THE UNIVERSITY of ADELAIDE

EXAMINATION FOR THE DEGREE OF B.E. and M.E

Semester 1 2016

# 105973 COMMUNICATIONS (ELEC ENG 4063)
# 105980 PRINCIPLES OF COMUNICATION SYSTEMS (ELEC ENG 7080)

| | |
|---|---|
| Official Reading Time: | 10 mins |
| Writing Time: | 120 mins |
| Total Duration: | 130 mins |

**Instructions:**

- This is a closed book examination.
- Attempt **ALL FOUR** questions.
- All questions carry equal marks; part marks are given in brackets where appropriate.
- **Explanations are expected and marks will be given for these.**
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.
- **ANSWERS TO QUESTIONS SHOULD BE EXPRESSED CLEARLY AND WRITTEN LEGIBLY. THESE ASPECTS OF PRESENTATION WILL BE TAKEN INTO ACCOUNT IN ASSESSMENT.**

**Materials:**

- One Blue book
- The use of calculators is permitted, this equipment to be supplied by the candidate. No pre-recorded material nor calculator instruction book is permitted, and calculators with remote communication links are not permitted.

**Attachments:**

- Fourier Transform Sheet
- Table of the Q Function
- Communications Data Sheet
- Optical Data Sheet

**DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO**

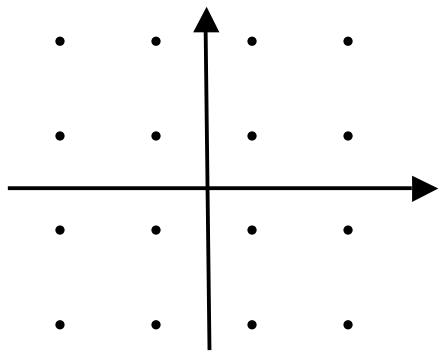
**Total number of pages (including attachments) = 15**

**Question 1 follows on page 2.**

# Question 1

**20 marks total**

**1a)** A 16QAM communications system uses a square 4 × 4 constellation, as shown below, such that the symbols are equally spaced by *d* both horizontally and vertically and are symmetrically placed with respect to the origin. The system transmits 2400 symbols/sec (ie. 9600 bits/sec) at a carrier frequency of 1 MHz. The receiver impedance is $R = 50\ \Omega$.



**(i)** Determine the *minimum* bandwidth $B$ required for transmission. **(2 marks)**

**(ii)** Calculate $E_b$, the average energy per bit (in $V^2s$), for the constellation in terms of the separation distance *d* (the horizontal and vertical separation of the symbols). **Hint:** the energy of each symbol in the constellation is given by the square of its distance from the origin.
**(3 marks)**

**(iii)** If the average received power $P_r = 1.2 \times 10^{-6}$ W, determine the symbol spacing *d*.
**(3 marks)**

**(iv)** If outermost corner symbols have an energy of $9 \times 10^{-10}$ J, write down an expression for the corner symbol energy $d^2$ in units of $V^2s$, as a function of the receiver input impedance *R*. If the noise power spectral density, $N_o = 1 \times 10^{-11}$ W/Hz, also write down an expression for the noise energy in units of $V^2s$, as a function of the receiver input impedance *R*. Then determine the probability of selecting an adjacent symbol in error during demodulation, given that a matched filter receiver is used.
**(4 marks)**

**1b)** A BPSK (binary phase shift keyed) system is used to transmit the same bit rate as in part a), but this time the receiver input impedance, *R*, is unknown.

**(i)** Determine the minimum bandwidth $B$ required for transmission. **(2 marks)**

**(ii)** For an average received power $P_r = 1.2 \times 10^{-6}$ W and noise spectral density $N_o = 1 \times 10^{-11}$ W/Hz, determine the probability of error for a matched filter receiver. **(4 marks)**

**(iii)** Comment on the differences between BPSK and 16QAM as revealed by your answers.
**(2 marks)**

**You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.**

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# Question 2 **20 marks total**

Consider a single mode fibre operating at 1550 nm. The specification is to transmit 400 Mbps NRZ data over the 100 km fibre link with a bit error rate (BER) of $10^{-9}$ or better.

**2a)** Given that $\tau = 1/R_{NRZ}$ and allowing for the system rise time to be 70% of $\tau$, compute the system rise time $t_S$.

**(1 mark)**

**2b)** Given a material dispersion of $M$ = -20 ps/(nm x km) and a waveguide dispersion of $M_g$ = 4.5 ps/(nm x km), compute the fibre rise time, $t_F$. The spectral linewidth is given as 0.15 nm.

**(1 mark)**

**2c)** If the rise time of the light source is $t_{LS} = 1$ ns, find an upper limit on the photodetector rise time $t_{PD}$.

**(4 marks)**

**2d)** Assume a basic BJT amplifier circuit at the receiver. If the photodetector has a transit time limited rise time of $t_{TR} = 0.5$ ns and a junction capacitance of $C_d = 1$ pF, calculate an upper limit on the load resistor.

**(2 marks)**

**2e)** Given that the fibre loss is 0.25 dB/km, the coupling efficiency to the fibre is 3 dB, there are two connectors with 1 dB loss each, there are 50 splices with 0.1 dB loss each and that the source power is 5 dBm, find the power at the receiver.

**(2 marks)**

**2f)** Calculate the optical power needed to achieve the specified BER assuming a quantum limited system. Comment on the result. You may assume dark current is negligible and therefore the expression for probability of an error $P_e = e^{-n_s}$ holds. The quantum efficiency of the detector is $\eta$ = 0.7.

**(4 marks)**

**2g)** Now calculate the optical power needed to achieve the specified BER assuming a thermal limited system. Comment on the result. You may assume a detector responsivity of $\rho$ = 1 A/W and a noise figure of $F$ = 2. Room temperature conditions hold, thus let $T$ = 300 K.

**(6 marks)**

**You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.**

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# Question 3 **20 marks total**

**3a)** A source $X$ has an alphabet {A, B, C, D, E, F} with corresponding probabilities {0.50, 0.15, 0.12, 0.10, 0.08, 0.05}.

