EXAMINATION FOR THE DEGREE OF B.E.

Semester I June 2010

101902 COMMUNICATIONS IV (ELEC ENG 4035)

Official Reading Time:	10 mins
Writing Time:	90 mins
Total Duration:	100 mins

Instructions:

- This is a closed book examination.
- Attempt **ALL THREE** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

Materials:

- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:

- Fourier Transform Sheet
- Table of the Q Function
- Communications IV Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Total number of pages (including attachments) = 10

Question 1

20 marks total

1a) A message signal $m(t) = \cos(2000\,\pi t) + 2\cos(4000\,\pi t)$ modulates a carrier $c(t) = 100\cos(2\pi f_c t)$ (where $f_c = 20$ kHz) to produce a double sideband suppressed carrier modulated (DSBSC) signal $v(t) = m(t)\,c(t)$.

(i) Calculate an expression for $v(t)$. **(1 mark)**

(ii) Calculate the Fourier transform $M(f)$ of $m(t)$ and sketch it, showing clearly the magnitudes of the various components. **(2 marks)**

(iii) Calculate the Fourier transform $V(f)$ of $v(t)$ and sketch it, showing clearly the magnitudes of the various components. **(3 marks)**

(iv) Determine an expression for the upper sideband component of $v(t)$. Note that this is a real signal. **(2 marks)**

(v) Determine an expression for the spectrum of the upper sideband component of $v(t)$ and sketch it, showing clearly the magnitudes of the various components.
(2 marks)

[Your sketches should show magnitudes of frequency components on the frequency axis].

1b) Consider a stationary random signal $x(t)$ with power spectral density $S_{xx}(f)$, which is passed through the system shown in Figure 1 giving rise to the output signal $y(t)$.

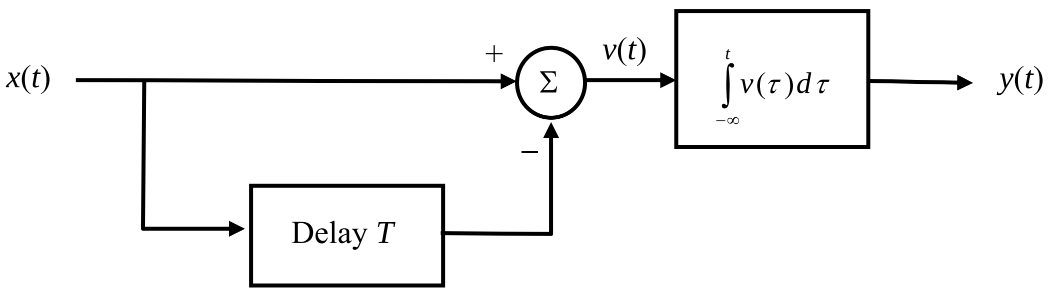


Figure 1

(i) Express the output $y(t)$ in terms of $x(t)$. **(2 marks)**

(ii) Calculate the transfer function $H(f)$ of the system.
Hint: you will need to recall the identity, $\sin\theta = (e^{j\theta} - e^{-j\theta})/2j$. **(4 marks)**

(iii) Calculate the power spectral density $S_{yy}(f)$ of $y(t)$ and sketch it for $S_{xx}(f) = 1$.
(3 marks)

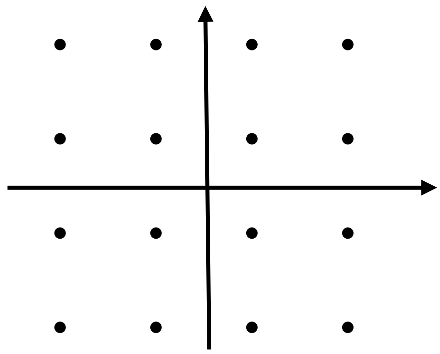
(iv) When $S_{xx}(f) = 1$, what frequency components are absent in the output?
(1 mark)

Question 2 follows on Page 3

Question 2

20 marks total

Q2a) A 16QAM communications system uses a square 4×4 constellation, as shown below, such that the symbols are equally spaced by *d* both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is $R = 50\ \Omega$.



