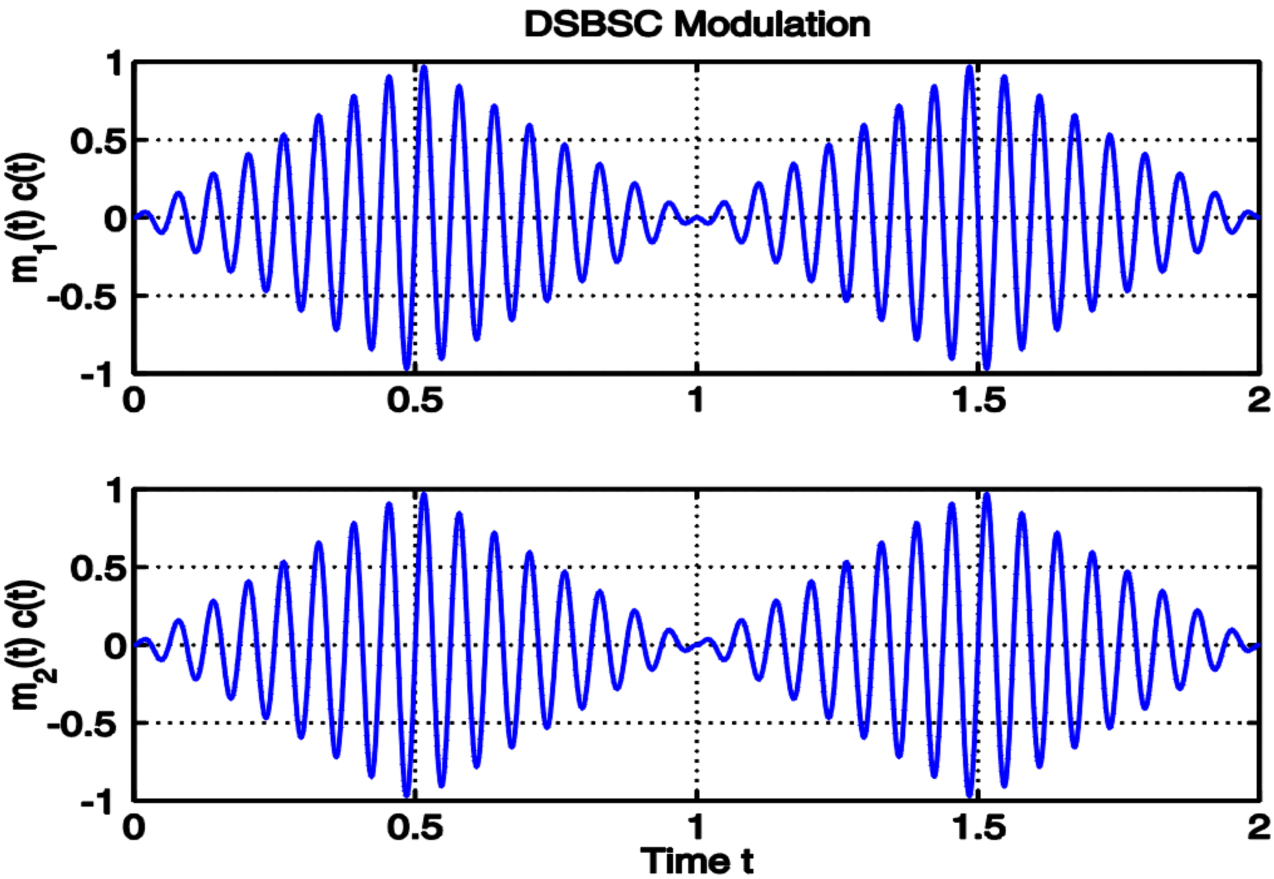
Communications IV Exercise Solutions

Exercise 3.3

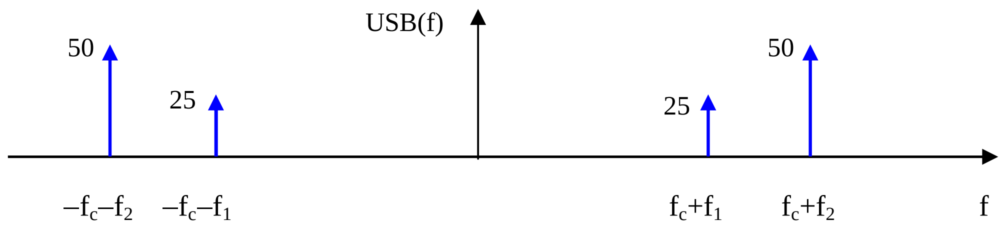
The modulated signals look the same and have the same envelope, but for $m_1(t)$ the phase of the carrier is reversed when $m_1(t)$ is negative. Hence a synchronous detector is required to correctly demodulate $m_1(t)$, but an envelope detector would suffice for $m_2(t)$.