**(i)** Calculate the source entropy $H(x)$ in bits and explain what this means. **(3 marks)**

**(ii)** Calculate the source entropy $H(x)$ in bits for a uniformly distributed alphabet of six symbols. (This means that each symbol has an equally likely probability of occurring). Explain why the result is larger than in (i). **(2 marks)**

**(iii)** Design a binary Huffman code for the symbol source, in part (i). Then calculate the *average code length* and the *code efficiency*. **(4 marks)**

**(iv)** Explain why it is possible to uniquely decipher a Huffman code. **(2 marks)**

**3b)** A BPSK (binary phase shift keyed) digital transmission system transmits data at $10^6$ symbols/sec on a 50 MHz carrier and has an uncorrected probability of a bit error equal to $P_e = Q\left\{\sqrt{2E_c / N_o}\right\} = 10^{-4}$, where $E_c$ is the energy per transmitted channel bit and $N_o/2$ is the spectral density of the accompanying additive white Gaussian noise (AWGN). Error correction is achieved by using a (15,11) Hamming block code.

**(i)** Calculate the value of $E_c/N_o$. **(1 mark)**

**(ii)** How many errors can the Hamming code *correct* in each block of 15? **(1 mark)**

**(iii)** Up to how many errors can the Hamming code *detect* in each block of 15?

**(iv)** What is the minimum bandwidth required to transmit the signal? **(1 mark)**

**(v)** Calculate the bit error probability after error correction. **(3 marks)**

**(vi)** If the BPSK system is redesigned so that the transmitter power is the same but the transmitted symbol rate with the Hamming coding is increased to $(15/11) \times 10^6$ symbols/sec so that the message bit rate is $10^6$ bits/sec, calculate the corrected probability of a bit error. (**Hint:** Calculate the reduced value of $E_c/N_o$, and hence the new value of $p$, the uncorrected probability of a bit error). **(3 marks)**

**You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.**

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# Question 4 **20 marks total**

**4a)** What is the difference (in Watts) between -65 dBm and 65 dBm? **(1 mark)**

**4b)** A fibre system operates with a carrier wavelength of 1.55 μm. Suppose that the system can handle digital information at a rate equal to 0.01% of the optical frequency. How many 20 Mbps HDTV compressed video channels can be multiplexed onto this fibre system? **(2 marks)**

**4c)** Prove that the power change $\gamma$ in dB/km and the attenuation coefficient α are related by $\gamma = -8.685\alpha$, where $\alpha$ is given in the units of $km^{-1}$. **(3 marks)**

**4d)** The power incident on a detector of light is 100 nW.

(i) Determine the number of photons per second incident on the detector if the wavelength is 800 nm. **(1.5 mark)**

(ii) If we carried out the above calculation for a longer wavelength, briefly state if the number of photons per second goes up or down, and why? **(1 mark)**

**4e)** A T3 system running at 45 Mbps has a BER of $10^{-9}$, compute the number of errors per minute. **(0.5 mark)**

**4f)** To operate properly, a particular fibre optic receiver requires -34 dBm of power. The system losses are 31 dB in total, from the light source to the receiver. Compute the power in mW that the light source needs to emit to meet the requirement. **(2 marks)**

**4g)** A cable contains 144 single-mode fibres, each operating at 2.3 Gb/s. How many digitised 64 kbps voice messages can be transmitted along this cable? **(1 mark)**

**4h)** A convex lens and a concave mirror are both dropped in a bucket of water. Briefly state in each case if the focal length increases, decreases, or is unchanged. Why? **(2 marks)**

**4i)** A long fibre, of arbitrary length, has 10 optical amplifiers equally spaced along its length. The amplifiers are used to compensate loss due to fibre attenuation. You may assume that the amplifier gain exactly equals the loss due to fibre attenuation. Each amplifier has a 3 dB noise figure, the SNR at the transmitter is $10^8$, and there is a 30 dB transmission loss between amplifiers along the fibre. Compute the SNR at the output of the fibre. **(3 marks)**

**4j)** You have microwave oven operating at 2.45 GHz. You have a cell phone with an 850 MHz carrier frequency. Calculate how many times larger the carrier wavelength is compared to the oven wavelength. You now put your phone in the oven, close the door, and find that it rings. Can the carrier wavelength penetrate the wire protective screen in the door? Can it penetrate the small gap around the rim of the door? Briefly explain. Also briefly explain why the microwaves inside the oven do not escape, whereas the carrier signal can get inside. **(3 marks)**

**You are reminded to clearly highlight your answers with a double underline, otherwise marks may be deducted.**

**End of Examination Questions**

**Data Sheets follow on Pages 7 – 15**

# Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

## Theorems

| | |
|---|---|
| $x(t)$ | $X(f)$ |
| $x(t/T)$ | $\lvert T \rvert X(fT)$ |
| $x(t-T)$ | $X(f)e^{-j2\pi f T}$ |
| $x(t)e^{j2\pi F t}$ | $X(f-F)$ |
| $x(-t)$ | $X(-f)$ |
| $\frac{dx(t)}{dt}$ | $j2\pi f\, X(f)$ |
| $\int_{-\infty}^{t} x(\lambda)\, d\lambda$ | $\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$ |
| $t\, x(t)$ | $-\frac{1}{j2\pi}\frac{dX(f)}{df}$ |
| $X(t)$ | $x(-f)$ |
| $\text{rep}_T\{x(t)\}$ | $\lvert F \rvert \text{comb}_F(f)\, X(f)$ |
| $\lvert T \rvert \text{comb}_T(t)\, x(t)$ | $\text{rep}_F\{X(f)\}$ |
| $x(t)\, y(t)$ | $X(f) \otimes Y(f)$ |
| $x(t) \otimes y(t)$ | $X(f)\, Y(f)$ |
| $x^*(t)$ | $X^*(-f)$ |

Note that $F$ and $T$ are real constants, with $FT = 1$.

## Transforms

| | |
|---|---|
| $u(t)e^{-at}$ | $\frac{1}{a + j2\pi f}$ ; $a > 0$ |
| $e^{-a\lvert t \rvert}$ | $\frac{2a}{a^2 + (2\pi f)^2}$ ; $a > 0$ |
| $\frac{1}{a^2 + t^2}$ | $\frac{\pi}{a} e^{-\lvert 2\pi f a \rvert}$ |
| $\delta(t)$ | $1$ |
| $1$ | $\delta(f)$ |
| $u(t)$ | $\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$ |
| $\text{sgn}(t)$ | $\frac{1}{j\pi f}$ |
| $\frac{1}{\pi t}$ | $-j\,\text{sgn}(f)$ |
| $\text{rect}(t/T)$ | $\lvert T \rvert \text{sinc}(fT)$ |
| $\text{sinc}(t/T)$ | $\lvert T \rvert \text{rect}(fT)$ |
| $\Delta(t/T)$ | $\lvert T \rvert \text{sinc}^2(fT)$ |
| $\text{comb}_T(t)$ | $\lvert F \rvert \text{comb}_F(f)$ |
| $e^{-t^2/2T^2}$ | $\lvert T \rvert \sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$ |
| $\text{sgn}(t)\,\text{rect}(t/T)$ | $\frac{1 - \cos(\pi f T)}{j\pi f}$ |

Note that $a$ is a real positive constant.

