THE UNIVERSITY of ADELAIDE

EXAMINATION FOR THE DEGREE OF B.E and M.E.

Semester 1 2014

105973 COMMUNICATIONS (ELEC ENG 4063)
105980 PRINCIPLES OF COMUNICATION SYSTEMS (ELEC ENG 7080)

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

Instructions:

- This is a closed book examination.
- Attempt **ALL FOUR** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

Materials:

- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

Attachments:

- Fourier Transform Sheet
- Table of the Q Function
- Communications Data Sheet
- Optical Data Sheet

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Total number of pages (including attachments) = 14

Question 1 begins on page 2.

Question 1 **20 marks total**

Consider a single mode fibre operating at 1550 nm. The specification is to transmit 400 Mbps NRZ data over the 100 km fibre link with a bit error rate (BER) of 10^{-9} or better.

1a) Given that $\tau = 1/R_{NRZ}$ and allowing for the system rise time to be 70% of τ, compute the system rise time t_S.

(1 mark)

1b) Given a material dispersion of M = -20 ps/(nm x km) and a waveguide dispersion of M_g = 4.5 ps/(nm x km), compute the fibre rise time, t_F. The spectral linewidth is given as 0.15 nm.

(1 mark)

1c) If the rise time of the light source is $t_{LS} = 1\,\text{ns}$, find an upper limit on the photodetector rise time t_{PD}.

(4 marks)

1d) Assume a basic BJT amplifier circuit at the receiver. If the photodetector has a transit time limited rise time of $t_{TR} = 0.5\,\text{ns}$ and a junction capacitance of $C_d = 1$ pF, calculate an upper limit on the load resistor.

(2 marks)

1e) Given that the fibre loss is 0.25 dB/km, the coupling efficiency to the fibre is 3 dB, there are two connectors with 1 dB loss each, there are 50 splices with 0.1 dB loss each and that the source power is 5 dBm, find the power at the receiver.

(2 marks)

1f) Calculate the optical power needed to achieve the specified BER assuming a quantum limited system. Comment on the result. You may assume dark current is negligible and therefore the expression for probability of an error $P_e = e^{-n_s}$ holds. The quantum efficiency of the detector is η = 0.7.

(4 marks)

1g) Now calculate the optical power needed to achieve the specified BER assuming a thermal limited system. Comment on the result. You may assume a detector responsivity of ρ = 1 A/W and a noise figure of F = 2. Room temperature conditions hold, thus let T = 300 K.

(6 marks)

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

Please See Next Page

Question 2

20 marks total

A video signal having a bandwidth of 4.8 MHz is transmitted over a 10-km path. We want to design a system so that the SNR at the receiver is 48 dB. Analog modulation is used. Spectral wavelength used is $\lambda_0 = 1.3$ μm.

The receiver is an InGaAs PIN photodiode.
Responsivity; $\rho = 0.6$ A/W
Dark Current; $I_d = 5$ nA
Junction capacitance; $C_d = 5$ pF
Noise figure; $F = 2$ at 300 K
Assume 100% modulation ($m = 1$).

2a) Calculate the load resistor R_L for the receiver. Comment on why you would not use this value in practice. **(2 marks)**

2b) Assume the system is thermal noise limited and hence calculate the power needed at the photodiode receiver to achieve the specified SNR, using the value of R_L calculated in (a). **(6 marks)**

2c) The available power from a laser diode source is $P_{ave} = 10$ mW. What is the available power budget left over for losses? **(3 marks)**

2d) Calculate the signal current. Assuming 4 V reverse bias on the photodiode, demonstrate if saturation and dark current will be negligible or not. **(4 marks)**

2e) Calculate the thermal noise and shot noise powers, hence demonstrate if the assumption in (b) was justified or not. **(5 marks)**

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

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Question 3

20 marks total

3a) A binary PAM system generates a signal

$$s(t) = \sum_{k=-\infty}^{\infty} A\, a_k p(t - kT)$$

where $A = 10$, $a_k = \pm 1$ is random uncorrelated binary data, the pulse shape is $p(t) = \Delta(t/T)$ and $T = 1$ ms is the bit period.

