Communications IV Exercises

Exercise 5.4

For m(t) uniformly distributed over (–1,1) we have $<m^2(t)> = 1/3$. You should derive this. We will find the average transmitter power required. $N_o = 1 \times 10^{-14}$ W/Hz.

1. For SSBSC,

$$SNR_o = \frac{P_r}{N_o W}$$

$$P_r = SNR_o \times N_o \times W = 10^3 \times 10^{-14} \times 1.5 \times 10^6 = 1.5 \times 10^{-5} \text{ W}$$

$$P_t = 10^9 P_r = 1.5 \times 10^4 = 15 \text{ kW}$$

2. For AM with a = 0.5,

$$SNR_o = \frac{a^2 < m^2 >}{1 + a^2 < m^2 >}\left(\frac{P_r}{N_o W}\right)$$

$$P_r = \frac{(1 + a^2 < m^2 >) \times SNR_o \times N_o \times W}{a^2 < m^2 >} = \frac{1.0833 \times 10^3 \times 10^{-14} \times 1.5 \times 10^6}{0.08333} = 1.95 \times 10^{-4} \text{ W}$$

$$P_t = 10^9 P_r = 1.95 \times 10^5 = 195 \text{ kW}$$

3. For DSBSC, same as for SSBSC so P_t = 15 kW.

Exercise 5.5

The noise bandwidth of the RF filter is 2000 + 1000/3 = 2333.3 Hz. You should derive this from the definition. $N_o = 2 \times 10^{-12}$ W/Hz.

1.

$$v(t) = 10^{-3}[1 + 0.5\cos pt]\cos(\omega_c t) + n(t)$$

$$v_p(t) = 10^{-3}[1 + 0.5\cos pt]\cos(\omega_c t) + n_p(t) = s_p(t) + n_p(t)$$

$$P\{s_p\} = 5 \times 10^{-7}(1 + 0.5^2 \times 0.5) = 5.625 \times 10^{-7} \text{ W}$$

$$P\{n_p\} = N_o \times B_n = 4.667 \times 10^{-9} \text{ W}$$

2.

$$v_p(t) = 10^{-3}[1 + 0.5\cos pt]\cos(\omega_c t) + n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$$

$$v_d(t) = 5 \times 10^{-4}\cos(pt) + n_c(t)$$

$$v_o(t) = 5 \times 10^{-4}\cos(pt) + n_o(t) = s_o(t) + n_o(t)$$

$$P_{so} = 1.25 \times 10^{-7}$$

$$P_{no} = 2N_o W = 2 \times 2 \times 10^{-12} \times 1000 = 4.00 \times 10^{-9}$$

$$SNR_o = 31.25 \text{ (14.9 dB)}$$

Exercise 5.9

Compare on basis of average powers.

$$SNR_{am} = \frac{a^2 < m^2 >}{1 + a^2 < m^2 >}\left(\frac{P_r}{N_o W}\right) = 0.067382\left(\frac{P_r}{N_o W}\right)$$

$$B_{fm} = 2(f_d + W) = 100\,\text{kHz} \Rightarrow f_d = 46\ \text{kHz.}$$

$$SNR_{fm} = 3 < m^2 > \left(\frac{f_d}{W}\right)^2\left(\frac{P_r}{N_o W}\right) = 39.675\left(\frac{P_r}{N_o W}\right)$$

$$\frac{SNR_{fm}}{SNR_{am}} = \frac{39.675}{.067382} = 588.8\ \ (27.7\ \text{dB})$$

Exercise 5.10

Compare on basis of average powers.

$$P_t = 10000$$

$$P_r = P_t \times 10^{-8} = 10^{-4}\ \text{W}$$

$$N_o = 10^{-12}\ \text{W / Hz}$$

$$W = 5000$$

$$B = 100\ \text{kHz} = 2(f_d + W)$$

$$f_d = 45\ \text{kHz}$$

$$< m^2 >= 0.1$$

$$a = 0.8$$

$$SNR_{am} = \frac{a^2 < m^2 >}{1 + a^2 < m^2 >}\frac{P_r}{N_o W} = 1203\ (30.8\ \text{dB})$$

$$SNR_{fm} = 3 < m^2 > \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_o W} = 4.860 \times 10^5\ \ (56.9\ \text{dB})$$