(i) Determine the *minimum* bandwidth *B* required for transmission. **(2 marks)**

(ii) Calculate E_b, the average energy per bit (in V^2s), for the constellation in terms of the separation distance *d* (the horizontal and vertical separation of the symbols). **Hint:** the energy of each symbol in the constellation is given by the square of its distance from the origin.
(3 marks)

(iii) If the average received power $P_r = 1.2 \times 10^{-6}$ W, determine the symbol spacing *d*.
(3 marks)

(iv) If outermost corner symbols have an energy of 9×10^{-10} J, write down an expression for the corner symbol energy d^2 in units of V^2s, as a function of the receiver input impedance *R*. If the noise power spectral density, $N_o = 1 \times 10^{-11}$ W/Hz, also write down an expression for the noise energy in units of V^2s, as a function of the receiver input impedance *R*. Then determine the probability of selecting an adjacent symbol in error during demodulation, given that a matched filter receiver is used. **(4 marks)**

Q2b) A BPSK (binary phase shift keyed) system is used to transmit the same bit rate as in part a), but this time the receiver input impedance, *R*, is unknown.

(i) Determine the minimum bandwidth *B* required for transmission. **(2 marks)**

(ii) For an average received power $P_r = 1.2 \times 10^{-6}$ W and noise spectral density $N_o = 1 \times 10^{-11}$ W/Hz, determine the probability of error for a matched filter receiver.
(4 marks)

(iii) Comment on the differences between BPSK and 16QAM as revealed by your answers.
(2 marks)

Question 3 follows on Page 4

Question3

20 marks total

3a) A source X has an alphabet {A, B, C, D, E, F} with corresponding probabilities {0.50, 0.15, 0.12, 0.10, 0.08, 0.05}.

(i) Calculate the source entropy $H(x)$ in bits and explain what this means.

(3 marks)

(ii) Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of six symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i). **(2 marks)**

(iii) Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency*. **(4 marks)**

(iv) Explain why it is possible to uniquely decipher a Huffman code. **(2 marks)**

3b) A BPSK (binary phase shift keyed) digital transmission system transmits data at 10^6 symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to $P_e = Q\left\{\sqrt{2E_c / N_o}\right\} = 10^{-4}$, where E_c is the energy per transmitted channel bit and $N_o/2$ is the spectral density of the accompanying additive white Gaussian noise (AWGN). Error correction is achieved by using a (15,11) Hamming block code.

(i) Calculate the value of E_c/N_o. **(1 mark)**

(ii) How many errors can the Hamming code *correct* in each block of 15?

(.5 mark)

(iii) Up to how many errors can the Hamming code *detect* in each block of 15?

(.5 mark)

(iv) What is the minimum bandwidth required to transmit the signal? **(1 mark)**

(v) Calculate the bit error probability after error correction. **(3 marks)**

(vi) If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to $(15/11) \times 10^6$ symbols/sec so that the message bit rate is 10^6 bits/sec, calculate the corrected probability of a bit error. (**Hint:** Calculate the reduced value of E_c/N_o, and hence the new value of p, the uncorrected probability of a bit error).

(3 marks)

End of Examination Questions

Data Sheets follow on Pages 5 – 10

Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\; dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\; df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\|T\|\,X(fT)$
$x(t-T)$	$X(f)\,e^{-j2\pi f T}$
$x(t)\,e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\dfrac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\,\delta(f)$
$t\, x(t)$	$-\dfrac{1}{j2\pi}\dfrac{dX(f)}{df}$
$X(t)$	$x(-f)$
$\mathrm{rep}_T\{x(t)\}$	$\|F\|\,\mathrm{comb}_F(f)\, X(f)$
$\|T\|\,\mathrm{comb}_T(t)\, x(t)$	$\mathrm{rep}_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)\, Y(f)$
$x^*(t)$	$X^*(-f)$

Note that F and T are real constants, with $FT = 1$.