(i) The signal $s(t)$ is cyclostationary. Explain clearly what stationary and cyclostationary means in relation to the mean and autocorrelation function of a signal. **(2 marks)**

(ii) Sketch the signal $s(t)$ for $0 \le t \le 4T$ if the data is $a_k = [-1\ \ +1\ \ +1\ \ -1\ \ +1]$ for $0 \le k \le 4$. **(3 marks)**

(iii) Calculate the power spectrum $S_{ss}(f)$ of $s(t)$ and sketch it showing clearly the amplitude and frequency scales. [Use the result on data sheet, derivation is not required]. **(3 marks)**

(iv) Write down the autocorrelation $\overline{R}_{ss}(\tau)$ expressing it in integral form. You are not required to evaluate the integral. **(1 mark)**

3b) A source X has an alphabet {A, B, C, D, E} with corresponding probabilities {0.20, 0.15, 0.05, 0.10, 0.50}.

(i) Calculate the source entropy $H(x)$ in bits and explain what this means. **(3 marks)**

(ii) Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of five symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i). **(2 marks)**

(iii) Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency.* **(4 marks)**

(iv) Explain why it is possible to uniquely decipher a Huffman code. **(2 marks)**

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

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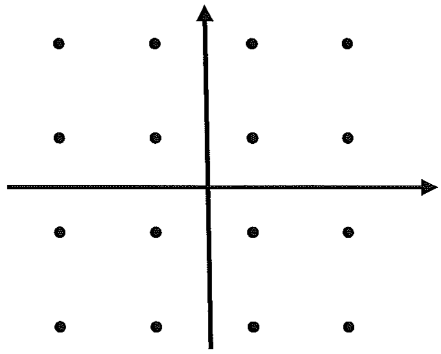
Question 4

20 marks total

4a) A 16QAM communications system uses a square 4 × 4 constellation, as shown below, such that the symbols are equally spaced by d both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is $R = 50\ \Omega$.



(i) Determine the *minimum* bandwidth B required for transmission. **(2 marks)**

(ii) Calculate E_b, the average energy per bit (in V^2s), for the constellation in terms of the separation distance d (the horizontal and vertical separation of the symbols). **Hint:** the energy of each symbol in the constellation is given by the square of its distance from the origin.
(3 marks)

(iii) If the average received power $P_r = 1.2 \times 10^{-6}$ W, determine the symbol spacing d.
(3 marks)

(iv) If outermost corner symbols have an energy of 9×10^{-10} J, write down an expression for the corner symbol energy d^2 in units of V^2s, as a function of the receiver input impedance R. If the noise power spectral density, $N_o = 1 \times 10^{-11}$ W/Hz, also write down an expression for the noise energy in units of V^2s, as a function of the receiver input impedance R. Then determine the probability of selecting an adjacent symbol in error during demodulation, given that a matched filter receiver is used.
(4 marks)

4b) A BPSK (binary phase shift keyed) system is used to transmit the same bit rate as in part a), but this time the receiver input impedance, R, is unknown.

(i) Determine the minimum bandwidth B required for transmission. **(2 marks)**

(ii) For an average received power $P_r = 1.2 \times 10^{-6}$ W and noise spectral density $N_o = 1 \times 10^{-11}$ W/Hz, determine the probability of error for a matched filter receiver. **(4 marks)**

(iii) Comment on the differences between BPSK and 16QAM as revealed by your answers.
(2 marks)

You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.

End of Examination Questions

Data Sheets follow on Pages 6 – 14

Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\lvert T \rvert X(fT)$
$x(t-T)$	$X(f)e^{-j2\pi f T}$
$x(t)e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\dfrac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$
$t\, x(t)$	$-\dfrac{1}{j2\pi}\dfrac{dX(f)}{df}$
$X(t)$	$x(-f)$
$\mathrm{rep}_T\{x(t)\}$	$\lvert F \rvert \mathrm{comb}_F(f)X(f)$
$\lvert T \rvert \mathrm{comb}_T(t)\, x(t)$	$\mathrm{rep}_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)Y(f)$
$x^*(t)$	$X^*(-f)$

Note that F and T are real constants, with $FT = 1$.