Exercise 3.8

We note that f_1 = 1000 Hz, f_2 = 2000 Hz and f_c = 1 MHz.

$$m(t) = \cos(2\pi f_1 t) + 2\cos(2\pi f_2 t)$$
$$c(t) = 100\cos(2\pi f_c t)$$
$$v(t) = m(t)c(t) = 50\cos[2\pi(f_c + f_1)t] + 50\cos[2\pi(f_c - f_1)t] + 100\cos[2\pi(f_c + f_2)t] + 100\cos[2\pi(f_c - f_2)t]$$
$$USB(t) = 50\cos[2\pi(f_c + f_1)t] + 100\cos[2\pi(f_c + f_2)t]$$
$$USB(f) = 25\delta(f - f_c - f_1) + 50\delta(f - f_c - f_2) + 25\delta(f + f_c + f_1) + 50\delta(f + f_c + f_2)$$



Exercise 3.9

$$c(t) = \frac{4}{\pi}\left\{\sin(\omega_c t) + \frac{1}{3}\sin(3\omega_c t) + \frac{1}{5}\sin(5\omega_c t) + \cdots\right\}$$
$$v(t) = m(t)\,c(t)$$
$$V(f) = \frac{2}{j\pi}\left\{M(f - f_c) - M(f + f_c) + \frac{M(f - 3f_c)}{3} - \frac{M(f + 3f_c)}{3} + \frac{M(f - 5f_c)}{5} - \frac{M(f + 5f_c)}{5} + \cdots\right\}$$
$$V_{filt}(f) = \frac{2}{j\pi}\{M(f - f_c) - M(f + f_c)\}$$
$$v_{filt}(t) = \frac{4}{\pi}m(t)\sin(\omega_c t)$$

Filtering will remove the components modulating multiples of the carrier frequency provided the carrier frequency $f_c > W$, where W is the bandwidth of m(t), because the highest frequency of the wanted signal is $f_c + W$ and the lowest frequency not of interest is $3f_c - W$.

Exercise 3.19

$$x(t) = m(t) + \cos(\omega_c t)$$
$$y(t) = a\,x(t) + b\,x^2(t)$$
$$= a\,m(t) + a\cos(\omega_c t) + b\,m^2(t) + 2\,b\,m(t)\cos(\omega_c t) + b\ \cos^2(\omega_c t)$$
$$= a\cos(\omega_c t) + 2\,b\,m(t)\cos(\omega_c t) + \left[a\,m(t) + b\,m^2(t) + b\ \cos^2(\omega_c t)\right]$$
$$v(t) = a\cos(\omega_c t) + 2\,b\,m(t)\cos(\omega_c t)$$

The terms in the square bracket have DC components from $m^2(t)$ and $\cos^2(\omega_c t)$ and low frequency components from m(t) and $m^2(t)$ plus high frequency components in the vicinity of $2f_c$ from $\cos^2(\omega_c t)$. The wanted signal v(t) is obtained using a bandpass filter centred at f_c of bandwidth twice that of m(t), and this will reject the low frequency terms and the components centred at $2f_c$.

Since m(t) has a maximum amplitude of $\pm A_m$, the modulation index is (largest modulation term)/(carrier term) = $2bA_m/a$.

Exercise 3.20

These are all straightforward.

Modulation Method	**Bandwidth of RF Signal**	**Comment**
SSBSC	10 kHz	Same for USB as LSB
DSBSC	20 kHz	Double the baseband
AM, a=0.8	20 kHz	Value of a is irrelevant
FM, f_d=60 kHz	140 kHz	Use Carson's rule

Solutions to Exercises on Slides

Slide 3.32

Let $m(t) = m_1(t) + m_2(t)$, where $m_1(t)$ contains frequency components for $|f| < W_1$ and $m_2(t)$ contains frequency components for $W_1 < |f| < W_2$. The vestigial sideband spectrum extends from W_1 below the carrier frequency to W_2 above it.

I will work with the phasor components. The transmitted signal is $s_T(t)$, the receiver filter (in phasor form) is $H(f)$, the received signal is $s_R(t)$ and the synchronous detector output is $s_d(t)$. Note that the synchronous detector produces the cosine component, which is the real part of the phasor.

Also note the 0.5 factor associated with the single sideband component. This is necessary to get the correct spectrum of the vestigial sideband signal (as given in the third line below).

$$\begin{aligned}
\tilde{s}_T(t) &= (\text{amplitude modulation part}) + (\text{single sideband part}) \\
&= A + Aa\,m_1(t) + 0.5Aa\,m_2(t) + 0.5jAa\,\hat{m}_2(t) \\
\tilde{S}_T(f) &= A\,\delta(f) + Aa\,M_1(f) + 0.5Aa\,M_2(f) + j0.5Aa[-j\,\mathrm{sgn}(f)]M_2(f) \\
&= A\,\delta(f) + Aa\,M_1(f) + Aa\,u(f)\,M_2(f)
\end{aligned}$$

$$H(f) = \begin{cases} \dfrac{f + W_1}{2W_1} & ;|f| < W_1 \\ 1 & ;W_1 < f < W_2 \end{cases}$$

$$\begin{aligned}
\tilde{S}_r(f) &= \tilde{S}_T(f)\,H(f) \\
&= \begin{cases} \dfrac{f + W_1}{2W_1}\left[A\,\delta(f) + Aa\,M_1(f)\right] & ;|f| < W_1 \\ Aa\,u(f)\,M_2(f) & ;W_1 < f < W_2 \end{cases}
\end{aligned}$$

$$\tilde{s}_r(t) = 0.5A + \frac{Aa}{j4\pi W_1}\frac{dm_1(t)}{dt} + 0.5Aa\,m_1(t) + 0.5Aa\,m_2(t) + j0.5Aa\,\hat{m}_2(t)$$

$$s_d(t) = \mathrm{Re}\{\tilde{s}_r(t)\} = 0.5A + 0.5Aa\,m_1(t) + 0.5Aa\,m_2(t)$$

Hence a synchronous detector correctly demodulates the signal. An envelope detector produces an almost correct result because the imaginary parts of the phasor are usually quite small in comparison with the (real) carrier component.