THE UNIVERSITY OF ADELAIDE
AUSTRALIA

EXAMINATION FOR THE DEGREE OF B.E.

Semester I June 2009

101902 COMMUNICATIONS IV (ELEC ENG 4035)

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

Instructions:

- This is a closed book examination.
- Attempt **ALL FOUR** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

Materials:

- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:

- Fourier Transform Sheet
- Table of the Q Function
- Communications IV Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Total number of pages (including attachments) = 11

<u>Question 1 follows on page 2</u>.

Question 1 **20 marks total**

Q1a) A band pass signal $x(t) = 20\ \text{sinc}(200t)\cos(2\pi 1000t)$ is applied to an ideal band pass filter centred at 1000 Hz and of bandwidth 100 Hz. The passband gain is unity.

(i) Calculate the Fourier transform $X(f)$ of $x(t)$ and sketch $X(f)$ showing clearly the amplitude and frequency scales. **(3 marks)**

(ii) Sketch the frequency response of the filter and find an expression for its transfer function $H(f)$. **(2 marks)**

(iii) Calculate the impulse response $h(t)$ of the bandpass filter. **(2 marks)**

(iv) Calculate the output $y(t)$ of the bandpass filter. **(3 marks)**

Q1b) A binary PAM system generates a signal

$$s(t) = \sum_{k=-\infty}^{\infty} A\, a_k\, p(t - kT)$$

where $A = 10$, $a_k = \pm 1$ is random uncorrelated binary data, the pulse shape is $p(t) = \Delta(t/T)$ and $T = 1$ ms is the bit period.

(i) The signal $s(t)$ is cyclostationary. Explain clearly what stationary and cyclostationary means in relation to the mean and autocorrelation function of a signal. **(2 marks)**

(ii) Sketch the signal $s(t)$ for $0 \le t \le 4T$ if the data is $a_k = [-1\ \ +1\ \ +1\ \ -1\ \ +1]$ for $0 \le k \le 4$. **(3 marks)**

(iii) Calculate the power spectrum $S_{ss}(f)$ of $s(t)$ and sketch it showing clearly the amplitude and frequency scales (use the result on data sheet, derivation is not required). **(3 marks)**

(iv) Write down the autocorrelation $\overline{R}_{ss}(\tau)$ expressing it in integral form. You are not required to evaluate the integral. **(2 marks)**

Question 2 follows on Page 3

Question 2

20 marks total

Q2a) In a broadcast communication system the transmitter power with modulation is 75 kW, the channel attenuation is 80 dB, the noise power spectral density is $S_{nn}(f) = N_o/2$ with $N_o = 10^{-10}$ W/Hz and the normalised baseband message signal $m(t)$ has a bandwidth of 10 kHz, $|m(t)| \leq 1$ and a mean square value $\langle m^2(t)\rangle = 0.1$.

(i) If amplitude modulation (AM) is used, with a modulation index $a = 0.90$, calculate the following:

The bandwidth of the signal, B
The predetection signal to noise ratio (SNR_p) in decibels
The output signal to noise ratio (SNR_o) in decibels **(4 marks)**

(ii) If the modulation used is double side band suppressed carrier (DSBSC), calculate:

The bandwidth of the signal
The predetection signal to noise ratio (SNR_p) in decibels
The output signal to noise ratio (SNR_o) in decibels **(3 marks)**

(iii) If the modulation used is frequency modulation (FM) with a peak frequency deviation of 75 kHz, calculate:

The bandwidth of the signal
The predetection signal to noise ratio (SNR_p) in decibels
The output signal to noise ratio (SNR_o) in decibels
Comment on how this FM case compares with the AM case in part (i). **(4 marks)**

(iv) What is the maximum channel attenuation (in decibels) allowed if the FM system in (iii) is to be above threshold? **(3 marks)**

2b) A communication channel has a bandwidth of $B = 250$ kHz. This channel is used to transmit a message signal $m(t)$ of bandwidth $W = 15$ kHz with $|m(t)| \leq 1$, $<m^2(t)> = 0.05$. At the receiver, the signal is received accompanied by white noise of power spectral density $S_{nn}(f) = N_o/2$ and the average received power is +30 dB relative to N_oW (the noise power in a bandwidth W), ie. $P_r/N_oW = 10^3$.