Transforms

$u(t)e^{-at}$	$\dfrac{1}{a + j2\pi f}$; $a > 0$
$e^{-a\lvert t \rvert}$	$\dfrac{2a}{a^2 + (2\pi f)^2}$; $a > 0$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a} e^{-\lvert 2\pi f a \rvert}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$\mathrm{sgn}(t)$	$\dfrac{1}{j\pi f}$
$\dfrac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$\mathrm{rect}(t/T)$	$\lvert T \rvert \mathrm{sinc}(fT)$
$\mathrm{sinc}(t/T)$	$\lvert T \rvert \mathrm{rect}(fT)$
$\Delta(t/T)$	$\lvert T \rvert \mathrm{sinc}^2(fT)$
$\mathrm{comb}_T(t)$	$\lvert F \rvert \mathrm{comb}_F(f)$
$e^{-t^2/2T^2}$	$\lvert T \rvert \sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$\mathrm{sgn}(t)\,\mathrm{rect}(t/T)$	$\dfrac{1 - \cos(\pi f T)}{j\pi f}$

Note that a is a real positive constant.

Definitions

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\mathrm{rep}_P\{f(x)\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\mathrm{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\mathrm{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of } x(t)$$

$$X(-f) = X^*(f) \quad \text{if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) = \int_{-\infty}^{\infty} X(f) Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.676E-04	9.354E-04	9.043E-04	8.740E-04	8.447E-04	8.164E-04	7.888E-04	7.622E-04	7.364E-04	7.114E-04
3.2	6.871E-04	6.637E-04	6.410E-04	6.190E-04	5.976E-04	5.770E-04	5.571E-04	5.377E-04	5.190E-04	5.009E-04
3.3	4.834E-04	4.665E-04	4.501E-04	4.342E-04	4.189E-04	4.041E-04	3.897E-04	3.758E-04	3.624E-04	3.495E-04
3.4	3.369E-04	3.248E-04	3.131E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.602E-04	2.507E-04	2.415E-04
3.5	2.326E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.926E-04	1.854E-04	1.785E-04	1.718E-04	1.653E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.363E-04	1.311E-04	1.261E-04	1.213E-04	1.166E-04	1.121E-04
3.7	1.078E-04	1.036E-04	9.961E-05	9.574E-05	9.201E-05	8.842E-05	8.496E-05	8.162E-05	7.841E-05	7.532E-05
3.8	7.235E-05	6.948E-05	6.673E-05	6.407E-05	6.152E-05	5.906E-05	5.669E-05	5.442E-05	5.223E-05	5.012E-05
3.9	4.810E-05	4.615E-05	4.427E-05	4.247E-05	4.074E-05	3.908E-05	3.747E-05	3.594E-05	3.446E-05	3.304E-05
4.0	3.167E-05	3.036E-05	2.910E-05	2.789E-05	2.673E-05	2.561E-05	2.454E-05	2.351E-05	2.252E-05	2.157E-05
4.1	2.066E-05	1.978E-05	1.894E-05	1.814E-05	1.737E-05	1.662E-05	1.591E-05	1.523E-05	1.458E-05	1.395E-05
4.2	1.335E-05	1.277E-05	1.222E-05	1.168E-05	1.118E-05	1.069E-05	1.022E-05	9.774E-06	9.345E-06	8.934E-06
4.3	8.540E-06	8.163E-06	7.801E-06	7.455E-06	7.124E-06	6.807E-06	6.503E-06	6.212E-06	5.934E-06	5.668E-06
4.4	5.413E-06	5.169E-06	4.935E-06	4.712E-06	4.498E-06	4.294E-06	4.098E-06	3.911E-06	3.732E-06	3.561E-06
4.5	3.398E-06	3.241E-06	3.092E-06	2.949E-06	2.813E-06	2.682E-06	2.558E-06	2.439E-06	2.325E-06	2.216E-06
4.6	2.112E-06	2.013E-06	1.919E-06	1.828E-06	1.742E-06	1.660E-06	1.581E-06	1.506E-06	1.434E-06	1.366E-06
4.7	1.301E-06	1.239E-06	1.179E-06	1.123E-06	1.069E-06	1.017E-06	9.680E-07	9.211E-07	8.765E-07	8.339E-07
4.8	7.933E-07	7.547E-07	7.178E-07	6.827E-07	6.492E-07	6.173E-07	5.869E-07	5.580E-07	5.304E-07	5.042E-07
4.9	4.792E-07	4.554E-07	4.327E-07	4.111E-07	3.906E-07	3.711E-07	3.525E-07	3.348E-07	3.179E-07	3.019E-07

Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	2.867E-07	2.722E-07	2.584E-07	2.452E-07	2.328E-07	2.209E-07	2.096E-07	1.989E-07	1.887E-07	1.790E-07
5.1	1.698E-07	1.611E-07	1.528E-07	1.449E-07	1.374E-07	1.302E-07	1.235E-07	1.170E-07	1.109E-07	1.051E-07
5.2	9.964E-08	9.442E-08	8.946E-08	8.476E-08	8.029E-08	7.605E-08	7.203E-08	6.821E-08	6.459E-08	6.116E-08
5.3	5.790E-08	5.481E-08	5.188E-08	4.911E-08	4.647E-08	4.398E-08	4.161E-08	3.937E-08	3.724E-08	3.523E-08
5.4	3.332E-08	3.151E-08	2.980E-08	2.818E-08	2.664E-08	2.518E-08	2.381E-08	2.250E-08	2.127E-08	2.010E-08
5.5	1.899E-08	1.794E-08	1.695E-08	1.601E-08	1.512E-08	1.428E-08	1.349E-08	1.274E-08	1.203E-08	1.135E-08
5.6	1.072E-08	1.012E-08	9.548E-09	9.010E-09	8.503E-09	8.022E-09	7.569E-09	7.140E-09	6.735E-09	6.352E-09
5.7	5.990E-09	5.649E-09	5.326E-09	5.022E-09	4.734E-09	4.462E-09	4.206E-09	3.964E-09	3.735E-09	3.519E-09
5.8	3.316E-09	3.124E-09	2.942E-09	2.771E-09	2.610E-09	2.458E-09	2.314E-09	2.179E-09	2.051E-09	1.931E-09
5.9	1.818E-09	1.711E-09	1.610E-09	1.515E-09	1.425E-09	1.341E-09	1.261E-09	1.186E-09	1.116E-09	1.049E-09
6.0	9.866E-10	9.276E-10	8.721E-10	8.198E-10	7.706E-10	7.242E-10	6.806E-10	6.396E-10	6.009E-10	5.646E-10
6.1	5.303E-10	4.982E-10	4.679E-10	4.394E-10	4.126E-10	3.874E-10	3.637E-10	3.414E-10	3.205E-10	3.008E-10
6.2	2.823E-10	2.649E-10	2.486E-10	2.332E-10	2.188E-10	2.052E-10	1.925E-10	1.805E-10	1.693E-10	1.587E-10
6.3	1.488E-10	1.395E-10	1.308E-10	1.226E-10	1.149E-10	1.077E-10	1.009E-10	9.451E-11	8.854E-11	8.294E-11
6.4	7.769E-11	7.276E-11	6.814E-11	6.380E-11	5.974E-11	5.593E-11	5.235E-11	4.900E-11	4.586E-11	4.292E-11
6.5	4.016E-11	3.758E-11	3.515E-11	3.288E-11	3.076E-11	2.877E-11	2.690E-11	2.516E-11	2.352E-11	2.199E-11
6.6	2.056E-11	1.922E-11	1.796E-11	1.678E-11	1.568E-11	1.465E-11	1.369E-11	1.279E-11	1.195E-11	1.116E-11
6.7	1.042E-11	9.731E-12	9.086E-12	8.483E-12	7.919E-12	7.392E-12	6.900E-12	6.439E-12	6.009E-12	5.607E-12
6.8	5.231E-12	4.880E-12	4.552E-12	4.246E-12	3.960E-12	3.692E-12	3.443E-12	3.210E-12	2.993E-12	2.790E-12
6.9	2.600E-12	2.423E-12	2.258E-12	2.104E-12	1.960E-12	1.826E-12	1.701E-12	1.585E-12	1.476E-12	1.374E-12
7.0	1.280E-12	1.192E-12	1.109E-12	1.033E-12	9.612E-13	8.946E-13	8.325E-13	7.747E-13	7.208E-13	6.706E-13
7.1	6.238E-13	5.802E-13	5.396E-13	5.018E-13	4.667E-13	4.339E-13	4.034E-13	3.750E-13	3.486E-13	3.240E-13
7.2	3.011E-13	2.798E-13	2.599E-13	2.415E-13	2.243E-13	2.084E-13	1.935E-13	1.797E-13	1.669E-13	1.550E-13
7.3	1.439E-13	1.336E-13	1.240E-13	1.151E-13	1.068E-13	9.910E-14	9.196E-14	8.531E-14	7.914E-14	7.341E-14
7.4	6.809E-14	6.315E-14	5.856E-14	5.430E-14	5.034E-14	4.667E-14	4.326E-14	4.010E-14	3.716E-14	3.444E-14
