THE UNIVERSITY OF ADELAIDE
AUSTRALIA

EXAMINATION FOR THE DEGREE OF B.E. and M.E.

Semester I 2012

105973 COMMUNICATIONS (ELEC ENG 4063)
105980 PRINCIPLES OF COMUNICATION SYSTEMS (ELEC ENG 7080)

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

Instructions:

- This is a closed book examination.
- Attempt **ALL FOUR** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

Materials:

- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:

- Fourier Transform Sheet
- Table of the Q Function
- Communications Data Sheet
- Optical Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Question 1 follows on page 2.

Question 1

20 marks total

A video signal having a bandwidth of 4.8 MHz is transmitted over a 10-km path. We want to design a system so that the SNR at the receiver is 48 dB. Analog modulation is used. Spectral wavelength used is $\lambda_0 = 1.3$ μm.

The receiver is an InGaAs PIN photodiode.
Responsivity; $\rho = 0.6$ A/W
Dark Current; $I_d = 5$ nA
Junction capacitance; $C_d = 5$ pF
Noise figure; $F = 2$ at 300 K
Assume 100% modulation ($m = 1$).

(a) Calculate the load resistor R_L for the receiver. Comment on why you would not use this value in practice. **(2 marks)**

(b) Assume the system is thermal noise limited and hence calculate the power needed at the photodiode receiver to achieve the specified SNR, using the value of R_L calculated in (a). **(6 marks)**

(c) The available power from a laser diode source is $P_{ave} = 10$ mW. What is the available power budget left over for losses? **(3 marks)**

(d) Calculate the signal current. Assuming 4 V reverse bias on the photodiode, demonstrate if saturation and dark current will be negligible or not. **(4 marks)**

(e) Calculate the thermal noise and shot noise powers, hence demonstrate if the assumption in (b) was justified or not. **(5 marks)**

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

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Question 2

20 marks total

Consider a single mode fibre operating at 1550 nm. The specification is to transmit 400 Mbps NRZ data over the 100 km fibre link with a bit error rate (BER) of 10^{-9} or better.

(a) Given that $\tau = 1/R_{NRZ}$ and allowing for the system rise time to be 70% of τ, compute the system rise time t_S.

(1 mark)

(b) Given a material dispersion of M = -20 ps/(nm x km) and a waveguide dispersion of M_g = 4.5 ps/(nm x km), compute the fibre rise time, t_F. The spectral linewidth is given as 0.15 nm.

(1 mark)

(c) If the rise time of the light source is $t_{LS} = 1$ ns, find an upper limit on the photodetector rise time t_{PD}.

(4 marks)

(d) Assume a basic BJT amplifier circuit at the receiver. If the photodetector has a transit time limited rise time of $t_{TR} = 0.5$ ns and a junction capacitance of $C_d = 1$ pF, calculate an upper limit on the load resistor.

(2 marks)

(e) Given that the fibre loss is 0.25 dB/km, the coupling efficiency to the fibre is 3 dB, there are two connectors with 1 dB loss each, there are 50 splices with 0.1 dB loss each and that the source power is 5 dBm, find the power at the receiver.

(2 marks)

(f) Calculate the optical power needed to achieve the specified BER assuming a quantum limited system. Comment on the result. You may assume dark current is negligible and therefore the expression for probability of an error $P_e = e^{-n_s}$ holds. The quantum efficiency of the detector is η = 0.7.

(4 marks)

(g) Now calculate the optical power needed to achieve the specified BER assuming a thermal limited system. Comment on the result. You may assume a detector responsivity of ρ = 1 A/W and a noise figure of F = 2. Room temperature conditions hold, thus let T = 300 K.

(6 marks)

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

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Question 3

20 marks total

3a) A BPSK (binary phase shift keyed) digital transmission system transmits data at 10^6 symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to $P_e = Q\left\{\sqrt{2E_c / N_o}\right\}$ = 10^{-5}, where E_c is the energy per transmitted channel bit and $N_o/2$ is the spectral density of the accompanying additive white Gaussian noise (AWGN). Error correction is achieved by using a (31,26) Hamming block code.