(i) If double sideband suppressed carrier (DSBSC) modulation is used, calculate the output signal to noise ratio (SNR_o) in decibels. **(1 mark)**

(ii) If frequency modulation (FM) is used so that the whole bandwidth is utilised, calculate the output signal to noise ratio (SNR_o) in decibels. **(2 marks)**

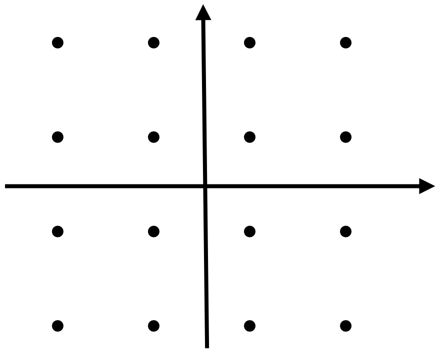
(iii) For the FM system in (ii), what is the value of P_r/N_oW in dB at threshold? Thus comment on whether or not the system is above threshold. **(3 marks)**

Question 3 follows on Page 4

Question 3

20 marks total

Q3a) A 16QAM communications system uses a square 4 × 4 constellation, as shown below, such that the symbols are equally spaced by *d* both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is $R = 50\ \Omega$.



(i) Determine the *minimum* bandwidth B required for transmission. **(2 marks)**

(ii) Calculate E_b, the average energy per bit (in V^2s), for the constellation in terms of the separation distance d (the horizontal and vertical separation of the symbols). **Hint:** the energy of each symbol in the constellation is given by the square of its distance from the origin. **(3 marks)**

(iii) If the average received power $P_r = 1.2 \times 10^{-6}$ W, determine the symbol spacing d. **(3 marks)**

(iv) If outermost corner symbols have an energy of 9×10^{-10} J, write down an expression for the corner symbol energy d^2 in units of V^2s, as a function of the receiver input impedance R. If the noise power spectral density, $N_o = 1 \times 10^{-11}$ W/Hz, also write down an expression for the noise energy in units of V^2s, as a function of the receiver input impedance R. Then determine the probability of selecting an adjacent symbol in error during demodulation, given that a matched filter receiver is used. **(4 marks)**

Q3b) A BPSK (binary phase shift keyed) system is used to transmit the same bit rate as in part a), but this time the receiver input impedance, R, is unknown.

(i) Determine the minimum bandwidth B required for transmission. **(2 marks)**

(ii) For an average received power $P_r = 1.2 \times 10^{-6}$ W and noise spectral density $N_o = 1 \times 10^{-11}$ W/Hz, determine the probability of error for a matched filter receiver. **(4 marks)**

(iii) Comment on the differences between BPSK and 16QAM as revealed by your answers. **(2 marks)**

Question 4 follows on Page 5

Question 4 **20 marks total**

4a) A source X has an alphabet {A, B, C, D, E} with corresponding probabilities {0.20, 0.15, 0.05, 0.10, 0.50}.

(i) Calculate the source entropy $H(x)$ in bits and explain what this means. **(3 marks)**

(ii) Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of five symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i). **(2 marks)**

(ii) Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency*. **(4 marks)**

(iii) Explain why it is possible to uniquely decipher a Huffman code. **(2 marks)**

4b) A BPSK (binary phase shift keyed) digital transmission system transmits data at 10^6 symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to $P_e = Q\left\{\sqrt{2E_c / N_o}\right\} = 10^{-4}$, where E_c is the energy per transmitted channel bit and $N_o/2$ is the spectral density of the accompanying additive white Gaussian noise (AWGN). Error correction is achieved by using a (15,11) Hamming block code.

(i) Calculate the value of E_c/N_o. **(1 mark)**

(ii) How many errors can the Hamming code *correct* in each block of 15? **(1 mark)**

(iii) Up to how many errors can the Hamming code *detect* in each block of 15?

(iv) What is the minimum bandwidth required to transmit the signal? **(1 mark)**

(v) Calculate the bit error probability after error correction. **(3 marks)**

(vi) If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to $(15/11) \times 10^6$ symbols/sec so that the message bit rate is 10^6 bits/sec, calculate the corrected probability of a bit error. (**Hint:** Calculate the reduced value of E_c/N_o, and hence the new value of p, the uncorrected probability of a bit error). **(3 marks)**

End of Examination Questions

Data Sheets follow on Pages 6 – 11

Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\|T\|X(fT)$
$x(t-T)$	$X(f)e^{-j2\pi f T}$
$x(t)e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\dfrac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
$t\, x(t)$	$-\dfrac{1}{j2\pi}\dfrac{dX(f)}{df}$
$X(t)$	$x(-f)$
$\mathrm{rep}_T\{x(t)\}$	$\|F\|\mathrm{comb}_F(f)\, X(f)$
$\|T\|\mathrm{comb}_T(t)\, x(t)$	$\mathrm{rep}_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)Y(f)$
$x^*(t)$	$X^*(-f)$

Note that F and T are real constants, with $FT = 1$.