7.5	3.191E-14	2.956E-14	2.739E-14	2.537E-14	2.350E-14	2.176E-14	2.015E-14	1.866E-14	1.728E-14	1.600E-14
7.6	1.481E-14	1.370E-14	1.268E-14	1.174E-14	1.086E-14	1.005E-14	9.297E-15	8.600E-15	7.954E-15	7.357E-15
7.7	6.803E-15	6.291E-15	5.816E-15	5.377E-15	4.971E-15	4.595E-15	4.246E-15	3.924E-15	3.626E-15	3.350E-15
7.8	3.095E-15	2.859E-15	2.641E-15	2.439E-15	2.253E-15	2.080E-15	1.921E-15	1.773E-15	1.637E-15	1.511E-15
7.9	1.395E-15	1.287E-15	1.188E-15	1.096E-15	1.011E-15	9.326E-16	8.602E-16	7.934E-16	7.317E-16	6.747E-16
8.0	6.221E-16	5.735E-16	5.287E-16	4.874E-16	4.492E-16	4.140E-16	3.815E-16	3.515E-16	3.238E-16	2.983E-16
8.1	2.748E-16	2.531E-16	2.331E-16	2.146E-16	1.976E-16	1.820E-16	1.675E-16	1.542E-16	1.419E-16	1.306E-16
8.2	1.202E-16	1.106E-16	1.018E-16	9.361E-17	8.611E-17	7.920E-17	7.284E-17	6.698E-17	6.159E-17	5.662E-17
8.3	5.206E-17	4.785E-17	4.398E-17	4.042E-17	3.715E-17	3.413E-17	3.136E-17	2.881E-17	2.646E-17	2.431E-17
8.4	2.232E-17	2.050E-17	1.882E-17	1.728E-17	1.587E-17	1.457E-17	1.337E-17	1.227E-17	1.126E-17	1.033E-17
8.5	9.480E-18	8.697E-18	7.978E-18	7.317E-18	6.711E-18	6.154E-18	5.643E-18	5.174E-18	4.744E-18	4.348E-18
8.6	3.986E-18	3.653E-18	3.348E-18	3.068E-18	2.811E-18	2.575E-18	2.359E-18	2.161E-18	1.979E-18	1.812E-18
8.7	1.659E-18	1.519E-18	1.391E-18	1.273E-18	1.166E-18	1.067E-18	9.763E-19	8.933E-19	8.174E-19	7.478E-19
8.8	6.841E-19	6.257E-19	5.723E-19	5.234E-19	4.786E-19	4.376E-19	4.001E-19	3.657E-19	3.343E-19	3.055E-19
8.9	2.792E-19	2.552E-19	2.331E-19	2.130E-19	1.946E-19	1.777E-19	1.623E-19	1.483E-19	1.354E-19	1.236E-19
9.0	1.129E-19	1.030E-19	9.404E-20	8.584E-20	7.834E-20	7.148E-20	6.523E-20	5.951E-20	5.429E-20	4.952E-20
9.1	4.517E-20	4.119E-20	3.756E-20	3.425E-20	3.123E-20	2.847E-20	2.595E-20	2.365E-20	2.155E-20	1.964E-20
9.2	1.790E-20	1.631E-20	1.486E-20	1.353E-20	1.232E-20	1.122E-20	1.022E-20	9.307E-21	8.474E-21	7.714E-21
9.3	7.022E-21	6.392E-21	5.817E-21	5.294E-21	4.817E-21	4.382E-21	3.987E-21	3.627E-21	3.299E-21	3.000E-21
9.4	2.728E-21	2.481E-21	2.255E-21	2.050E-21	1.864E-21	1.694E-21	1.540E-21	1.399E-21	1.271E-21	1.155E-21
9.5	1.049E-21	9.533E-22	8.659E-22	7.864E-22	7.142E-22	6.485E-22	5.888E-22	5.345E-22	4.852E-22	4.404E-22
9.6	3.997E-22	3.627E-22	3.292E-22	2.986E-22	2.709E-22	2.458E-22	2.229E-22	2.022E-22	1.834E-22	1.663E-22
9.7	1.507E-22	1.367E-22	1.239E-22	1.123E-22	1.018E-22	9.223E-23	8.358E-23	7.573E-23	6.861E-23	6.215E-23
9.8	5.629E-23	5.098E-23	4.617E-23	4.181E-23	3.786E-23	3.427E-23	3.102E-23	2.808E-23	2.542E-23	2.300E-23
9.9	2.081E-23	1.883E-23	1.704E-23	1.541E-23	1.394E-23	1.261E-23	1.140E-23	1.031E-23	9.323E-24	8.429E-24