## Definitions

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\text{rep}_P\{f(x)\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\text{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\text{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\text{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

## Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of x(t)}$$

$$X(-f) = X^*(f) \text{ if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t) = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Unless otherwise stated, these relations are true for $x(t)$ real or complex.

# Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

| | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 5.000E-01 | 4.960E-01 | 4.920E-01 | 4.880E-01 | 4.840E-01 | 4.801E-01 | 4.761E-01 | 4.721E-01 | 4.681E-01 | 4.641E-01 |
| **0.1** | 4.602E-01 | 4.562E-01 | 4.522E-01 | 4.483E-01 | 4.443E-01 | 4.404E-01 | 4.364E-01 | 4.325E-01 | 4.286E-01 | 4.247E-01 |
| **0.2** | 4.207E-01 | 4.168E-01 | 4.129E-01 | 4.090E-01 | 4.052E-01 | 4.013E-01 | 3.974E-01 | 3.936E-01 | 3.897E-01 | 3.859E-01 |
| **0.3** | 3.821E-01 | 3.783E-01 | 3.745E-01 | 3.707E-01 | 3.669E-01 | 3.632E-01 | 3.594E-01 | 3.557E-01 | 3.520E-01 | 3.483E-01 |
| **0.4** | 3.446E-01 | 3.409E-01 | 3.372E-01 | 3.336E-01 | 3.300E-01 | 3.264E-01 | 3.228E-01 | 3.192E-01 | 3.156E-01 | 3.121E-01 |
| **0.5** | 3.085E-01 | 3.050E-01 | 3.015E-01 | 2.981E-01 | 2.946E-01 | 2.912E-01 | 2.877E-01 | 2.843E-01 | 2.810E-01 | 2.776E-01 |
| **0.6** | 2.743E-01 | 2.709E-01 | 2.676E-01 | 2.643E-01 | 2.611E-01 | 2.578E-01 | 2.546E-01 | 2.514E-01 | 2.483E-01 | 2.451E-01 |
| **0.7** | 2.420E-01 | 2.389E-01 | 2.358E-01 | 2.327E-01 | 2.296E-01 | 2.266E-01 | 2.236E-01 | 2.206E-01 | 2.177E-01 | 2.148E-01 |
| **0.8** | 2.119E-01 | 2.090E-01 | 2.061E-01 | 2.033E-01 | 2.005E-01 | 1.977E-01 | 1.949E-01 | 1.922E-01 | 1.894E-01 | 1.867E-01 |
| **0.9** | 1.841E-01 | 1.814E-01 | 1.788E-01 | 1.762E-01 | 1.736E-01 | 1.711E-01 | 1.685E-01 | 1.660E-01 | 1.635E-01 | 1.611E-01 |
| **1.0** | 1.587E-01 | 1.562E-01 | 1.539E-01 | 1.515E-01 | 1.492E-01 | 1.469E-01 | 1.446E-01 | 1.423E-01 | 1.401E-01 | 1.379E-01 |
| **1.1** | 1.357E-01 | 1.335E-01 | 1.314E-01 | 1.292E-01 | 1.271E-01 | 1.251E-01 | 1.230E-01 | 1.210E-01 | 1.190E-01 | 1.170E-01 |
| **1.2** | 1.151E-01 | 1.131E-01 | 1.112E-01 | 1.093E-01 | 1.075E-01 | 1.056E-01 | 1.038E-01 | 1.020E-01 | 1.003E-01 | 9.853E-02 |
| **1.3** | 9.680E-02 | 9.510E-02 | 9.342E-02 | 9.176E-02 | 9.012E-02 | 8.851E-02 | 8.692E-02 | 8.534E-02 | 8.379E-02 | 8.226E-02 |
| **1.4** | 8.076E-02 | 7.927E-02 | 7.780E-02 | 7.636E-02 | 7.493E-02 | 7.353E-02 | 7.215E-02 | 7.078E-02 | 6.944E-02 | 6.811E-02 |
| **1.5** | 6.681E-02 | 6.552E-02 | 6.426E-02 | 6.301E-02 | 6.178E-02 | 6.057E-02 | 5.938E-02 | 5.821E-02 | 5.705E-02 | 5.592E-02 |
| **1.6** | 5.480E-02 | 5.370E-02 | 5.262E-02 | 5.155E-02 | 5.050E-02 | 4.947E-02 | 4.846E-02 | 4.746E-02 | 4.648E-02 | 4.551E-02 |
| **1.7** | 4.457E-02 | 4.363E-02 | 4.272E-02 | 4.182E-02 | 4.093E-02 | 4.006E-02 | 3.920E-02 | 3.836E-02 | 3.754E-02 | 3.673E-02 |
| **1.8** | 3.593E-02 | 3.515E-02 | 3.438E-02 | 3.362E-02 | 3.288E-02 | 3.216E-02 | 3.144E-02 | 3.074E-02 | 3.005E-02 | 2.938E-02 |
| **1.9** | 2.872E-02 | 2.807E-02 | 2.743E-02 | 2.680E-02 | 2.619E-02 | 2.559E-02 | 2.500E-02 | 2.442E-02 | 2.385E-02 | 2.330E-02 |
| **2.0** | 2.275E-02 | 2.222E-02 | 2.169E-02 | 2.118E-02 | 2.068E-02 | 2.018E-02 | 1.970E-02 | 1.923E-02 | 1.876E-02 | 1.831E-02 |
| **2.1** | 1.786E-02 | 1.743E-02 | 1.700E-02 | 1.659E-02 | 1.618E-02 | 1.578E-02 | 1.539E-02 | 1.500E-02 | 1.463E-02 | 1.426E-02 |