Transforms

$u(t)e^{-at}$	$\dfrac{1}{a + j2\pi f}$; $a > 0$
$e^{-a\|t\|}$	$\dfrac{2a}{a^2 + (2\pi f)^2}$; $a > 0$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a} e^{-\|2\pi f a\|}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$\mathrm{sgn}(t)$	$\dfrac{1}{j\pi f}$
$\dfrac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$\mathrm{rect}(t/T)$	$\|T\|\mathrm{sinc}(fT)$
$\mathrm{sinc}(t/T)$	$\|T\|\mathrm{rect}(fT)$
$\Delta(t/T)$	$\|T\|\mathrm{sinc}^2(fT)$
$\mathrm{comb}_T(t)$	$\|F\|\mathrm{comb}_F(f)$
$e^{-t^2/2T^2}$	$\|T\|\sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$\mathrm{sgn}(t)\,\mathrm{rect}(t/T)$	$\dfrac{1-\cos(\pi f T)}{j\pi f}$

Note that a is a real positive constant.

Definitions

$$\mathrm{sinc}(x) = \frac{\sin(\pi\, x)}{\pi\, x}$$

$$\mathrm{rep}_{\mathrm{P}}\{\mathrm{f(x)}\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\mathrm{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\mathrm{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of x(t)}$$

$$X(-f) = X^*(f) \quad \text{if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t) = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.676E-04	9.354E-04	9.043E-04	8.740E-04	8.447E-04	8.164E-04	7.888E-04	7.622E-04	7.364E-04	7.114E-04
3.2	6.871E-04	6.637E-04	6.410E-04	6.190E-04	5.976E-04	5.770E-04	5.571E-04	5.377E-04	5.190E-04	5.009E-04
3.3	4.834E-04	4.665E-04	4.501E-04	4.342E-04	4.189E-04	4.041E-04	3.897E-04	3.758E-04	3.624E-04	3.495E-04
3.4	3.369E-04	3.248E-04	3.131E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.602E-04	2.507E-04	2.415E-04
3.5	2.326E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.926E-04	1.854E-04	1.785E-04	1.718E-04	1.653E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.363E-04	1.311E-04	1.261E-04	1.213E-04	1.166E-04	1.121E-04
3.7	1.078E-04	1.036E-04	9.961E-05	9.574E-05	9.201E-05	8.842E-05	8.496E-05	8.162E-05	7.841E-05	7.532E-05
3.8	7.235E-05	6.948E-05	6.673E-05	6.407E-05	6.152E-05	5.906E-05	5.669E-05	5.442E-05	5.223E-05	5.012E-05
3.9	4.810E-05	4.615E-05	4.427E-05	4.247E-05	4.074E-05	3.908E-05	3.747E-05	3.594E-05	3.446E-05	3.304E-05
4.0	3.167E-05	3.036E-05	2.910E-05	2.789E-05	2.673E-05	2.561E-05	2.454E-05	2.351E-05	2.252E-05	2.157E-05
4.1	2.066E-05	1.978E-05	1.894E-05	1.814E-05	1.737E-05	1.662E-05	1.591E-05	1.523E-05	1.458E-05	1.395E-05
4.2	1.335E-05	1.277E-05	1.222E-05	1.168E-05	1.118E-05	1.069E-05	1.022E-05	9.774E-06	9.345E-06	8.934E-06
4.3	8.540E-06	8.163E-06	7.801E-06	7.455E-06	7.124E-06	6.807E-06	6.503E-06	6.212E-06	5.934E-06	5.668E-06
4.4	5.413E-06	5.169E-06	4.935E-06	4.712E-06	4.498E-06	4.294E-06	4.098E-06	3.911E-06	3.732E-06	3.561E-06
4.5	3.398E-06	3.241E-06	3.092E-06	2.949E-06	2.813E-06	2.682E-06	2.558E-06	2.439E-06	2.325E-06	2.216E-06
4.6	2.112E-06	2.013E-06	1.919E-06	1.828E-06	1.742E-06	1.660E-06	1.581E-06	1.506E-06	1.434E-06	1.366E-06
4.7	1.301E-06	1.239E-06	1.179E-06	1.123E-06	1.069E-06	1.017E-06	9.680E-07	9.211E-07	8.765E-07	8.339E-07
4.8	7.933E-07	7.547E-07	7.178E-07	6.827E-07	6.492E-07	6.173E-07	5.869E-07	5.580E-07	5.304E-07	5.042E-07
4.9	4.792E-07	4.554E-07	4.327E-07	4.111E-07	3.906E-07	3.711E-07	3.525E-07	3.348E-07	3.179E-07	3.019E-07