(i) Calculate the value of E_c/N_o. **(1 mark)**

(ii) How many errors can the Hamming code *correct* in each block of 31? **(1 mark)**

(iii) Up to how many errors can the Hamming code *detect* in each block of 31? **(1 mark)**

(iv) What is the minimum bandwidth required to transmit the signal? **(1 mark)**

(v) Calculate the bit error probability after error correction. **(2 marks)**

(vi) If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to (31/26) × 10^6 symbols/sec so that the message bit rate is 10^6 bits/sec, calculate the corrected probability of a bit error. (**Hint:** Calculate the reduced value of E_c/N_o, and hence the new value of p, the uncorrected probability of a bit error). **(3 marks)**

3b) A source X has an alphabet {A, B, C, D, E} with corresponding probabilities {0.20, 0.15, 0.05, 0.10, 0.50}.

(i) Calculate the source entropy $H(x)$ in bits and explain what this means. **(3 marks)**

(ii) Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of five symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i). **(2 marks)**

(iii) Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency*. **(4 marks)**

(iv) Explain why it is possible to uniquely decipher a Huffman code. **(2 marks)**

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

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Question 4 **20 marks total**

4a)

(i) State three facts that any given two-dimensional signal constellation diagram indicates about a digital modulation scheme. **(3 marks)**

(ii) A 16QAM digital modulation system consists of $M = 16$ symbols. If the system can transmit 38400 bits per second, determine the minimum bandwidth required for transmission.

(2 marks)

(iii) A BPSK (Binary Phase Shift Keyed) system is used to transmit data at a rate of 4800 bits per second. For an average received power $P = 6.00 \times 10^{-7}$ W and additive white Gaussian noise, with power spectral density with $N_o = 1.56 \times 10^{-7}$ W/Hz, determine the probability of error for a matched filter receiver. **(3 marks)**

4b)

(i) Briefly state and describe two factors that are important when choosing the shape of a pulse for the transmission of digital data in a bandlimited communications system.

(2 marks)

(ii) An 'eye diagram' can provide an indication of the Inter-Symbol Interference (ISI) properties of a pulse shape. Briefly describe what the width of the 'eye' in an eye diagram indicates, and what the height of the 'eye' in an eye diagram indicates.

(2 marks)

4c) A voice band telephone channel passes frequencies in the range 300 Hz to 3300 Hz. It is desired to design a modem using Nyquist pulses and which is capable of transmitting digital data at a symbol rate of 2400 symbols per second and a data rate of 4800 bits per second.

(i) Select an appropriate QAM (Quadrature Amplitude Modulation) signal constellation, a suitable sub-carrier frequency (i.e. in the range 300-3300 Hz) and an appropriate roll-off factor ρ for Nyquist pulses, assuming the whole frequency band is utilised. **(5 marks)**

(ii) Sketch the spectrum of the transmitted signal, showing clearly the carrier frequency and the frequencies where the Nyquist roll-off begins and ends. **(3 marks)**

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

End of Examination Questions

Data Sheets follow

Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\lvert T\rvert X(fT)$
$x(t-T)$	$X(f)e^{-j2\pi f T}$
$x(t)e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\dfrac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
$t\,x(t)$	$-\dfrac{1}{j2\pi}\dfrac{dX(f)}{df}$
$X(t)$	$x(-f)$
$\text{rep}_T\{x(t)\}$	$\lvert F\rvert \text{comb}_F(f)\, X(f)$
$\lvert T\rvert \text{comb}_T(t)\, x(t)$	$\text{rep}_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)\, Y(f)$
$x^*(t)$	$X^*(-f)$

Note that F and T are real constants, with $FT = 1$.

