tilde{m}G, \; G = [I\; P] = generator \; matrix$$

$$\tilde{y} = \tilde{x} + \tilde{e}, \; \tilde{s} = \tilde{y}H = syndrome$$

$$H = \begin{bmatrix} P \\ I \end{bmatrix} = parity \; check \; matrix$$

Hamming codes $n = 2^q - 1$ (correct one error per block)

$$2^q \geq \sum_{i=0}^{t}\binom{n}{i} (necessary \; for \; existence \; of \; code)$$

$$X(p) = p^q M(p) + C(p) \; (cyclic \; code \; generation)$$

$$C(p) = \mathrm{rem}\left\{\frac{p^q M(p)}{G(p)}\right\}(check \; digits)$$

$$S(p) = \mathrm{rem}\left\{\frac{Y(p)}{G(p)}\right\}(syndrome, cyclic \; code)$$

11. Gaussian Probability

One dimension (mean η, variance σ^2):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\eta)^2/2\sigma^2}$$

$$P\{x > V\} = Q\left\{\frac{V-\eta}{\sigma}\right\}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\, dt$$

Two dimensions (zero means):

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2}-\frac{2\rho xy}{\sigma_x\sigma_y}+\frac{y^2}{\sigma_y^2}\right]}$$

$$\sigma_x^2 = E\{x^2\}, \; \sigma_y^2 = E\{y^2\}, \; \rho = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

End of Examination Paper