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	2.867E-07	2.722E-07	2.584E-07	2.452E-07	2.328E-07	2.209E-07	2.096E-07	1.989E-07	1.887E-07	1.790E-07
5.1	1.698E-07	1.611E-07	1.528E-07	1.449E-07	1.374E-07	1.302E-07	1.235E-07	1.170E-07	1.109E-07	1.051E-07
5.2	9.964E-08	9.442E-08	8.946E-08	8.476E-08	8.029E-08	7.605E-08	7.203E-08	6.821E-08	6.459E-08	6.116E-08
5.3	5.790E-08	5.481E-08	5.188E-08	4.911E-08	4.647E-08	4.398E-08	4.161E-08	3.937E-08	3.724E-08	3.523E-08
5.4	3.332E-08	3.151E-08	2.980E-08	2.818E-08	2.664E-08	2.518E-08	2.381E-08	2.250E-08	2.127E-08	2.010E-08
5.5	1.899E-08	1.794E-08	1.695E-08	1.601E-08	1.512E-08	1.428E-08	1.349E-08	1.274E-08	1.203E-08	1.135E-08
5.6	1.072E-08	1.012E-08	9.548E-09	9.010E-09	8.503E-09	8.022E-09	7.569E-09	7.140E-09	6.735E-09	6.352E-09
5.7	5.990E-09	5.649E-09	5.326E-09	5.022E-09	4.734E-09	4.462E-09	4.206E-09	3.964E-09	3.735E-09	3.519E-09
5.8	3.316E-09	3.124E-09	2.942E-09	2.771E-09	2.610E-09	2.458E-09	2.314E-09	2.179E-09	2.051E-09	1.931E-09
5.9	1.818E-09	1.711E-09	1.610E-09	1.515E-09	1.425E-09	1.341E-09	1.261E-09	1.186E-09	1.116E-09	1.049E-09
6.0	9.866E-10	9.276E-10	8.721E-10	8.198E-10	7.706E-10	7.242E-10	6.806E-10	6.396E-10	6.009E-10	5.646E-10
6.1	5.303E-10	4.982E-10	4.679E-10	4.394E-10	4.126E-10	3.874E-10	3.637E-10	3.414E-10	3.205E-10	3.008E-10
6.2	2.823E-10	2.649E-10	2.486E-10	2.332E-10	2.188E-10	2.052E-10	1.925E-10	1.805E-10	1.693E-10	1.587E-10
6.3	1.488E-10	1.395E-10	1.308E-10	1.226E-10	1.149E-10	1.077E-10	1.009E-10	9.451E-11	8.854E-11	8.294E-11
6.4	7.769E-11	7.276E-11	6.814E-11	6.380E-11	5.974E-11	5.593E-11	5.235E-11	4.900E-11	4.586E-11	4.292E-11
6.5	4.016E-11	3.758E-11	3.515E-11	3.288E-11	3.076E-11	2.877E-11	2.690E-11	2.516E-11	2.352E-11	2.199E-11
6.6	2.056E-11	1.922E-11	1.796E-11	1.678E-11	1.568E-11	1.465E-11	1.369E-11	1.279E-11	1.195E-11	1.116E-11
6.7	1.042E-11	9.731E-12	9.086E-12	8.483E-12	7.919E-12	7.392E-12	6.900E-12	6.439E-12	6.009E-12	5.607E-12
6.8	5.231E-12	4.880E-12	4.552E-12	4.246E-12	3.960E-12	3.692E-12	3.443E-12	3.210E-12	2.993E-12	2.790E-12
6.9	2.600E-12	2.423E-12	2.258E-12	2.104E-12	1.960E-12	1.826E-12	1.701E-12	1.585E-12	1.476E-12	1.374E-12
7.0	1.280E-12	1.192E-12	1.109E-12	1.033E-12	9.612E-13	8.946E-13	8.325E-13	7.747E-13	7.208E-13	6.706E-13
7.1	6.238E-13	5.802E-13	5.396E-13	5.018E-13	4.667E-13	4.339E-13	4.034E-13	3.750E-13	3.486E-13	3.240E-13
7.2	3.011E-13	2.798E-13	2.599E-13	2.415E-13	2.243E-13	2.084E-13	1.935E-13	1.797E-13	1.669E-13	1.550E-13
7.3	1.439E-13	1.336E-13	1.240E-13	1.151E-13	1.068E-13	9.910E-14	9.196E-14	8.531E-14	7.914E-14	7.341E-14
7.4	6.809E-14	6.315E-14	5.856E-14	5.430E-14	5.034E-14	4.667E-14	4.326E-14	4.010E-14	3.716E-14	3.444E-14