Transforms

$u(t)e^{-at}$	$\dfrac{1}{a + j2\pi f}$;$a > 0$
$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2 + (2\pi f)^2}$;$a > 0$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a} e^{-\lvert 2\pi f a\rvert}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$\text{sgn}(t)$	$\dfrac{1}{j\pi f}$
$\dfrac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$\text{rect}(t/T)$	$\lvert T\rvert \text{sinc}(fT)$
$\text{sinc}(t/T)$	$\lvert T\rvert \text{rect}(fT)$
$\Delta(t/T)$	$\lvert T\rvert \text{sinc}^2(fT)$
$\text{comb}_T(t)$	$\lvert F\rvert \text{comb}_F(f)$
$e^{-t^2/2T^2}$	$\lvert T\rvert \sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$\text{sgn}(t)\,\text{rect}(t/T)$	$\dfrac{1 - \cos(\pi f T)}{j\pi f}$

Note that a is a real positive constant.

Definitions

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\mathrm{rep}_P\{f(x)\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\mathrm{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\mathrm{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of } x(t)$$

$$X(-f) = X^*(f) \quad \text{if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) = \int_{-\infty}^{\infty} X(f) Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.676E-04	9.354E-04	9.043E-04	8.740E-04	8.447E-04	8.164E-04	7.888E-04	7.622E-04	7.364E-04	7.114E-04
3.2	6.871E-04	6.637E-04	6.410E-04	6.190E-04	5.976E-04	5.770E-04	5.571E-04	5.377E-04	5.190E-04	5.009E-04
3.3	4.834E-04	4.665E-04	4.501E-04	4.342E-04	4.189E-04	4.041E-04	3.897E-04	3.758E-04	3.624E-04	3.495E-04
3.4	3.369E-04	3.248E-04	3.131E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.602E-04	2.507E-04	2.415E-04
3.5	2.326E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.926E-04	1.854E-04	1.785E-04	1.718E-04	1.653E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.363E-04	1.311E-04	1.261E-04	1.213E-04	1.166E-04	1.121E-04
3.7	1.078E-04	1.036E-04	9.961E-05	9.574E-05	9.201E-05	8.842E-05	8.496E-05	8.162E-05	7.841E-05	7.532E-05
3.8	7.235E-05	6.948E-05	6.673E-05	6.407E-05	6.152E-05	5.906E-05	5.669E-05	5.442E-05	5.223E-05	5.012E-05
3.9	4.810E-05	4.615E-05	4.427E-05	4.247E-05	4.074E-05	3.908E-05	3.747E-05	3.594E-05	3.446E-05	3.304E-05
4.0	3.167E-05	3.036E-05	2.910E-05	2.789E-05	2.673E-05	2.561E-05	2.454E-05	2.351E-05	2.252E-05	2.157E-05
4.1	2.066E-05	1.978E-05	1.894E-05	1.814E-05	1.737E-05	1.662E-05	1.591E-05	1.523E-05	1.458E-05	1.395E-05
4.2	1.335E-05	1.277E-05	1.222E-05	1.168E-05	1.118E-05	1.069E-05	1.022E-05	9.774E-06	9.345E-06	8.934E-06
4.3	8.540E-06	8.163E-06	7.801E-06	7.455E-06	7.124E-06	6.807E-06	6.503E-06	6.212E-06	5.934E-06	5.668E-06
4.4	5.413E-06	5.169E-06	4.935E-06	4.712E-06	4.498E-06	4.294E-06	4.098E-06	3.911E-06	3.732E-06	3.561E-06
4.5	3.398E-06	3.241E-06	3.092E-06	2.949E-06	2.813E-06	2.682E-06	2.558E-06	2.439E-06	2.325E-06	2.216E-06
4.6	2.112E-06	2.013E-06	1.919E-06	1.828E-06	1.742E-06	1.660E-06	1.581E-06	1.506E-06	1.434E-06	1.366E-06
4.7	1.301E-06	1.239E-06	1.179E-06	1.123E-06	1.069E-06	1.017E-06	9.680E-07	9.211E-07	8.765E-07	8.339E-07
4.8	7.933E-07	7.547E-07	7.178E-07	6.827E-07	6.492E-07	6.173E-07	5.869E-07	5.580E-07	5.304E-07	5.042E-07
4.9	4.792E-07	4.554E-07	4.327E-07	4.111E-07	3.906E-07	3.711E-07	3.525E-07	3.348E-07	3.179E-07	3.019E-07

