Communications IV Exercise Solutions

Exercise 7.10

Assuming a matched filter is used:

$$S_{nn}(f) = \alpha = \frac{N_o}{2} = 5\times10^{-3}$$

$$SNR = \frac{E_b}{\alpha} = \frac{A^2T_b}{\alpha} = \frac{A^2\times10^{-5}}{5\times10^{-3}} = 0.002\,A^2 = 4.753^2 \text{ for } P_e = 10^{-6}$$

$$A = 106.3$$

Exercise 7.15

$$H(f) = \frac{1-e^{-j2\pi fT}}{j2\pi f}$$

$$h(t) = u(t) - u(t-T)$$

$$p(t) = h(T-t) = u(T-t) - u(-t) = u(t) - u(t-T)$$

Exercise 7.17

1.

$$s_o(T) = T\sqrt{E_b/T}$$

$$n_o(T) = \int_0^T n(\lambda)\,d\lambda$$

$$E\left\{n_o^2(T)\right\} = \int_0^T\int_0^T E\{n(\lambda)\,n(\mu)\}\,d\lambda\,d\mu$$

$$= \frac{N_o}{2}\int_0^T\int_0^T \delta(\lambda-\mu)\,d\lambda\,d\mu = \frac{N_oT}{2}$$

$$SNR = \frac{2E_b}{N_o}$$

By hindsight we see that this is a matched filter. But note that the integrator must be “dumped” (reset to zero) after each pulse.

2.

$$s_o(t) = \pm\sqrt{E_b/T}\left(1-e^{-t/RC}\right) \quad ;0\le t\le T$$

$$s_o(T) = \pm\sqrt{E_b/T}\left(1-e^{-T/RC}\right)$$

$$h(t) = u(t)\frac{1}{RC}e^{-t/RC}$$

$$n_o(T) = \int_0^T \frac{1}{RC}e^{-\lambda/RC}\,n(T-\lambda)\,d\lambda$$

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$$n_o(T) = \int_0^T \frac{1}{RC}e^{-\lambda/RC}\, n(T-\lambda)\, d\lambda$$

$$E\{n_o^2\} = \int_0^T\int_0^T \frac{1}{R^2C^2}e^{-\lambda/RC}\, e^{-\mu/RC}\, E\{n(T-\lambda)\, n(T-\mu)\}\, d\lambda d\mu$$

$$= \frac{N_o}{2R^2C^2}\int_0^T\int_0^T e^{-\lambda/RC}\, e^{-\mu/RC}\, \delta(\lambda-\mu)\, d\lambda d\mu$$

$$= \frac{N_o}{2R^2C^2}\int_0^T e^{-2\lambda/RC}\, d\lambda$$

$$= \frac{N_o\left(1 - e^{-2T/RC}\right)}{4RC}$$

$$SNR = \frac{4RC\,E_b\left(1 - e^{-T/RC}\right)^2}{N_o T\left(1 - e^{-2T/RC}\right)} = \frac{2E_b}{N_o} \text{ if } T << RC$$

The circuit shows that the capacitor must be “dumped” before the current symbol is applied, otherwise there will be significant ISI. If you plot the SNR a function of x = T/RC, it has a maximum at x = 0 and is monotonically decreasing. Hence the optimum RC is a value large compared with T, but this is equivalent to an integrator (ie. the same as in the first part).

Exercise 7.34

Assuming a matched filter,

$$N_o = 2\times10^{-10} \text{ W/Hz}$$

$$SNR = \frac{2E_b}{N_o} = \frac{A^2T}{N_o} = 4.753^2 \text{ if } P_b = 10^{-6}$$

$$A = 4.753\sqrt{N_o/T} = \begin{cases} 6.722\times10^{-3} & ;10\text{ kb/s} \\ 2.126\times10^{-2} & ;100\text{ kb/s} \\ 6.722\times10^{-2} & ;1\text{ Mb/s} \end{cases}$$

If the impedance of the system is R, the values of A must be multiplied by √(R).

Exercise 7.43

For constellation (a) the outer symbols have coordinates (±2A,±2A). The average energy per symbol is:

$$E_s = \frac{1}{8}\left(4 \times 4A^2 + 4 \times 8A^2\right) = 6.0A^2$$

For constellation (b) the symbols on the X axis have coordinates (±A,0), the two on the Y axis have coordinates (0,±A√(3)) and the outer symbols have coordinates (±2A,±A√(3)). The average energy per symbol is:

$$E_s = \frac{1}{8}\left(2 \times A^2 + 2 \times 3A^2 + 4 \times (4A^2 + 3A^2)\right) = 4.5A^2$$

These would have the same error performance, so clearly constellation (b) is more power efficient.

Solutions to Exercises on Slides

Slide 7.52

In MPSK the symbols are on a circle of radius r = $\sqrt{E_s}$ and are spaced d = 2r sin(π/M) and $E_b = E_s/K$. Hence

$$d^2 = 4KE_b \sin^2(\pi / M)$$

$$P_e = Q\left\{\sqrt{\frac{d^2}{2N_o}}\right\} = Q\left\{\sqrt{\frac{2KE_b \sin^2(\pi / M)}{N_o}}\right\}$$

To get the same error probability for 4PSK and 8PSK requires

$$E_{s4PSK} \sin^2(\pi / 4) = E_{s8PSK} \sin^2(\pi / 8)$$

$$\frac{E_{s8PSK}}{E_{s4PSK}} = 3.4142 \quad (5.3\,\text{dB})$$

$$\frac{E_{b8PSK}}{E_{b8PSK}} = \frac{2}{3} \times 3.4142 = 2.2761 \quad (3.6\,\text{dB})$$

Slide 7.53

The inner circle has radius $r_1 = d/\sqrt{2} = 0.7071d$. The symbol in the first quadrant on the outer circle has coordinates (x,x) and we must have $d^2 = x^2 + (x - r_1)^2$ giving

$$2x^2 - 2r_1 x + r_1^2 - d^2 = 0$$

$$x^2 - 0.7071dx - 0.25d^2 = 0$$

$$x = 0.96593\,d$$

$$r_2 = x\sqrt{2} = 1.3660\,d = 0.5(1+\sqrt{3})\,d$$

$$E_s(av) = 0.5(r_1^2 + r_2^2) = 1.1830\,d^2$$

$$E_b(av) = E_s(av)/3 = 0.39432\,d^2 = 19.953\,N_o$$

$$P_{sym} = Q\left\{\sqrt{\frac{d^2}{2N_o}}\right\} = Q\left\{\sqrt{\frac{19.953}{2 \times 0.39432}}\right\} = Q\{5.030\} = 2.45 \times 10^{-7}$$

since we have E_b/N_o = 13 dB = 19.953.