Transforms

$u(t)\, e^{-at}$	$\dfrac{1}{a + j2\pi f}$; $a > 0$
$e^{-a\|t\|}$	$\dfrac{2a}{a^2 + (2\pi f)^2}$; $a > 0$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a} e^{-\|2\pi f a\|}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$\mathrm{sgn}(t)$	$\dfrac{1}{j\pi f}$
$\dfrac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$\mathrm{rect}(t/T)$	$\|T\|\,\mathrm{sinc}(fT)$
$\mathrm{sinc}(t/T)$	$\|T\|\,\mathrm{rect}(fT)$
$\Delta(t/T)$	$\|T\|\,\mathrm{sinc}^2(fT)$
$\mathrm{comb}_T(t)$	$\|F\|\,\mathrm{comb}_F(f)$
$e^{-t^2/2T^2}$	$\|T\|\sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$\mathrm{sgn}(t)\,\mathrm{rect}(t/T)$	$\dfrac{1 - \cos(\pi f T)}{j\pi f}$

Note that a is a real positive constant.

Definitions

$$\text{sinc}(x) = \frac{\sin(\pi\, x)}{\pi\, x}$$

$$\text{rep}_{\text{P}}\{\text{f(x)}\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\text{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\text{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\text{rect}(x) = \begin{cases} 1 & ; |\,x\,| < 0.5 \\ 0 & ; |\,x\,| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |\,x\,| & ; |\,x\,| < 1 \\ 0 & ; |\,x\,| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of x(t)}$$

$$X(-f) = X^*(f) \text{ if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t) = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |\,x(t)\,|^2\, dt = \int_{-\infty}^{\infty} |\,X(f)\,|^2\, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.676E-04	9.354E-04	9.043E-04	8.740E-04	8.447E-04	8.164E-04	7.888E-04	7.622E-04	7.364E-04	7.114E-04
3.2	6.871E-04	6.637E-04	6.410E-04	6.190E-04	5.976E-04	5.770E-04	5.571E-04	5.377E-04	5.190E-04	5.009E-04
3.3	4.834E-04	4.665E-04	4.501E-04	4.342E-04	4.189E-04	4.041E-04	3.897E-04	3.758E-04	3.624E-04	3.495E-04
3.4	3.369E-04	3.248E-04	3.131E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.602E-04	2.507E-04	2.415E-04
3.5	2.326E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.926E-04	1.854E-04	1.785E-04	1.718E-04	1.653E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.363E-04	1.311E-04	1.261E-04	1.213E-04	1.166E-04	1.121E-04
3.7	1.078E-04	1.036E-04	9.961E-05	9.574E-05	9.201E-05	8.842E-05	8.496E-05	8.162E-05	7.841E-05	7.532E-05
3.8	7.235E-05	6.948E-05	6.673E-05	6.407E-05	6.152E-05	5.906E-05	5.669E-05	5.442E-05	5.223E-05	5.012E-05
3.9	4.810E-05	4.615E-05	4.427E-05	4.247E-05	4.074E-05	3.908E-05	3.747E-05	3.594E-05	3.446E-05	3.304E-05
4.0	3.167E-05	3.036E-05	2.910E-05	2.789E-05	2.673E-05	2.561E-05	2.454E-05	2.351E-05	2.252E-05	2.157E-05
4.1	2.066E-05	1.978E-05	1.894E-05	1.814E-05	1.737E-05	1.662E-05	1.591E-05	1.523E-05	1.458E-05	1.395E-05
4.2	1.335E-05	1.277E-05	1.222E-05	1.168E-05	1.118E-05	1.069E-05	1.022E-05	9.774E-06	9.345E-06	8.934E-06
4.3	8.540E-06	8.163E-06	7.801E-06	7.455E-06	7.124E-06	6.807E-06	6.503E-06	6.212E-06	5.934E-06	5.668E-06
4.4	5.413E-06	5.169E-06	4.935E-06	4.712E-06	4.498E-06	4.294E-06	4.098E-06	3.911E-06	3.732E-06	3.561E-06
4.5	3.398E-06	3.241E-06	3.092E-06	2.949E-06	2.813E-06	2.682E-06	2.558E-06	2.439E-06	2.325E-06	2.216E-06
4.6	2.112E-06	2.013E-06	1.919E-06	1.828E-06	1.742E-06	1.660E-06	1.581E-06	1.506E-06	1.434E-06	1.366E-06
4.7	1.301E-06	1.239E-06	1.179E-06	1.123E-06	1.069E-06	1.017E-06	9.680E-07	9.211E-07	8.765E-07	8.339E-07
4.8	7.933E-07	7.547E-07	7.178E-07	6.827E-07	6.492E-07	6.173E-07	5.869E-07	5.580E-07	5.304E-07	5.042E-07
4.9	4.792E-07	4.554E-07	4.327E-07	4.111E-07	3.906E-07	3.711E-07	3.525E-07	3.348E-07	3.179E-07	3.019E-07