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	2.867E-07	2.722E-07	2.584E-07	2.452E-07	2.328E-07	2.209E-07	2.096E-07	1.989E-07	1.887E-07	1.790E-07
5.1	1.698E-07	1.611E-07	1.528E-07	1.449E-07	1.374E-07	1.302E-07	1.235E-07	1.170E-07	1.109E-07	1.051E-07
5.2	9.964E-08	9.442E-08	8.946E-08	8.476E-08	8.029E-08	7.605E-08	7.203E-08	6.821E-08	6.459E-08	6.116E-08
5.3	5.790E-08	5.481E-08	5.188E-08	4.911E-08	4.647E-08	4.398E-08	4.161E-08	3.937E-08	3.724E-08	3.523E-08
5.4	3.332E-08	3.151E-08	2.980E-08	2.818E-08	2.664E-08	2.518E-08	2.381E-08	2.250E-08	2.127E-08	2.010E-08
5.5	1.899E-08	1.794E-08	1.695E-08	1.601E-08	1.512E-08	1.428E-08	1.349E-08	1.274E-08	1.203E-08	1.135E-08
5.6	1.072E-08	1.012E-08	9.548E-09	9.010E-09	8.503E-09	8.022E-09	7.569E-09	7.140E-09	6.735E-09	6.352E-09
5.7	5.990E-09	5.649E-09	5.326E-09	5.022E-09	4.734E-09	4.462E-09	4.206E-09	3.964E-09	3.735E-09	3.519E-09
5.8	3.316E-09	3.124E-09	2.942E-09	2.771E-09	2.610E-09	2.458E-09	2.314E-09	2.179E-09	2.051E-09	1.931E-09
5.9	1.818E-09	1.711E-09	1.610E-09	1.515E-09	1.425E-09	1.341E-09	1.261E-09	1.186E-09	1.116E-09	1.049E-09
6.0	9.866E-10	9.276E-10	8.721E-10	8.198E-10	7.706E-10	7.242E-10	6.806E-10	6.396E-10	6.009E-10	5.646E-10
6.1	5.303E-10	4.982E-10	4.679E-10	4.394E-10	4.126E-10	3.874E-10	3.637E-10	3.414E-10	3.205E-10	3.008E-10
6.2	2.823E-10	2.649E-10	2.486E-10	2.332E-10	2.188E-10	2.052E-10	1.925E-10	1.805E-10	1.693E-10	1.587E-10
6.3	1.488E-10	1.395E-10	1.308E-10	1.226E-10	1.149E-10	1.077E-10	1.009E-10	9.451E-11	8.854E-11	8.294E-11
6.4	7.769E-11	7.276E-11	6.814E-11	6.380E-11	5.974E-11	5.593E-11	5.235E-11	4.900E-11	4.586E-11	4.292E-11
6.5	4.016E-11	3.758E-11	3.515E-11	3.288E-11	3.076E-11	2.877E-11	2.690E-11	2.516E-11	2.352E-11	2.199E-11
6.6	2.056E-11	1.922E-11	1.796E-11	1.678E-11	1.568E-11	1.465E-11	1.369E-11	1.279E-11	1.195E-11	1.116E-11
6.7	1.042E-11	9.731E-12	9.086E-12	8.483E-12	7.919E-12	7.392E-12	6.900E-12	6.439E-12	6.009E-12	5.607E-12
6.8	5.231E-12	4.880E-12	4.552E-12	4.246E-12	3.960E-12	3.692E-12	3.443E-12	3.210E-12	2.993E-12	2.790E-12
6.9	2.600E-12	2.423E-12	2.258E-12	2.104E-12	1.960E-12	1.826E-12	1.701E-12	1.585E-12	1.476E-12	1.374E-12
7.0	1.280E-12	1.192E-12	1.109E-12	1.033E-12	9.612E-13	8.946E-13	8.325E-13	7.747E-13	7.208E-13	6.706E-13
7.1	6.238E-13	5.802E-13	5.396E-13	5.018E-13	4.667E-13	4.339E-13	4.034E-13	3.750E-13	3.486E-13	3.240E-13
7.2	3.011E-13	2.798E-13	2.599E-13	2.415E-13	2.243E-13	2.084E-13	1.935E-13	1.797E-13	1.669E-13	1.550E-13
7.3	1.439E-13	1.336E-13	1.240E-13	1.151E-13	1.068E-13	9.910E-14	9.196E-14	8.531E-14	7.914E-14	7.341E-14
7.4	6.809E-14	6.315E-14	5.856E-14	5.430E-14	5.034E-14	4.667E-14	4.326E-14	4.010E-14	3.716E-14	3.444E-14