Exercise 5.11

1.

$$W = 8\,\text{kHz}$$

$$< m^2 >= 0.5$$

$$N_o = 2 \times 10^{-12}\ \text{W / Hz}$$

$$B = 60\ \text{kHz} = 2(f_d + W)$$

$$f_d = 22\ \text{kHz}$$

$$SNR_{fm} = 3 < m^2 > \left(\frac{f_d}{W}\right)^2 \frac{P_r}{N_o W} = 10^4$$

$$P_r = 1.411 \times 10^{-5}\ \text{W} \Rightarrow P_t = 141.1\ \text{mW}$$

Should check $P_r/N_oB = 117.6 > 10$ (so system is above threshold).

2. If SNR increased to 60 dB, then $P_t = 14.11$ W.

3. Using pre-emphasis,

$$f_e = \frac{1}{2\pi 75 \times 10^{-6}} = 2.122 \text{ kHz}$$
$$W = 8 \text{ kHz}$$
$$\frac{N_{o1}}{N_{o2}} = 3\left(\frac{f_e}{W}\right)^2 - 2\left(\frac{f_e}{W}\right)^3 = 0.1738$$
$$P_t = 14.11 \times 0.1738 = 2.452 \text{ W}$$

This is a saving of 7.6 dB.

Solutions to Exercises on Slides

Slide 5.20

The peak power is the power in the modulated signal at the maximum of the modulation (ie. when m(t) = 1).

For a baseband system:

$$P_r = A^2 < m^2 > \qquad P_{pk} = A^2 \quad (\text{since } |m(t)|_{pk} = 1)$$
$$SNR_o = \frac{A^2 < m^2 >}{N_o W} = \left(\frac{P_r}{N_o W}\right) = \left(\frac{P_{pk}}{N_o W}\right) < m^2 >$$

For an AM system:

$$P_r = \frac{A^2 (1 + a^2 < m^2 >)}{2} \qquad P_{pk} = \frac{A^2 (1+a)^2}{2}$$
$$SNR_o = \frac{A^2 a^2 < m^2 >}{2 N_o W} = \left(\frac{P_r}{N_o W}\right)\left(\frac{a^2 < m^2 >}{1 + a^2 < m^2 >}\right) = \left(\frac{P_{pk}}{N_o W}\right)\left(\frac{a^2 < m^2 >}{(1+a)^2}\right)$$

For the same peak power, the SNR for an AM system with 100% modulation is 0.25 of that for a baseband system. Peak power is important in practice because most transmitters have a limit on the peak power, not the average power. Peak power was not considered in lectures this year.

Slide 5.50

$$\frac{N_{o2}}{N_{o1}} = 3\left(\frac{f_e}{W}\right)^2 - 2\left(\frac{f_e}{W}\right)^3 = 3\left(\frac{3.183}{15}\right)^2 - 2\left(\frac{3.183}{15}\right)^3 = 0.1160 \quad (-9.4\,\text{dB})$$

Putting H(f) = 1/(1+jf/f_e) gives:

$$N_{o1} = \frac{2}{3} c W^3 \quad \text{(as before)}$$

$$N_{o2} = \int_{-W}^{W} \frac{cf^2}{1 + f^2 / f_e^2} df = \int_{-W}^{W} \frac{cf^2 f_e^2}{f_e^2 + f^2} df$$

$$= \int_{-W}^{W} \left\{ cf_e^2 - \frac{cf_e^4}{f_e^2 + f^2} \right\} df = 2cf_e^2 W - 2cf_e^3 \arctan\left(\frac{W}{f_e}\right)$$

$$\frac{N_{o2}}{N_{o1}} = 2\left(\frac{f_e}{W}\right)^2 - 3\left(\frac{f_e}{W}\right)^3 \arctan\left(\frac{W}{f_e}\right) = 0.09605 \ (-10.2\,\text{dB})$$