Communications IV Data Sheet

1. Correlation and Power Spectrum

$$R_{xy}(t_1,t_2) = E\{y(t_1)y^*(t_2)\}$$
$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$
$$R_{xx}(-\tau) = R_{xx}(\tau)$$
$$R_{yx}(\tau) = R^*_{xy}(-\tau)$$
$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$
$$S_{yx}(f) = S^*_{xy}(f)$$

2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda = h(t)\otimes x(t)$$
$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$
$$S_{xy}(f) = H^*(f)S_{xx}(f)$$
$$S_{yx}(f) = H(f)S_{xx}(f)$$

3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\hat{x}(t) \qquad (analytic\ signal)$$
$$X^+(f) = 2u(f)X(f)$$
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t-\lambda}\,d\lambda \qquad (hilbert\ transform)$$
$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty}|H(f)|^2\,df$$

5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$
$$S^{(+)}_{nn}(f) = u(f)S_{nn}(f)$$
$$S^{(-)}_{nn}(f) = u(-f)S_{nn}(f)$$
$$S_{n_cn_c}(f) = S_{n_sn_s}(f) = S^{(+)}_{nn}(f+f_o) + S^{(-)}_{nn}(f-f_o)$$
$$S_{n_cn_s}(f) = j\left\{S^{(+)}_{nn}(f+f_o) - S^{(-)}_{nn}(f-f_o)\right\}$$

6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \le 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

$$s_{base}(t) = Am(t)$$
$$B = W$$
$$SNR_{base} = \frac{A^2<m^2>}{N_oW} = \frac{P_r}{N_oW}$$

$$s_{am}(t) = A\{1+am(t)\}\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{am} = \frac{A^2a^2<m^2>}{2N_oW} = \frac{a^2<m^2>}{1+a^2<m^2>}\frac{P_r}{N_oW}$$

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{dsbsc} = \frac{A^2<m^2>}{2N_oW} = \frac{P_r}{N_oW}$$

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$
$$B = W$$
$$SNR_{ssbsc} = \frac{A^2<m^2>}{N_oW} = \frac{P_r}{N_oW}$$

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d\int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$
$$B = 2(f_d + W)$$
$$SNR_{fm} = \frac{3A^2f_d^2<m^2>}{2N_oW^3} = 3<m^2>\left(\frac{f_d}{W}\right)^2\frac{P_r}{N_oW}$$

Threshold at $\frac{P_r}{N_oB} = 10$ (10 dB)

---

7. Information Theory

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$$H(x) = -\sum_i P(x_i)\log_2 P(x_i)\ \textit{bits / symbol}$$

$$H(x,y) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(x_i,y_j)$$

$$H(y\mid x) = -\sum_i\sum_j P(x_i,y_j)\log_2 P(y_j\mid x_i)$$

$$= H(x,y) - H(x)$$

$$I(x,y) = H(x) - H(x\mid y) = H(y) - H(y\mid x)$$

$$C = r\max I(x,y)\ \textit{bits}/\text{sec}\ (\textit{discrete channel})$$

$$C = B\log_2(1 + P_s/P_n)\,\text{bits/sec (continuous channel)}$$

8. Digital Modulation

$$s_o(t_o) = \pm V,\ \Gamma = \frac{V^2}{< n_o^2 >}\ \text{(binary antipodal signals)}$$

$$P_b = Q\left\{\sqrt{\Gamma}\right\}\ \text{(bit error, binary antipodal system)}$$

$$h(t) = c\,s(t_o - t)\ \text{(matched filter)}$$

$$\Gamma = \frac{2E_s}{N_o}\ (\textit{matched filter, baseband \& BPSK})$$

$$P_{sym} \approx Q\left\{\sqrt{d^2/2N_o}\right\}\ \text{(matched filter, spacing d)}$$

$$\textit{Constellation radius} = \sqrt{\text{(symbol energy)}}$$

$$S_{xx}(f) = \frac{E\{a_k^2\}}{T}|P(f)|^2 \quad ;\ x(t) = \sum_{k=-\infty}^{\infty} a_k\, p(t-kT)$$