7.5	3.191E-14	2.956E-14	2.739E-14	2.537E-14	2.350E-14	2.176E-14	2.015E-14	1.866E-14	1.728E-14	1.600E-14
7.6	1.481E-14	1.370E-14	1.268E-14	1.174E-14	1.086E-14	1.005E-14	9.297E-15	8.600E-15	7.954E-15	7.357E-15
7.7	6.803E-15	6.291E-15	5.816E-15	5.377E-15	4.971E-15	4.595E-15	4.246E-15	3.924E-15	3.626E-15	3.350E-15
7.8	3.095E-15	2.859E-15	2.641E-15	2.439E-15	2.253E-15	2.080E-15	1.921E-15	1.773E-15	1.637E-15	1.511E-15
7.9	1.395E-15	1.287E-15	1.188E-15	1.096E-15	1.011E-15	9.326E-16	8.602E-16	7.934E-16	7.317E-16	6.747E-16
8.0	6.221E-16	5.735E-16	5.287E-16	4.874E-16	4.492E-16	4.140E-16	3.815E-16	3.515E-16	3.238E-16	2.983E-16
8.1	2.748E-16	2.531E-16	2.331E-16	2.146E-16	1.976E-16	1.820E-16	1.675E-16	1.542E-16	1.419E-16	1.306E-16
8.2	1.202E-16	1.106E-16	1.018E-16	9.361E-17	8.611E-17	7.920E-17	7.284E-17	6.698E-17	6.159E-17	5.662E-17
8.3	5.206E-17	4.785E-17	4.398E-17	4.042E-17	3.715E-17	3.413E-17	3.136E-17	2.881E-17	2.646E-17	2.431E-17
8.4	2.232E-17	2.050E-17	1.882E-17	1.728E-17	1.587E-17	1.457E-17	1.337E-17	1.227E-17	1.126E-17	1.033E-17
8.5	9.480E-18	8.697E-18	7.978E-18	7.317E-18	6.711E-18	6.154E-18	5.643E-18	5.174E-18	4.744E-18	4.348E-18
8.6	3.986E-18	3.653E-18	3.348E-18	3.068E-18	2.811E-18	2.575E-18	2.359E-18	2.161E-18	1.979E-18	1.812E-18
8.7	1.659E-18	1.519E-18	1.391E-18	1.273E-18	1.166E-18	1.067E-18	9.763E-19	8.933E-19	8.174E-19	7.478E-19
8.8	6.841E-19	6.257E-19	5.723E-19	5.234E-19	4.786E-19	4.376E-19	4.001E-19	3.657E-19	3.343E-19	3.055E-19
8.9	2.792E-19	2.552E-19	2.331E-19	2.130E-19	1.946E-19	1.777E-19	1.623E-19	1.483E-19	1.354E-19	1.236E-19
9.0	1.129E-19	1.030E-19	9.404E-20	8.584E-20	7.834E-20	7.148E-20	6.523E-20	5.951E-20	5.429E-20	4.952E-20
9.1	4.517E-20	4.119E-20	3.756E-20	3.425E-20	3.123E-20	2.847E-20	2.595E-20	2.365E-20	2.155E-20	1.964E-20
9.2	1.790E-20	1.631E-20	1.486E-20	1.353E-20	1.232E-20	1.122E-20	1.022E-20	9.307E-21	8.474E-21	7.714E-21
9.3	7.022E-21	6.392E-21	5.817E-21	5.294E-21	4.817E-21	4.382E-21	3.987E-21	3.627E-21	3.299E-21	3.000E-21
9.4	2.728E-21	2.481E-21	2.255E-21	2.050E-21	1.864E-21	1.694E-21	1.540E-21	1.399E-21	1.271E-21	1.155E-21
9.5	1.049E-21	9.533E-22	8.659E-22	7.864E-22	7.142E-22	6.485E-22	5.888E-22	5.345E-22	4.852E-22	4.404E-22
9.6	3.997E-22	3.627E-22	3.292E-22	2.986E-22	2.709E-22	2.458E-22	2.229E-22	2.022E-22	1.834E-22	1.663E-22
9.7	1.507E-22	1.367E-22	1.239E-22	1.123E-22	1.018E-22	9.223E-23	8.358E-23	7.573E-23	6.861E-23	6.215E-23
9.8	5.629E-23	5.098E-23	4.617E-23	4.181E-23	3.786E-23	3.427E-23	3.102E-23	2.808E-23	2.542E-23	2.300E-23
9.9	2.081E-23	1.883E-23	1.704E-23	1.541E-23	1.394E-23	1.261E-23	1.140E-23	1.031E-23	9.323E-24	8.429E-24