| **2.2** | 1.390E-02 | 1.355E-02 | 1.321E-02 | 1.287E-02 | 1.255E-02 | 1.222E-02 | 1.191E-02 | 1.160E-02 | 1.130E-02 | 1.101E-02 |
| **2.3** | 1.072E-02 | 1.044E-02 | 1.017E-02 | 9.903E-03 | 9.642E-03 | 9.387E-03 | 9.137E-03 | 8.894E-03 | 8.656E-03 | 8.424E-03 |
| **2.4** | 8.198E-03 | 7.976E-03 | 7.760E-03 | 7.549E-03 | 7.344E-03 | 7.143E-03 | 6.947E-03 | 6.756E-03 | 6.569E-03 | 6.387E-03 |
| **2.5** | 6.210E-03 | 6.037E-03 | 5.868E-03 | 5.703E-03 | 5.543E-03 | 5.386E-03 | 5.234E-03 | 5.085E-03 | 4.940E-03 | 4.799E-03 |
| **2.6** | 4.661E-03 | 4.527E-03 | 4.397E-03 | 4.269E-03 | 4.145E-03 | 4.025E-03 | 3.907E-03 | 3.793E-03 | 3.681E-03 | 3.573E-03 |
| **2.7** | 3.467E-03 | 3.364E-03 | 3.264E-03 | 3.167E-03 | 3.072E-03 | 2.980E-03 | 2.890E-03 | 2.803E-03 | 2.718E-03 | 2.635E-03 |
| **2.8** | 2.555E-03 | 2.477E-03 | 2.401E-03 | 2.327E-03 | 2.256E-03 | 2.186E-03 | 2.118E-03 | 2.052E-03 | 1.988E-03 | 1.926E-03 |
| **2.9** | 1.866E-03 | 1.807E-03 | 1.750E-03 | 1.695E-03 | 1.641E-03 | 1.589E-03 | 1.538E-03 | 1.489E-03 | 1.441E-03 | 1.395E-03 |
| **3.0** | 1.350E-03 | 1.306E-03 | 1.264E-03 | 1.223E-03 | 1.183E-03 | 1.144E-03 | 1.107E-03 | 1.070E-03 | 1.035E-03 | 1.001E-03 |
| **3.1** | 9.676E-04 | 9.354E-04 | 9.043E-04 | 8.740E-04 | 8.447E-04 | 8.164E-04 | 7.888E-04 | 7.622E-04 | 7.364E-04 | 7.114E-04 |
| **3.2** | 6.871E-04 | 6.637E-04 | 6.410E-04 | 6.190E-04 | 5.976E-04 | 5.770E-04 | 5.571E-04 | 5.377E-04 | 5.190E-04 | 5.009E-04 |
| **3.3** | 4.834E-04 | 4.665E-04 | 4.501E-04 | 4.342E-04 | 4.189E-04 | 4.041E-04 | 3.897E-04 | 3.758E-04 | 3.624E-04 | 3.495E-04 |
| **3.4** | 3.369E-04 | 3.248E-04 | 3.131E-04 | 3.018E-04 | 2.909E-04 | 2.803E-04 | 2.701E-04 | 2.602E-04 | 2.507E-04 | 2.415E-04 |
| **3.5** | 2.326E-04 | 2.241E-04 | 2.158E-04 | 2.078E-04 | 2.001E-04 | 1.926E-04 | 1.854E-04 | 1.785E-04 | 1.718E-04 | 1.653E-04 |
| **3.6** | 1.591E-04 | 1.531E-04 | 1.473E-04 | 1.417E-04 | 1.363E-04 | 1.311E-04 | 1.261E-04 | 1.213E-04 | 1.166E-04 | 1.121E-04 |
| **3.7** | 1.078E-04 | 1.036E-04 | 9.961E-05 | 9.574E-05 | 9.201E-05 | 8.842E-05 | 8.496E-05 | 8.162E-05 | 7.841E-05 | 7.532E-05 |
| **3.8** | 7.235E-05 | 6.948E-05 | 6.673E-05 | 6.407E-05 | 6.152E-05 | 5.906E-05 | 5.669E-05 | 5.442E-05 | 5.223E-05 | 5.012E-05 |
| **3.9** | 4.810E-05 | 4.615E-05 | 4.427E-05 | 4.247E-05 | 4.074E-05 | 3.908E-05 | 3.747E-05 | 3.594E-05 | 3.446E-05 | 3.304E-05 |
| **4.0** | 3.167E-05 | 3.036E-05 | 2.910E-05 | 2.789E-05 | 2.673E-05 | 2.561E-05 | 2.454E-05 | 2.351E-05 | 2.252E-05 | 2.157E-05 |
| **4.1** | 2.066E-05 | 1.978E-05 | 1.894E-05 | 1.814E-05 | 1.737E-05 | 1.662E-05 | 1.591E-05 | 1.523E-05 | 1.458E-05 | 1.395E-05 |
| **4.2** | 1.335E-05 | 1.277E-05 | 1.222E-05 | 1.168E-05 | 1.118E-05 | 1.069E-05 | 1.022E-05 | 9.774E-06 | 9.345E-06 | 8.934E-06 |
| **4.3** | 8.540E-06 | 8.163E-06 | 7.801E-06 | 7.455E-06 | 7.124E-06 | 6.807E-06 | 6.503E-06 | 6.212E-06 | 5.934E-06 | 5.668E-06 |
| **4.4** | 5.413E-06 | 5.169E-06 | 4.935E-06 | 4.712E-06 | 4.498E-06 | 4.294E-06 | 4.098E-06 | 3.911E-06 | 3.732E-06 | 3.561E-06 |
| **4.5** | 3.398E-06 | 3.241E-06 | 3.092E-06 | 2.949E-06 | 2.813E-06 | 2.682E-06 | 2.558E-06 | 2.439E-06 | 2.325E-06 | 2.216E-06 |
| **4.6** | 2.112E-06 | 2.013E-06 | 1.919E-06 | 1.828E-06 | 1.742E-06 | 1.660E-06 | 1.581E-06 | 1.506E-06 | 1.434E-06 | 1.366E-06 |
| **4.7** | 1.301E-06 | 1.239E-06 | 1.179E-06 | 1.123E-06 | 1.069E-06 | 1.017E-06 | 9.680E-07 | 9.211E-07 | 8.765E-07 | 8.339E-07 |
| **4.8** | 7.933E-07 | 7.547E-07 | 7.178E-07 | 6.827E-07 | 6.492E-07 | 6.173E-07 | 5.869E-07 | 5.580E-07 | 5.304E-07 | 5.042E-07 |
| **4.9** | 4.792E-07 | 4.554E-07 | 4.327E-07 | 4.111E-07 | 3.906E-07 | 3.711E-07 | 3.525E-07 | 3.348E-07 | 3.179E-07 | 3.019E-07 |