$$s_{QAM}(t) = \sum_{k=-\infty}^{\infty} A\,p(t-kT)\left[a_k\cos\omega_o t - b_k\sin\omega_o t\right]$$

$$s_{PSK}(t) = \sum_{k=-\infty}^{\infty} A\,p(t-kT)\cos(\omega_o t + \theta_k)$$

9. Nyquist Pulse

$$W = \frac{1}{2T}$$

$$P(f) = \begin{cases} T & ;|f| < (1-\rho)W \\ 0.5T - 0.5T\sin\left(\frac{\pi(|f|-W)}{2\rho W}\right) & ;||f|-W| < \rho W \\ 0 & ;\textit{elsewhere} \end{cases}$$

$$p(t) = \frac{\pi}{4}\mathrm{sinc}\left(\frac{t}{T}\right)\left\{\mathrm{sinc}\left(\frac{\rho t}{T}+\frac{1}{2}\right) + \mathrm{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)\right\}$$

10. Coding

$$P(i) = \binom{n}{i}P_b^i Q_b^{n-i}\ \text{(binomial distribution)}$$

$$P_b = Q\left\{\sqrt{RE_b/\alpha}\right\}\ \text{(BER before correction)}$$

$$P_{cbe} \approx \frac{2t+1}{n}\binom{n}{t+1}P_b^{t+1}\ \text{(BER, correcting t errors)}$$

$$\tilde{x} = \tilde{m}G,\ G = [I\ P] = \textit{generator matrix}$$

$$\tilde{y} = \tilde{x} + \tilde{e},\ \tilde{s} = \tilde{y}H = \text{syndrome}$$

$$H = \begin{bmatrix} P \\ I \end{bmatrix} = \text{parity check matrix}$$

Hamming codes $n = 2^q - 1$ (correct one error per block)

$$2^q \ge \sum_{i=0}^{t}\binom{n}{i}\ \text{(necessary for existence of code)}$$

$$X(p) = p^q M(p) + C(p)\ (\textit{cyclic}\ \text{code generation})$$

$$C(p) = \text{rem}\left\{\frac{p^q M(p)}{G(p)}\right\}\ \text{(check digits)}$$

$$S(p) = \text{rem}\left\{\frac{Y(p)}{G(p)}\right\}\ \text{(syndrome, cyclic code)}$$

11. Gaussian Probability

One dimension (mean η, variance σ^2):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\eta)^2/2\sigma^2}$$

$$P\{x > V\} = Q\left\{\frac{V-\eta}{\sigma}\right\}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\,dt$$

Two dimensions (zero means):

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right]}$$

$$\sigma_x^2 = E\{x^2\},\ \sigma_y^2 = E\{y^2\},\ \rho = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

OPTICAL COMMUNICATIONS
SUMMARY OF SOME USEFUL FORMULAE

General

$$\begin{aligned}
\Delta(\tau/L)_{\text{dis}} &= -(M+M_g)\Delta\lambda \\
f_{3dB}(\text{opt.})\times L &= \frac{0.5}{\Delta(\tau/L)} \\
f_{3dB}(\text{elec.})\times L &= \frac{0.35}{\Delta(\tau/L)} \\
R_{RZ}\times L &= \frac{0.35}{\Delta(\tau/L)} \\
R_{NRZ}\times L &= \frac{0.7}{\Delta(\tau/L)}
\end{aligned}$$

Resonant Cavities

$$\Delta f_c = \frac{c}{2Ln}$$

Optics

$$\begin{aligned}
R(\text{normal incidence}) &= \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 \\
\sin\theta_c &= \frac{n_2}{n_1}
\end{aligned}$$

Optical Fibres

Step Index:

$$\begin{aligned}
\Delta &= \frac{n_1-n_2}{n_1} \\
\text{NA} &= \sqrt{n_1^2-n_2^2} \\
\Delta(\tau/L)_{\text{mod}} &= \frac{n_1\Delta}{c} \\
\text{Single mode:}\quad a/\lambda_0 &< \frac{2.405}{2\pi.\text{NA}}
\end{aligned}$$