Communications IV Data Sheet

1. Correlation and Power Spectrum

$$R_{xy}(t_1,t_2) = E\{y(t_1)y^*(t_2)\}$$
$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$
$$R_{xx}(-\tau) = R_{xx}(\tau)$$
$$R_{yx}(\tau) = R_{xy}^*(-\tau)$$
$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$
$$S_{yx}(f) = S_{xy}^*(f)$$

2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda = h(t)\otimes x(t)$$
$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$
$$S_{xy}(f) = H^*(f)S_{xx}(f)$$
$$S_{yx}(f) = H(f)S_{xx}(f)$$

3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\hat{x}(t) \qquad (analytic\ signal)$$
$$X^+(f) = 2u(f)X(f)$$
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t-\lambda}\,d\lambda \qquad (hilbert\ transform)$$
$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty}|H(f)|^2\,df$$

5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$
$$S_{nn}^{(+)}(f) = u(f)S_{nn}(f)$$
$$S_{nn}^{(-)}(f) = u(-f)S_{nn}(f)$$
$$S_{n_cn_c}(f) = S_{n_sn_s}(f) = S_{nn}^{(+)}(f+f_o) + S_{nn}^{(-)}(f-f_o)$$
$$S_{n_cn_s}(f) = j\{S_{nn}^{(+)}(f+f_o) - S_{nn}^{(-)}(f-f_o)\}$$

6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \le 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

$$s_{base}(t) = Am(t)$$
$$B = W$$
$$SNR_{base} = \frac{A^2 <m^2>}{N_oW} = \frac{P_r}{N_oW}$$

$$s_{am}(t) = A\{1 + am(t)\}\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{am} = \frac{A^2a^2 <m^2>}{2N_oW} = \frac{a^2 <m^2>}{1+a^2 <m^2>}\frac{P_r}{N_oW}$$

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{dsbsc} = \frac{A^2 <m^2>}{2N_oW} = \frac{P_r}{N_oW}$$

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$
$$B = W$$
$$SNR_{ssbsc} = \frac{A^2 <m^2>}{N_oW} = \frac{P_r}{N_oW}$$

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$
$$B = 2(f_d + W)$$
$$SNR_{fm} = \frac{3A^2 f_d^2 <m^2>}{2N_oW^3} = 3 <m^2> \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_oW}$$

Threshold at $\frac{P_r}{N_oB} = 10$ (10 dB)