7.5	3.191E-14	2.956E-14	2.739E-14	2.537E-14	2.350E-14	2.176E-14	2.015E-14	1.866E-14	1.728E-14	1.600E-14
7.6	1.481E-14	1.370E-14	1.268E-14	1.174E-14	1.086E-14	1.005E-14	9.297E-15	8.600E-15	7.954E-15	7.357E-15
7.7	6.803E-15	6.291E-15	5.816E-15	5.377E-15	4.971E-15	4.595E-15	4.246E-15	3.924E-15	3.626E-15	3.350E-15
7.8	3.095E-15	2.859E-15	2.641E-15	2.439E-15	2.253E-15	2.080E-15	1.921E-15	1.773E-15	1.637E-15	1.511E-15
7.9	1.395E-15	1.287E-15	1.188E-15	1.096E-15	1.011E-15	9.326E-16	8.602E-16	7.934E-16	7.317E-16	6.747E-16
8.0	6.221E-16	5.735E-16	5.287E-16	4.874E-16	4.492E-16	4.140E-16	3.815E-16	3.515E-16	3.238E-16	2.983E-16
8.1	2.748E-16	2.531E-16	2.331E-16	2.146E-16	1.976E-16	1.820E-16	1.675E-16	1.542E-16	1.419E-16	1.306E-16
8.2	1.202E-16	1.106E-16	1.018E-16	9.361E-17	8.611E-17	7.920E-17	7.284E-17	6.698E-17	6.159E-17	5.662E-17
8.3	5.206E-17	4.785E-17	4.398E-17	4.042E-17	3.715E-17	3.413E-17	3.136E-17	2.881E-17	2.646E-17	2.431E-17
8.4	2.232E-17	2.050E-17	1.882E-17	1.728E-17	1.587E-17	1.457E-17	1.337E-17	1.227E-17	1.126E-17	1.033E-17
8.5	9.480E-18	8.697E-18	7.978E-18	7.317E-18	6.711E-18	6.154E-18	5.643E-18	5.174E-18	4.744E-18	4.348E-18
8.6	3.986E-18	3.653E-18	3.348E-18	3.068E-18	2.811E-18	2.575E-18	2.359E-18	2.161E-18	1.979E-18	1.812E-18
8.7	1.659E-18	1.519E-18	1.391E-18	1.273E-18	1.166E-18	1.067E-18	9.763E-19	8.933E-19	8.174E-19	7.478E-19
8.8	6.841E-19	6.257E-19	5.723E-19	5.234E-19	4.786E-19	4.376E-19	4.001E-19	3.657E-19	3.343E-19	3.055E-19
8.9	2.792E-19	2.552E-19	2.331E-19	2.130E-19	1.946E-19	1.777E-19	1.623E-19	1.483E-19	1.354E-19	1.236E-19
9.0	1.129E-19	1.030E-19	9.404E-20	8.584E-20	7.834E-20	7.148E-20	6.523E-20	5.951E-20	5.429E-20	4.952E-20
9.1	4.517E-20	4.119E-20	3.756E-20	3.425E-20	3.123E-20	2.847E-20	2.595E-20	2.365E-20	2.155E-20	1.964E-20
9.2	1.790E-20	1.631E-20	1.486E-20	1.353E-20	1.232E-20	1.122E-20	1.022E-20	9.307E-21	8.474E-21	7.714E-21
9.3	7.022E-21	6.392E-21	5.817E-21	5.294E-21	4.817E-21	4.382E-21	3.987E-21	3.627E-21	3.299E-21	3.000E-21
9.4	2.728E-21	2.481E-21	2.255E-21	2.050E-21	1.864E-21	1.694E-21	1.540E-21	1.399E-21	1.271E-21	1.155E-21
9.5	1.049E-21	9.533E-22	8.659E-22	7.864E-22	7.142E-22	6.485E-22	5.888E-22	5.345E-22	4.852E-22	4.404E-22
9.6	3.997E-22	3.627E-22	3.292E-22	2.986E-22	2.709E-22	2.458E-22	2.229E-22	2.022E-22	1.834E-22	1.663E-22
9.7	1.507E-22	1.367E-22	1.239E-22	1.123E-22	1.018E-22	9.223E-23	8.358E-23	7.573E-23	6.861E-23	6.215E-23
9.8	5.629E-23	5.098E-23	4.617E-23	4.181E-23	3.786E-23	3.427E-23	3.102E-23	2.808E-23	2.542E-23	2.300E-23
9.9	2.081E-23	1.883E-23	1.704E-23	1.541E-23	1.394E-23	1.261E-23	1.140E-23	1.031E-23	9.323E-24	8.429E-24