# Table of the Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2}\, dt$$

| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **5.0** | 2.867E-07 | 2.722E-07 | 2.584E-07 | 2.452E-07 | 2.328E-07 | 2.209E-07 | 2.096E-07 | 1.989E-07 | 1.887E-07 | 1.790E-07 |
| **5.1** | 1.698E-07 | 1.611E-07 | 1.528E-07 | 1.449E-07 | 1.374E-07 | 1.302E-07 | 1.235E-07 | 1.170E-07 | 1.109E-07 | 1.051E-07 |
| **5.2** | 9.964E-08 | 9.442E-08 | 8.946E-08 | 8.476E-08 | 8.029E-08 | 7.605E-08 | 7.203E-08 | 6.821E-08 | 6.459E-08 | 6.116E-08 |
| **5.3** | 5.790E-08 | 5.481E-08 | 5.188E-08 | 4.911E-08 | 4.647E-08 | 4.398E-08 | 4.161E-08 | 3.937E-08 | 3.724E-08 | 3.523E-08 |
| **5.4** | 3.332E-08 | 3.151E-08 | 2.980E-08 | 2.818E-08 | 2.664E-08 | 2.518E-08 | 2.381E-08 | 2.250E-08 | 2.127E-08 | 2.010E-08 |
| **5.5** | 1.899E-08 | 1.794E-08 | 1.695E-08 | 1.601E-08 | 1.512E-08 | 1.428E-08 | 1.349E-08 | 1.274E-08 | 1.203E-08 | 1.135E-08 |
| **5.6** | 1.072E-08 | 1.012E-08 | 9.548E-09 | 9.010E-09 | 8.503E-09 | 8.022E-09 | 7.569E-09 | 7.140E-09 | 6.735E-09 | 6.352E-09 |
| **5.7** | 5.990E-09 | 5.649E-09 | 5.326E-09 | 5.022E-09 | 4.734E-09 | 4.462E-09 | 4.206E-09 | 3.964E-09 | 3.735E-09 | 3.519E-09 |
| **5.8** | 3.316E-09 | 3.124E-09 | 2.942E-09 | 2.771E-09 | 2.610E-09 | 2.458E-09 | 2.314E-09 | 2.179E-09 | 2.051E-09 | 1.931E-09 |
| **5.9** | 1.818E-09 | 1.711E-09 | 1.610E-09 | 1.515E-09 | 1.425E-09 | 1.341E-09 | 1.261E-09 | 1.186E-09 | 1.116E-09 | 1.049E-09 |
| **6.0** | 9.866E-10 | 9.276E-10 | 8.721E-10 | 8.198E-10 | 7.706E-10 | 7.242E-10 | 6.806E-10 | 6.396E-10 | 6.009E-10 | 5.646E-10 |
| **6.1** | 5.303E-10 | 4.982E-10 | 4.679E-10 | 4.394E-10 | 4.126E-10 | 3.874E-10 | 3.637E-10 | 3.414E-10 | 3.205E-10 | 3.008E-10 |
| **6.2** | 2.823E-10 | 2.649E-10 | 2.486E-10 | 2.332E-10 | 2.188E-10 | 2.052E-10 | 1.925E-10 | 1.805E-10 | 1.693E-10 | 1.587E-10 |
| **6.3** | 1.488E-10 | 1.395E-10 | 1.308E-10 | 1.226E-10 | 1.149E-10 | 1.077E-10 | 1.009E-10 | 9.451E-11 | 8.854E-11 | 8.294E-11 |
| **6.4** | 7.769E-11 | 7.276E-11 | 6.814E-11 | 6.380E-11 | 5.974E-11 | 5.593E-11 | 5.235E-11 | 4.900E-11 | 4.586E-11 | 4.292E-11 |
| **6.5** | 4.016E-11 | 3.758E-11 | 3.515E-11 | 3.288E-11 | 3.076E-11 | 2.877E-11 | 2.690E-11 | 2.516E-11 | 2.352E-11 | 2.199E-11 |
| **6.6** | 2.056E-11 | 1.922E-11 | 1.796E-11 | 1.678E-11 | 1.568E-11 | 1.465E-11 | 1.369E-11 | 1.279E-11 | 1.195E-11 | 1.116E-11 |
| **6.7** | 1.042E-11 | 9.731E-12 | 9.086E-12 | 8.483E-12 | 7.919E-12 | 7.392E-12 | 6.900E-12 | 6.439E-12 | 6.009E-12 | 5.607E-12 |
| **6.8** | 5.231E-12 | 4.880E-12 | 4.552E-12 | 4.246E-12 | 3.960E-12 | 3.692E-12 | 3.443E-12 | 3.210E-12 | 2.993E-12 | 2.790E-12 |
| **6.9** | 2.600E-12 | 2.423E-12 | 2.258E-12 | 2.104E-12 | 1.960E-12 | 1.826E-12 | 1.701E-12 | 1.585E-12 | 1.476E-12 | 1.374E-12 |
| **7.0** | 1.280E-12 | 1.192E-12 | 1.109E-12 | 1.033E-12 | 9.612E-13 | 8.946E-13 | 8.325E-13 | 7.747E-13 | 7.208E-13 | 6.706E-13 |
| **7.1** | 6.238E-13 | 5.802E-13 | 5.396E-13 | 5.018E-13 | 4.667E-13 | 4.339E-13 | 4.034E-13 | 3.750E-13 | 3.486E-13 | 3.240E-13 |
| **7.2** | 3.011E-13 | 2.798E-13 | 2.599E-13 | 2.415E-13 | 2.243E-13 | 2.084E-13 | 1.935E-13 | 1.797E-13 | 1.669E-13 | 1.550E-13 |
| **7.3** | 1.439E-13 | 1.336E-13 | 1.240E-13 | 1.151E-13 | 1.068E-13 | 9.910E-14 | 9.196E-14 | 8.531E-14 | 7.914E-14 | 7.341E-14 |
| **7.4** | 6.809E-14 | 6.315E-14 | 5.856E-14 | 5.430E-14 | 5.034E-14 | 4.667E-14 | 4.326E-14 | 4.010E-14 | 3.716E-14 | 3.444E-14 |