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	2.867E-07	2.722E-07	2.584E-07	2.452E-07	2.328E-07	2.209E-07	2.096E-07	1.989E-07	1.887E-07	1.790E-07
5.1	1.698E-07	1.611E-07	1.528E-07	1.449E-07	1.374E-07	1.302E-07	1.235E-07	1.170E-07	1.109E-07	1.051E-07
5.2	9.964E-08	9.442E-08	8.946E-08	8.476E-08	8.029E-08	7.605E-08	7.203E-08	6.821E-08	6.459E-08	6.116E-08
5.3	5.790E-08	5.481E-08	5.188E-08	4.911E-08	4.647E-08	4.398E-08	4.161E-08	3.937E-08	3.724E-08	3.523E-08
5.4	3.332E-08	3.151E-08	2.980E-08	2.818E-08	2.664E-08	2.518E-08	2.381E-08	2.250E-08	2.127E-08	2.010E-08
5.5	1.899E-08	1.794E-08	1.695E-08	1.601E-08	1.512E-08	1.428E-08	1.349E-08	1.274E-08	1.203E-08	1.135E-08
5.6	1.072E-08	1.012E-08	9.548E-09	9.010E-09	8.503E-09	8.022E-09	7.569E-09	7.140E-09	6.735E-09	6.352E-09
5.7	5.990E-09	5.649E-09	5.326E-09	5.022E-09	4.734E-09	4.462E-09	4.206E-09	3.964E-09	3.735E-09	3.519E-09
5.8	3.316E-09	3.124E-09	2.942E-09	2.771E-09	2.610E-09	2.458E-09	2.314E-09	2.179E-09	2.051E-09	1.931E-09
5.9	1.818E-09	1.711E-09	1.610E-09	1.515E-09	1.425E-09	1.341E-09	1.261E-09	1.186E-09	1.116E-09	1.049E-09
6.0	9.866E-10	9.276E-10	8.721E-10	8.198E-10	7.706E-10	7.242E-10	6.806E-10	6.396E-10	6.009E-10	5.646E-10
6.1	5.303E-10	4.982E-10	4.679E-10	4.394E-10	4.126E-10	3.874E-10	3.637E-10	3.414E-10	3.205E-10	3.008E-10
6.2	2.823E-10	2.649E-10	2.486E-10	2.332E-10	2.188E-10	2.052E-10	1.925E-10	1.805E-10	1.693E-10	1.587E-10
6.3	1.488E-10	1.395E-10	1.308E-10	1.226E-10	1.149E-10	1.077E-10	1.009E-10	9.451E-11	8.854E-11	8.294E-11
6.4	7.769E-11	7.276E-11	6.814E-11	6.380E-11	5.974E-11	5.593E-11	5.235E-11	4.900E-11	4.586E-11	4.292E-11
6.5	4.016E-11	3.758E-11	3.515E-11	3.288E-11	3.076E-11	2.877E-11	2.690E-11	2.516E-11	2.352E-11	2.199E-11
6.6	2.056E-11	1.922E-11	1.796E-11	1.678E-11	1.568E-11	1.465E-11	1.369E-11	1.279E-11	1.195E-11	1.116E-11
6.7	1.042E-11	9.731E-12	9.086E-12	8.483E-12	7.919E-12	7.392E-12	6.900E-12	6.439E-12	6.009E-12	5.607E-12
6.8	5.231E-12	4.880E-12	4.552E-12	4.246E-12	3.960E-12	3.692E-12	3.443E-12	3.210E-12	2.993E-12	2.790E-12
6.9	2.600E-12	2.423E-12	2.258E-12	2.104E-12	1.960E-12	1.826E-12	1.701E-12	1.585E-12	1.476E-12	1.374E-12
7.0	1.280E-12	1.192E-12	1.109E-12	1.033E-12	9.612E-13	8.946E-13	8.325E-13	7.747E-13	7.208E-13	6.706E-13
7.1	6.238E-13	5.802E-13	5.396E-13	5.018E-13	4.667E-13	4.339E-13	4.034E-13	3.750E-13	3.486E-13	3.240E-13
7.2	3.011E-13	2.798E-13	2.599E-13	2.415E-13	2.243E-13	2.084E-13	1.935E-13	1.797E-13	1.669E-13	1.550E-13
7.3	1.439E-13	1.336E-13	1.240E-13	1.151E-13	1.068E-13	9.910E-14	9.196E-14	8.531E-14	7.914E-14	7.341E-14
7.4	6.809E-14	6.315E-14	5.856E-14	5.430E-14	5.034E-14	4.667E-14	4.326E-14	4.010E-14	3.716E-14	3.444E-14