Communications IV Data Sheet

1. Correlation and Power Spectrum

$$R_{xy}(t_1,t_2) = E\{y(t_1)y^*(t_2)\}$$
$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$
$$R_{xx}(-\tau) = R_{xx}(\tau)$$
$$R_{yx}(\tau) = R^*_{xy}(-\tau)$$
$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$
$$S_{yx}(f) = S^*_{xy}(f)$$

2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\,x(t-\lambda)\,d\lambda = h(t)\otimes x(t)$$
$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$
$$S_{xy}(f) = H^*(f)S_{xx}(f)$$
$$S_{yx}(f) = H(f)S_{xx}(f)$$

3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\,\hat{x}(t) \qquad (\textit{analytic signal})$$
$$X^+(f) = 2u(f)X(f)$$
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t-\lambda}\,d\lambda \qquad (\textit{hilbert transform})$$
$$\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)$$

4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty}|H(f)|^2\,df$$

5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$
$$S^{(+)}_{nn}(f) = u(f)S_{nn}(f)$$
$$S^{(-)}_{nn}(f) = u(-f)S_{nn}(f)$$
$$S_{n_c n_c}(f) = S_{n_s n_s}(f) = S^{(+)}_{nn}(f+f_o) + S^{(-)}_{nn}(f-f_o)$$
$$S_{n_c n_s}(f) = j\left\{S^{(+)}_{nn}(f+f_o) - S^{(-)}_{nn}(f-f_o)\right\}$$

6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \le 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

$$s_{base}(t) = Am(t)$$
$$B = W$$
$$SNR_{base} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

$$s_{am}(t) = A\{1 + am(t)\}\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{am} = \frac{A^2 a^2 <m^2>}{2N_o W} = \frac{a^2 <m^2>}{1+a^2 <m^2>}\frac{P_r}{N_o W}$$

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{dsbsc} = \frac{A^2 <m^2>}{2N_o W} = \frac{P_r}{N_o W}$$

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$
$$B = W$$
$$SNR_{ssbsc} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d\int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$
$$B = 2(f_d + W)$$
$$SNR_{fm} = \frac{3A^2 f_d^2 <m^2>}{2N_o W^3} = 3 <m^2> \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_o W}$$

Threshold at $\frac{P_r}{N_o B} = 10$ (10 dB)