| **7.5** | 3.191E-14 | 2.956E-14 | 2.739E-14 | 2.537E-14 | 2.350E-14 | 2.176E-14 | 2.015E-14 | 1.866E-14 | 1.728E-14 | 1.600E-14 |
| **7.6** | 1.481E-14 | 1.370E-14 | 1.268E-14 | 1.174E-14 | 1.086E-14 | 1.005E-14 | 9.297E-15 | 8.600E-15 | 7.954E-15 | 7.357E-15 |
| **7.7** | 6.803E-15 | 6.291E-15 | 5.816E-15 | 5.377E-15 | 4.971E-15 | 4.595E-15 | 4.246E-15 | 3.924E-15 | 3.626E-15 | 3.350E-15 |
| **7.8** | 3.095E-15 | 2.859E-15 | 2.641E-15 | 2.439E-15 | 2.253E-15 | 2.080E-15 | 1.921E-15 | 1.773E-15 | 1.637E-15 | 1.511E-15 |
| **7.9** | 1.395E-15 | 1.287E-15 | 1.188E-15 | 1.096E-15 | 1.011E-15 | 9.326E-16 | 8.602E-16 | 7.934E-16 | 7.317E-16 | 6.747E-16 |
| **8.0** | 6.221E-16 | 5.735E-16 | 5.287E-16 | 4.874E-16 | 4.492E-16 | 4.140E-16 | 3.815E-16 | 3.515E-16 | 3.238E-16 | 2.983E-16 |
| **8.1** | 2.748E-16 | 2.531E-16 | 2.331E-16 | 2.146E-16 | 1.976E-16 | 1.820E-16 | 1.675E-16 | 1.542E-16 | 1.419E-16 | 1.306E-16 |
| **8.2** | 1.202E-16 | 1.106E-16 | 1.018E-16 | 9.361E-17 | 8.611E-17 | 7.920E-17 | 7.284E-17 | 6.698E-17 | 6.159E-17 | 5.662E-17 |
| **8.3** | 5.206E-17 | 4.785E-17 | 4.398E-17 | 4.042E-17 | 3.715E-17 | 3.413E-17 | 3.136E-17 | 2.881E-17 | 2.646E-17 | 2.431E-17 |
| **8.4** | 2.232E-17 | 2.050E-17 | 1.882E-17 | 1.728E-17 | 1.587E-17 | 1.457E-17 | 1.337E-17 | 1.227E-17 | 1.126E-17 | 1.033E-17 |
| **8.5** | 9.480E-18 | 8.697E-18 | 7.978E-18 | 7.317E-18 | 6.711E-18 | 6.154E-18 | 5.643E-18 | 5.174E-18 | 4.744E-18 | 4.348E-18 |
| **8.6** | 3.986E-18 | 3.653E-18 | 3.348E-18 | 3.068E-18 | 2.811E-18 | 2.575E-18 | 2.359E-18 | 2.161E-18 | 1.979E-18 | 1.812E-18 |
| **8.7** | 1.659E-18 | 1.519E-18 | 1.391E-18 | 1.273E-18 | 1.166E-18 | 1.067E-18 | 9.763E-19 | 8.933E-19 | 8.174E-19 | 7.478E-19 |
| **8.8** | 6.841E-19 | 6.257E-19 | 5.723E-19 | 5.234E-19 | 4.786E-19 | 4.376E-19 | 4.001E-19 | 3.657E-19 | 3.343E-19 | 3.055E-19 |
| **8.9** | 2.792E-19 | 2.552E-19 | 2.331E-19 | 2.130E-19 | 1.946E-19 | 1.777E-19 | 1.623E-19 | 1.483E-19 | 1.354E-19 | 1.236E-19 |
| **9.0** | 1.129E-19 | 1.030E-19 | 9.404E-20 | 8.584E-20 | 7.834E-20 | 7.148E-20 | 6.523E-20 | 5.951E-20 | 5.429E-20 | 4.952E-20 |
| **9.1** | 4.517E-20 | 4.119E-20 | 3.756E-20 | 3.425E-20 | 3.123E-20 | 2.847E-20 | 2.595E-20 | 2.365E-20 | 2.155E-20 | 1.964E-20 |
| **9.2** | 1.790E-20 | 1.631E-20 | 1.486E-20 | 1.353E-20 | 1.232E-20 | 1.122E-20 | 1.022E-20 | 9.307E-21 | 8.474E-21 | 7.714E-21 |
| **9.3** | 7.022E-21 | 6.392E-21 | 5.817E-21 | 5.294E-21 | 4.817E-21 | 4.382E-21 | 3.987E-21 | 3.627E-21 | 3.299E-21 | 3.000E-21 |
| **9.4** | 2.728E-21 | 2.481E-21 | 2.255E-21 | 2.050E-21 | 1.864E-21 | 1.694E-21 | 1.540E-21 | 1.399E-21 | 1.271E-21 | 1.155E-21 |
| **9.5** | 1.049E-21 | 9.533E-22 | 8.659E-22 | 7.864E-22 | 7.142E-22 | 6.485E-22 | 5.888E-22 | 5.345E-22 | 4.852E-22 | 4.404E-22 |
| **9.6** | 3.997E-22 | 3.627E-22 | 3.292E-22 | 2.986E-22 | 2.709E-22 | 2.458E-22 | 2.229E-22 | 2.022E-22 | 1.834E-22 | 1.663E-22 |
| **9.7** | 1.507E-22 | 1.367E-22 | 1.239E-22 | 1.123E-22 | 1.018E-22 | 9.223E-23 | 8.358E-23 | 7.573E-23 | 6.861E-23 | 6.215E-23 |
| **9.8** | 5.629E-23 | 5.098E-23 | 4.617E-23 | 4.181E-23 | 3.786E-23 | 3.427E-23 | 3.102E-23 | 2.808E-23 | 2.542E-23 | 2.300E-23 |
| **9.9** | 2.081E-23 | 1.883E-23 | 1.704E-23 | 1.541E-23 | 1.394E-23 | 1.261E-23 | 1.140E-23 | 1.031E-23 | 9.323E-24 | 8.429E-24 |