7.5	3.191E-14	2.956E-14	2.739E-14	2.537E-14	2.350E-14	2.176E-14	2.015E-14	1.866E-14	1.728E-14	1.600E-14
7.6	1.481E-14	1.370E-14	1.268E-14	1.174E-14	1.086E-14	1.005E-14	9.297E-15	8.600E-15	7.954E-15	7.357E-15
7.7	6.803E-15	6.291E-15	5.816E-15	5.377E-15	4.971E-15	4.595E-15	4.246E-15	3.924E-15	3.626E-15	3.350E-15
7.8	3.095E-15	2.859E-15	2.641E-15	2.439E-15	2.253E-15	2.080E-15	1.921E-15	1.773E-15	1.637E-15	1.511E-15
7.9	1.395E-15	1.287E-15	1.188E-15	1.096E-15	1.011E-15	9.326E-16	8.602E-16	7.934E-16	7.317E-16	6.747E-16
8.0	6.221E-16	5.735E-16	5.287E-16	4.874E-16	4.492E-16	4.140E-16	3.815E-16	3.515E-16	3.238E-16	2.983E-16
8.1	2.748E-16	2.531E-16	2.331E-16	2.146E-16	1.976E-16	1.820E-16	1.675E-16	1.542E-16	1.419E-16	1.306E-16
8.2	1.202E-16	1.106E-16	1.018E-16	9.361E-17	8.611E-17	7.920E-17	7.284E-17	6.698E-17	6.159E-17	5.662E-17
8.3	5.206E-17	4.785E-17	4.398E-17	4.042E-17	3.715E-17	3.413E-17	3.136E-17	2.881E-17	2.646E-17	2.431E-17
8.4	2.232E-17	2.050E-17	1.882E-17	1.728E-17	1.587E-17	1.457E-17	1.337E-17	1.227E-17	1.126E-17	1.033E-17
8.5	9.480E-18	8.697E-18	7.978E-18	7.317E-18	6.711E-18	6.154E-18	5.643E-18	5.174E-18	4.744E-18	4.348E-18
8.6	3.986E-18	3.653E-18	3.348E-18	3.068E-18	2.811E-18	2.575E-18	2.359E-18	2.161E-18	1.979E-18	1.812E-18
8.7	1.659E-18	1.519E-18	1.391E-18	1.273E-18	1.166E-18	1.067E-18	9.763E-19	8.933E-19	8.174E-19	7.478E-19
8.8	6.841E-19	6.257E-19	5.723E-19	5.234E-19	4.786E-19	4.376E-19	4.001E-19	3.657E-19	3.343E-19	3.055E-19
8.9	2.792E-19	2.552E-19	2.331E-19	2.130E-19	1.946E-19	1.777E-19	1.623E-19	1.483E-19	1.354E-19	1.236E-19
9.0	1.129E-19	1.030E-19	9.404E-20	8.584E-20	7.834E-20	7.148E-20	6.523E-20	5.951E-20	5.429E-20	4.952E-20
9.1	4.517E-20	4.119E-20	3.756E-20	3.425E-20	3.123E-20	2.847E-20	2.595E-20	2.365E-20	2.155E-20	1.964E-20
9.2	1.790E-20	1.631E-20	1.486E-20	1.353E-20	1.232E-20	1.122E-20	1.022E-20	9.307E-21	8.474E-21	7.714E-21
9.3	7.022E-21	6.392E-21	5.817E-21	5.294E-21	4.817E-21	4.382E-21	3.987E-21	3.627E-21	3.299E-21	3.000E-21
9.4	2.728E-21	2.481E-21	2.255E-21	2.050E-21	1.864E-21	1.694E-21	1.540E-21	1.399E-21	1.271E-21	1.155E-21
9.5	1.049E-21	9.533E-22	8.659E-22	7.864E-22	7.142E-22	6.485E-22	5.888E-22	5.345E-22	4.852E-22	4.404E-22
9.6	3.997E-22	3.627E-22	3.292E-22	2.986E-22	2.709E-22	2.458E-22	2.229E-22	2.022E-22	1.834E-22	1.663E-22
9.7	1.507E-22	1.367E-22	1.239E-22	1.123E-22	1.018E-22	9.223E-23	8.358E-23	7.573E-23	6.861E-23	6.215E-23
9.8	5.629E-23	5.098E-23	4.617E-23	4.181E-23	3.786E-23	3.427E-23	3.102E-23	2.808E-23	2.542E-23	2.300E-23
9.9	2.081E-23	1.883E-23	1.704E-23	1.541E-23	1.394E-23	1.261E-23	1.140E-23	1.031E-23	9.323E-24	8.429E-24