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7. Information Theory

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$$H(x) = -\sum_i P(x_i)\log_2 P(x_i)\ bits/symbol$$

$$H(x,y) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(x_i,y_j)$$

$$H(y\,|\,x) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(y_j\,|\,x_i)$$

$$= H(x,y) - H(x)$$

$$I(x,y) = H(x) - H(x\,|\,y) = H(y) - H(y\,|\,x)$$

$$C = r\max I(x,y)\ \ bits/\sec\ (discrete\ channel)$$

$$C = B\log_2(1+P_s/P_n)\ \text{bits/sec (continuous channel)}$$

8. Digital Modulation

$$s_o(t_o) = \pm V,\ \Gamma = \frac{V^2}{<n_o^2>}\ \text{(binary antipodal signals)}$$

$$P_b = Q\left\{\sqrt{\Gamma}\right\}\ \text{(bit error, binary antipodal system)}$$

$$h(t) = c\,s(t_o - t)\ \ \text{(matched filter)}$$

$$\Gamma = \frac{2\mathrm{E_s}}{\mathrm{N_o}}\ \ (matched\ \ filter, baseband\ \&\ BPSK)$$

$$P_{sym} \approx Q\left\{\sqrt{d^2/2N_o}\right\}\ \text{(matched filter, spacing d)}$$

$$Constellation\ radius = \sqrt{\text{(symbol energy)}}$$

$$S_{xx}(f) = \frac{E\left\{a_k^2\right\}}{T}\left|P(f)\right|^2 \quad ;\ x(t) = \sum_{k=-\infty}^{\infty} a_k\ p(t-kT)$$

$$s_{QAM}(\mathrm{t}) = \sum_{\mathrm{k}=-\infty}^{\infty} A\,p(t-kT)\left[a_k\cos\omega_o t - b_k\sin\omega_o t\right]$$

$$\mathrm{s_{PSK}}(\mathrm{t}) = \sum_{\mathrm{k}=-\infty}^{\infty} A\,p(t-kT)\cos(\omega_o t + \theta_k)$$

9. Nyquist Pulse

$$W = \frac{1}{2T}$$

$$P(f) = \begin{cases} T & ;|\ f\ |<(1-\rho)W \\ 0.5T - 0.5T\sin\left(\frac{\pi(|f|-W)}{2\rho W}\right) & ;||\ f\ |-W\ |<\rho W \\ 0 & ; elsewhere \end{cases}$$

$$p(t) = \frac{\pi}{4}\mathrm{sinc}\left(\frac{t}{T}\right)\left\{\mathrm{sinc}\left(\frac{\rho t}{\mathrm{T}}+\frac{1}{2}\right) + \mathrm{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)\right\}$$

10. Coding

$$P(i) = \binom{n}{i}P_b^i\,Q_b^{n-i}\ \ \text{(binomial distribution)}$$

$$\mathrm{P_b} = \mathrm{Q}\left\{\sqrt{\mathrm{RE_b}/\alpha}\right\}\ \text{(BER before correction)}$$

$$P_{cbe} \approx \frac{2t+1}{n}\binom{n}{t+1}P_b^{t+1}\ \ \text{(BER, correcting t errors)}$$

$$\tilde{\mathrm{x}} = \tilde{\mathrm{m}}\mathrm{G},\ \ \mathrm{G} = [\mathrm{I}\ \ \mathrm{P}] = generator\ matrix$$

$$\tilde{y} = \tilde{x} + \tilde{e},\ \tilde{s} = \tilde{y}H = \text{syndrome}$$

$$H = \begin{bmatrix} P \\ I \end{bmatrix} = \text{parity check matrix}$$

$$\text{Hamming codes } \mathrm{n} = 2^{\mathrm{q}} - 1\ \text{(correct one error per block)}$$

$$2^{\mathrm{q}} \geq \sum_{\mathrm{i}=0}^{\mathrm{t}}\binom{n}{i}\text{(necessary for existence of code)}$$

$$\mathrm{X(p)} = \mathrm{p^q M(p)} + \mathrm{C(p)}\ (\textit{cyclic}\ \text{code generation})$$

$$\mathrm{C(p)} = \mathrm{rem}\left\{\frac{\mathrm{p^q}M(p)}{G(p)}\right\}\text{(check digits)}$$

$$\mathrm{S(p)} = \mathrm{rem}\left\{\frac{\mathrm{Y(p)}}{\mathrm{G(p)}}\right\}\text{(syndrome, cyclic code)}$$

11. Gaussian Probability

One dimension (mean η, variance σ^2):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-(\mathrm{x}-\eta)^2/2\sigma^2}$$

$$P\{x>V\} = Q\left\{\frac{V-\eta}{\sigma}\right\}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\,dt$$

Two dimensions (zero means):

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}\,e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right]}$$

$$\sigma_x^2 = E\{x^2\},\ \sigma_y^2 = E\{y^2\},\ \rho = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

End of Examination Paper