## Communications IV Data Sheet

### 1. Correlation and Power Spectrum

$$R_{xy}(t_1,t_2) = E\{y(t_1)y^*(t_2)\}$$
$$R_{xy}(\tau) = E\{x(t)y^*(t-\tau)\}$$
$$R_{xx}(-\tau) = R_{xx}(\tau)$$
$$R_{yx}(\tau) = R^*_{xy}(-\tau)$$
$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}\,d\tau$$
$$S_{yx}(f) = S^*_{xy}(f)$$

### 2. Linear Time Invariant Systems

$$Y(f) = H(f)X(f)$$
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda = h(t)\otimes x(t)$$
$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$
$$S_{xy}(f) = H^*(f)S_{xx}(f)$$
$$S_{yx}(f) = H(f)S_{xx}(f)$$

### 3. Analytic Signal and Hilbert Transform

$$x^+(t) = x(t) + j\hat{x}(t) \qquad (analytic\ signal)$$
$$X^+(f) = 2u(f)X(f)$$
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t-\lambda}\,d\lambda \qquad (hilbert\ transform)$$
$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

### 4. Noise Bandwidth

$$B_n = \frac{1}{|H_o|^2}\int_0^{\infty}|H(f)|^2\,df$$

### 5. Narrowband Noise

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t)$$
$$S^{(+)}_{nn}(f) = u(f)S_{nn}(f)$$
$$S^{(-)}_{nn}(f) = u(-f)S_{nn}(f)$$
$$S_{n_c n_c}(f) = S_{n_s n_s}(f) = S^{(+)}_{nn}(f+f_o) + S^{(-)}_{nn}(f-f_o)$$
$$S_{n_c n_s}(f) = j\left\{S^{(+)}_{nn}(f+f_o) - S^{(-)}_{nn}(f-f_o)\right\}$$

### 6. Analog Modulation

Baseband signal $m(t)$, $|m(t)| \le 1$, Bandwidth $= W$

Noise added to signal s(t) is $n(t)$, $S_{nn}(f) = N_o/2$

---

$$s_{base}(t) = Am(t)$$
$$B = W$$
$$SNR_{base} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

---

$$s_{am}(t) = A\{1 + am(t)\}\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{am} = \frac{A^2 a^2 <m^2>}{2N_o W} = \frac{a^2 <m^2>}{1+a^2 <m^2>}\frac{P_r}{N_o W}$$

---

$$s_{dsbsc}(t) = Am(t)\cos(2\pi f_o t)$$
$$B = 2W$$
$$SNR_{dsbsc} = \frac{A^2 <m^2>}{2N_o W} = \frac{P_r}{N_o W}$$

---

$$s_{ssbsc}(t) = A\{m(t)\cos(2\pi f_o t) - \hat{m}(t)\sin(2\pi f_o t)\}$$
$$B = W$$
$$SNR_{ssbsc} = \frac{A^2 <m^2>}{N_o W} = \frac{P_r}{N_o W}$$

---

$$s_{fm}(t) = A\cos\left(2\pi f_o t + 2\pi f_d \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$
$$B = 2(f_d + W)$$
$$SNR_{fm} = \frac{3A^2 f_d^2 <m^2>}{2N_o W^3} = 3 <m^2> \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_o W}$$

Threshold at $\frac{P_r}{N_o B} = 10$ $(10\ dB)$

---

**7. Information Theory**

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$$H(x) = -\sum_i P(x_i)\log_2 P(x_i)\ bits/symbol$$

$$H(x,y) = -\sum_i \sum_j P(x_i,y_j)\log_2 P(x_i,y_j)$$

$$H(y|x) = -\sum_i \sum_j P(x_i,y_j)\log_2 P(y_j|x_i)$$

$$= H(x,y) - H(x)$$

$$I(x,y) = H(x) - H(x|y) = H(y) - H(y|x)$$

$$C = r\max I(x,y)\ \ bits/\text{sec}\ (discrete\ channel)$$

$$C = B\log_2(1 + P_s/P_n)\ \text{bits/sec (continuous channel)}$$

**8. Digital Modulation**

$$s_o(t_o) = \pm V,\ \Gamma = \frac{V^2}{<n_o^2>}\ \text{(binary antipodal signals)}$$

$$P_b = Q\left\{\sqrt{\Gamma}\right\}\ \text{(bit error, binary antipodal system)}$$

$$h(t) = c\,s(t_o - t)\ \text{(matched filter)}$$

$$\Gamma = \frac{2E_s}{N_o}\ (matched\ \ filter, baseband\ \&\ BPSK)$$

$$P_{sym} \approx Q\left\{\sqrt{d^2/2N_o}\right\}\ \text{(matched filter, spacing d)}$$

$$Constellation\ radius = \sqrt{\text{(symbol energy)}}$$

$$S_{xx}(f) = \frac{E\{a_k^2\}}{T}|P(f)|^2 \quad ;\ x(t) = \sum_{k=-\infty}^{\infty} a_k\ p(t-kT)$$

$$s_{QAM}(t) = \sum_{k=-\infty}^{\infty} A\,p(t-kT)\left[a_k\cos\omega_o t - b_k\sin\omega_o t\right]$$

$$s_{PSK}(t) = \sum_{k=-\infty}^{\infty} A\,p(t-kT)\cos(\omega_o t + \theta_k)$$

**9. Nyquist Pulse**

$$W = \frac{1}{2T}$$

$$P(f) = \begin{cases} T & ;|f| < (1-\rho)W \\ 0.5T - 0.5T\sin\left(\frac{\pi(|f|-W)}{2\rho W}\right) & ;||f| - W| < \rho W \\ 0 & ;elsewhere \end{cases}$$

$$p(t) = \frac{\pi}{4}\text{sinc}\left(\frac{t}{T}\right)\left\{\text{sinc}\left(\frac{\rho t}{T}+\frac{1}{2}\right) + \text{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)\right\}$$

**10. Coding**

$$P(i) = \binom{n}{i}P_b^i Q_b^{n-i}\ \text{(binomial distribution)}$$

$$P_b = Q\left\{\sqrt{RE_b/\alpha}\right\}\text{(BER before correction)}$$

$$P_{cbe} \approx \frac{2t+1}{n}\binom{n}{t+1}P_b^{t+1}\ \text{(BER, correcting t errors)}$$

$$\tilde{x} = \tilde{m}G,\ \ G = [I\ \ P] = generator\ matrix$$

$$\tilde{y} = \tilde{x} + \tilde{e},\ \tilde{s} = \tilde{y}H = \text{syndrome}$$

$$H = \begin{bmatrix} P \\ I \end{bmatrix} = \text{parity check matrix}$$

Hamming codes $n = 2^q - 1$ (correct one error per block)

$$2^q \geq \sum_{i=0}^{t}\binom{n}{i}\text{(necessary for existence of code)}$$

$$X(p) = p^q M(p) + C(p)\ (\textit{cyclic}\ \text{code generation})$$

$$C(p) = \text{rem}\left\{\frac{p^q M(p)}{G(p)}\right\}\text{(check digits)}$$

$$S(p) = \text{rem}\left\{\frac{Y(p)}{G(p)}\right\}\text{(syndrome, cyclic code)}$$