Communications IV Data Sheet

1. Correlation and Power Spectrum

$$R_{xy}(t_1,t_2) = E\{y(t_1)y^*(t_2)\}$$
$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$
$$R_{xx}(-\tau) = R_{xx}(\tau)$$
$$R_{yx}(\tau) = R_{xy}^*(-\tau)$$
$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$
$$S_{yx}(f) = S_{xy}^*(f)$$

2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\,x(t-\lambda)\,d\lambda = h(t)\otimes x(t)$$
$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$
$$S_{xy}(f) = H^*(f)S_{xx}(f)$$
$$S_{yx}(f) = H(f)S_{xx}(f)$$

3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\,\hat{x}(t) \qquad (analytic\ signal)$$
$$X^+(f) = 2u(f)X(f)$$
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t-\lambda}d\lambda \qquad (hilbert\ transform)$$
$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty}|H(f)|^2\,df$$

5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$
$$S_{nn}^{(+)}(f) = u(f)S_{nn}(f)$$
$$S_{nn}^{(-)}(f) = u(-f)S_{nn}(f)$$
$$S_{n_c n_c}(f) = S_{n_s n_s}(f) = S_{nn}^{(+)}(f+f_o) + S_{nn}^{(-)}(f-f_o)$$
$$S_{n_c n_s}(f) = j\left\{S_{nn}^{(+)}(f+f_o) - S_{nn}^{(-)}(f-f_o)\right\}$$

6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \le 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

$$s_{base}(t) = Am(t)$$
$$B = W$$
$$SNR_{base} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

$$s_{am}(t) = A\{1 + am(t)\}\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{am} = \frac{A^2a^2<m^2>}{2N_oW} = \frac{a^2<m^2>}{1+a^2<m^2>}\frac{P_r}{N_oW}$$

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{dsbsc} = \frac{A^2<m^2>}{2N_oW} = \frac{P_r}{N_oW}$$

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$
$$B = W$$
$$SNR_{ssbsc} = \frac{A^2<m^2>}{N_oW} = \frac{P_r}{N_oW}$$

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d\int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$
$$B = 2(f_d + W)$$
$$SNR_{fm} = \frac{3A^2 f_d^2 <m^2>}{2N_oW^3} = 3<m^2>\left(\frac{f_d}{W}\right)^2\frac{P_r}{N_oW}$$

Threshold at $\frac{P_r}{N_oB} = 10$ (10 dB)

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7. Information Theory

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$$H(x) = -\sum_i P(x_i)\log_2 P(x_i)\ bits/symbol$$

$$H(x,y) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(x_i,y_j)$$

$$H(y|x) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(y_j|x_i)$$

$$= H(x,y) - H(x)$$

$$I(x,y) = H(x) - H(x|y) = H(y) - H(y|x)$$

$$C = r\max I(x,y)\ \ bits/\text{sec}\ (discrete\ channel)$$

$$C = B\log_2(1+P_s/P_n)\,\text{bits/sec (continuous channel)}$$

8. Digital Modulation

$$s_o(t_o) = \pm V,\ \Gamma = \frac{V^2}{<n_o^2>}\ \text{(binary antipodal signals)}$$