Graded Index (Parabolic Profile):

$$\begin{aligned}
\Delta &= \frac{n_1^2-n_2^2}{2n_1^2} \\
n(r) &= n_1\sqrt{1-2(r/a)^2\Delta}\ \text{for}\ r\le a \\
\text{NA} &= n_1\sqrt{2\Delta}\sqrt{1-(r/a)^2} \\
\Delta(\tau/L)_{\text{mod}} &= \frac{n_1\Delta^2}{2c} \\
n_{\text{eff}} &= n_1-(p+q+1)\frac{\sqrt{2\Delta}}{k_0a}
\end{aligned}$$

Source Bandwidth

$$P_{SP} = \frac{a_1 I_{SP}}{\sqrt{1+\omega^2\tau^2}}$$
$$f_{3dB} = \frac{1}{2\pi\tau}$$
$$t_r = \frac{0.35}{f_{3dB}}$$

Source Coupling

For a source with pattern $\cos^m\theta$:

$$\eta_{\text{SI}} = 1 - \left[1-(\text{NA})^2\right]^{(m+1)/2} \text{ to SI Fibre}$$
$$\eta_{\text{GRIN}} = \eta_{\text{SI}}\left\{1 - 0.5\left(\frac{a_e}{a_f}\right)^2\right\} \text{ to GRIN Fibre where } a_e \le a_f$$

Optical Detectors

$$\rho = i/P = \frac{M\eta e\lambda}{hc}$$
$$f_{3dB} = \frac{0.35}{t_r}$$
$$t_r = 2.19 R_L C_d$$

Analogue Modulation

AM/IM:

$$i = I_0 + I_s(1 + m\cos\omega_m t)\cos\omega_{sc} t$$
$$P = P_0 + P_s(1 + m\cos\omega_m t)\cos\omega_{sc} t$$

FM/IM:

$$i = I_0 + I_s\cos(\omega_{sc} t + \beta\sin\omega_m t)$$
$$P = P_0 + P_s\cos(\omega_{sc} t + \beta\sin\omega_m t)$$
$$B = \frac{\omega_m}{\pi}(1+\beta)$$

LED Operating Characteristics

$$\lambda_0 = \frac{1.24}{W_g(\text{eV})}$$
$$P = \eta i W_g(\text{eV})$$
$$P_s = \frac{a_1 I_s}{\sqrt{1+\omega^2\tau^2}}$$

Harmonic Distortion

$$
\begin{aligned}
P &= P_{dc} + a_1 i_s + a_2 i_s^2 + \ldots \\
\text{THD} &= 0.25 \left(\frac{a_2 I}{a_1} \right)^2 \quad \text{for } i_s = I \sin \omega t
\end{aligned}
$$

Receiver Design

General Signal to Noise Ratio:

$$
S/N = \frac{(m^2/2)(M\rho P)^2 R_L}{M^n 2eR_L \Delta f (I_D + \rho P) + 4kT\Delta f F}
$$

Thermal noise limited probability of error:

$$
\begin{aligned}
P_e &= \frac{1}{2} - \frac{1}{2}\text{erf}\left(0.354\sqrt{S/N}\right) \\
\text{erf}(x) &\approx 1 - \frac{e^{-x^2}}{\sqrt{\pi}x} \quad \text{for } x \geq 3
\end{aligned}
$$

Peak S/N (with threshold set at $0.5 i_s$) for specified bit error rates.

P_e	S/N
10^{-4}	17.4dB
10^{-6}	19.5dB
10^{-9}	21.5dB

Quantum noise limited probability of error:

$$
\begin{aligned}
P_e &= e^{-n_s} \\
n_s &= \frac{\eta P_T}{hf}
\end{aligned}
$$

Heterodyne Receivers

$$
\begin{aligned}
i_{dc} &= \frac{\eta e}{hf}(P_L + P_S) \\
i_{IF} &= \frac{2\eta e}{hf}\sqrt{P_L P_S}\cos\left[\omega_{IF} t - \theta(t)\right] \\
S/N &= \frac{2(\eta e/hf)^2 R_L P_S P_L}{2eR_L \Delta f \left[I_D + (\eta e P_L/hf)(1 + P_S/P_L)\right] + 4kT\Delta f}
\end{aligned}
$$

Some Physical Constants

Speed of Light in Free Space	$c = 3 \times 10^8$ m/s
Boltzmann's Constant	$k = 1.38 \times 10^{-23}$ J/K
Planck's Constant	$h = 6.626 \times 10^{-34}$ Js
Electron Charge	$e = 1.6 \times 10^{-19}$ C