**11. Gaussian Probability**

One dimension (mean $\eta$, variance $\sigma^2$):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-(x-\eta)^2/2\sigma^2}$$

$$P\{x > V\} = Q\left\{\frac{V-\eta}{\sigma}\right\}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\,dt$$

Two dimensions (zero means):

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}\ e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right]}$$

$$\sigma_x^2 = E\{x^2\},\ \sigma_y^2 = E\{y^2\},\ \rho = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

# OPTICAL COMMUNICATIONS
# SUMMARY OF SOME USEFUL FORMULAE

## General

$$\Delta(\tau/L)_{\text{dis}} = -(M + M_g)\Delta\lambda$$

$$f_{3dB}(\text{opt.}) \times L = \frac{0.5}{\Delta(\tau/L)}$$

$$f_{3dB}(\text{elec.}) \times L = \frac{0.35}{\Delta(\tau/L)}$$

$$R_{RZ} \times L = \frac{0.35}{\Delta(\tau/L)}$$

$$R_{NRZ} \times L = \frac{0.7}{\Delta(\tau/L)}$$

## Resonant Cavities

$$\Delta f_c = \frac{c}{2Ln}$$

## Optics

$$R(\text{normal incidence}) = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

## Optical Fibres

Step Index:

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\Delta(\tau/L)_{\text{mod}} = \frac{n_1\Delta}{c}$$

$$\text{Single mode:} \quad a/\lambda_0 < \frac{2.405}{2\pi.\text{NA}}$$

Graded Index (Parabolic Profile):

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$n(r) = n_1\sqrt{1 - 2(r/a)^2\Delta} \text{ for } r \leq a$$

$$\text{NA} = n_1\sqrt{2\Delta}\sqrt{1 - (r/a)^2}$$

$$\Delta(\tau/L)_{\text{mod}} = \frac{n_1\Delta^2}{2c}$$

$$n_{\text{eff}} = n_1 - (p + q + 1)\frac{\sqrt{2\Delta}}{k_0 a}$$

## Source Bandwidth

$$
\begin{aligned}
P_{SP} &= \frac{a_1 I_{SP}}{\sqrt{1+\omega^2\tau^2}} \\
f_{3dB} &= \frac{1}{2\pi\tau} \\
t_r &= \frac{0.35}{f_{3dB}}
\end{aligned}
$$

## Source Coupling

For a source with pattern $\cos^m \theta$:

$$
\begin{aligned}
\eta_{\text{SI}} &= 1 - \left[1-(\text{NA})^2\right]^{(m+1)/2} \text{ to SI Fibre} \\
\eta_{\text{GRIN}} &= \eta_{\text{SI}} \left\{1 - 0.5\left(\frac{a_e}{a_f}\right)^2\right\} \text{ to GRIN Fibre where } a_e \leq a_f
\end{aligned}
$$

## Optical Detectors

$$
\begin{aligned}
\rho &= i/P = \frac{M\eta e\lambda}{hc} \\
f_{3dB} &= \frac{0.35}{t_r} \\
t_r &= 2.19 R_L C_d
\end{aligned}
$$

## Analogue Modulation

AM/IM:

$$
\begin{aligned}
i &= I_0 + I_s(1+m\cos\omega_m t)\cos\omega_{sc} t \\
P &= P_0 + P_s(1+m\cos\omega_m t)\cos\omega_{sc} t
\end{aligned}
$$

FM/IM:

$$
\begin{aligned}
i &= I_0 + I_s\cos(\omega_{sc} t + \beta\sin\omega_m t) \\
P &= P_0 + P_s\cos(\omega_{sc} t + \beta\sin\omega_m t) \\
B &= \frac{\omega_m}{\pi}(1+\beta)
\end{aligned}
$$

## LED Operating Characteristics

$$
\begin{aligned}
\lambda_0 &= \frac{1.24}{W_g(\text{eV})} \\
P &= \eta i W_g(\text{eV}) \\
P_s &= \frac{a_1 I_s}{\sqrt{1+\omega^2\tau^2}}
\end{aligned}
$$

## Harmonic Distortion

$$P = P_{dc} + a_1 i_s + a_2 i_s^2 + \dots$$
$$\text{THD} = 0.25\left(\frac{a_2 I}{a_1}\right)^2 \quad \text{for } i_s = I\sin\omega t$$

## Receiver Design

General Signal to Noise Ratio:

$$S/N = \frac{(m^2/2)(M\rho P)^2 R_L}{M^n 2eR_L\Delta f(I_D + \rho P) + 4kT\Delta f F}$$

Thermal noise limited probability of error:

$$P_e = \frac{1}{2} - \frac{1}{2}\text{erf}\left(0.354\sqrt{S/N}\right)$$
$$\text{erf}(x) \approx 1 - \frac{e^{-x^2}}{\sqrt{\pi}x} \quad \text{for } x \geq 3$$

Peak S/N (with threshold set at $0.5i_s$) for specified bit error rates.

| $P_s$ | S/N |
|---|---|
| $10^{-4}$ | 17.4dB |
| $10^{-6}$ | 19.5dB |
| $10^{-9}$ | 21.5dB |

Quantum noise limited probability of error:

$$P_e = e^{-n_s}$$
$$n_s = \frac{\eta P\tau}{hf}$$

## Heterodyne Receivers

$$i_{dc} = \frac{\eta e}{hf}(P_L + P_S)$$
$$i_{IF} = \frac{2\eta e}{hf}\sqrt{P_L P_S}\cos\left[\omega_{IF} t - \theta(t)\right]$$
$$S/N = \frac{2\left(\eta e/hf\right)^2 R_L P_S P_L}{2eR_L\Delta f\left[I_D + (\eta e P_L/hf)(1 + P_S/P_L)\right] + 4kT\Delta f}$$

## Some Physical Constants

| | |
|---|---|
| Speed of Light in Free Space | $c = 3\times10^8$ m/s |
| Boltzmann's Constant | $k = 1.38\times10^{-23}$ J/K |
| Planck's Constant | $h = 6.626\times10^{-34}$ Js |
| Electron Charge | $e = 1.6\times10^{-19}$ C |