$$P_b = Q\left\{\sqrt{\Gamma}\right\}\ \text{(bit error, binary antipodal system)}$$

$$h(t) = c\,s(t_o - t)\ \ \text{(matched filter)}$$

$$\Gamma = \frac{2\mathrm{E_s}}{\mathrm{N_o}}\ \ (matched\ \ filter, baseband\ \&\ BPSK)$$

$$P_{sym} \approx Q\left\{\sqrt{d^2/2N_o}\right\}\ \text{(matched filter, spacing d)}$$

$$Constellation\ radius = \sqrt{\text{(symbol energy)}}$$

$$S_{xx}(f) = \frac{E\left\{a_k^2\right\}}{T}\left|P(f)\right|^2 \quad ;\ x(t) = \sum_{k=-\infty}^{\infty} a_k\ p(t-kT)$$

$$s_{QAM}(\mathrm{t}) = \sum_{\mathrm{k}=-\infty}^{\infty} A\,p(t-kT)\left[a_k\cos\omega_o t - b_k\sin\omega_o t\right]$$

$$\mathrm{s_{PSK}}(\mathrm{t}) = \sum_{\mathrm{k}=-\infty}^{\infty} A\,p(t-kT)\cos(\omega_o t + \theta_k)$$

9. Nyquist Pulse

$$W = \frac{1}{2T}$$

$$P(f) = \begin{cases} T & ;|\ f\ | < (1-\rho)W \\ 0.5T - 0.5T\sin\left(\frac{\pi(|f|-W)}{2\rho W}\right) & ;||\ f\ | - W| < \rho W \\ 0 & ;elsewhere \end{cases}$$

$$p(t) = \frac{\pi}{4}\mathrm{sinc}\left(\frac{t}{T}\right)\left\{\mathrm{sinc}\left(\frac{\rho t}{\mathrm{T}}+\frac{1}{2}\right) + \mathrm{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)\right\}$$

10. Coding

$$P(i) = \binom{n}{i}P_b^i\,Q_b^{n-i}\ \ \text{(binomial distribution)}$$

$$\mathrm{P_b} = \mathrm{Q}\left\{\sqrt{\mathrm{RE_b}/\alpha}\right\}\text{(BER before correction)}$$

$$P_{cbe} \approx \tfrac{2t+1}{n}\binom{n}{t+1}P_b^{t+1}\ \text{(BER, correcting t errors)}$$

$$\tilde{\mathrm{x}} = \tilde{\mathrm{m}}\mathrm{G},\ \ \mathrm{G} = [\mathrm{I\ \ P}] = generator\ matrix$$

$$\tilde{y} = \tilde{x} + \tilde{e},\ \tilde{s} = \tilde{y}H = \text{syndrome}$$

$$H = \begin{bmatrix} P \\ I \end{bmatrix} = \text{parity check matrix}$$

$$\text{Hamming codes } \mathrm{n} = 2^{\mathrm{q}} - 1\ \text{(correct one error per block)}$$

$$2^{\mathrm{q}} \geq \sum_{\mathrm{i}=0}^{\mathrm{t}}\binom{n}{i}\text{(necessary for existence of code)}$$

$$\mathrm{X(p)} = \mathrm{p^q M(p)} + \mathrm{C(p)}\ (cyclic\ \text{code generation})$$

$$\mathrm{C(p)} = \mathrm{rem}\left\{\frac{\mathrm{p^q}M(p)}{G(p)}\right\}\text{(check digits)}$$

$$\mathrm{S(p)} = \mathrm{rem}\left\{\frac{\mathrm{Y(p)}}{\mathrm{G(p)}}\right\}\text{(syndrome, cyclic code)}$$

11. Gaussian Probability

One dimension (mean η, variance σ^2):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-(\mathrm{x}-\eta)^2/2\sigma^2}$$

$$P\{x > V\} = Q\left\{\frac{V-\eta}{\sigma}\right\}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\,dt$$

Two dimensions (zero means):

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}\,e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right]}$$

$$\sigma_x^2 = E\{x^2\},\ \sigma_y^2 = E\{y^2\},\ \rho = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

End of Examination Paper