Communications IV Exercise Solutions

Exercise 8.2

If $a_m = 1$, the possible sample values in the absence of noise are $1 + i_m$. With additive gaussian noise of rms value $\sigma_n$, the probability of error for a decision boundary of 0 volt is:

$$P_e(i_m) = Q\left(\frac{1 + i_m}{\sigma_n}\right)$$

$$P_e(\text{avg}) = 0.25 Q\left(\frac{0.5}{\sigma_n}\right) + 0.5 Q\left(\frac{1.0}{\sigma_n}\right) + 0.25 Q\left(\frac{1.5}{\sigma_n}\right)$$

For $a_m = -1$, the result is the same. I used Matlab to generate the plot below.
Exercise 8.3
If the signal pulse is \( p(t) = u(t) - u(t-T) \), the matched filter output is \( \Delta I(t-T)/T \) which reaches its maximum value at time \( t = T \). If the sample occurs 10% early at \( t = 0.9T \), then the amplitude will be 90% of the ideal value. This will reduce the signal to noise ratio by 0.81 which is –0.92 dB. If the error rate was originally \( Q(3.09) = 1.00 \times 10^{-3} \), this would become \( Q(0.90 \times 3.09) = 2.71 \times 10^{-3} \). However, the situation is actually worse than that, as there is intersymbol interference from the previous data pulse, and this contributes ±0.1 to the sample value. Hence the possible sample values would be 0.8 giving an error of \( Q(0.8 \times 3.09) = 6.72 \times 10^{-3} \), or \( Q(1.0 \times 3.09) = 1.00 \times 10^{-3} \) for an average error probability of \( 3.86 \times 10^{-3} \).

The same occurs if the timing is 10% late.

![Matched filter output](image)

Exercise 8.10
1. The bandwidth of 1200 Hz would allow a maximum rate of 2400 bits/sec, but more realistically might be 1600 bits/sec if Nyquist pulses with \( \rho = 0.5 \) were used.

2. To achieve \( P_e = 10^{-7} \), with a matched filter we a signal to noise ratio of \( 5.199^2 = 2E_b/N_o \), giving \( E_b/N_o = 13.51 \) (11.3 dB).

3. If \( N_o = 4.1 \times 10^{-21} \) W/Hz, then \( E_b = 5.539 \times 10^{-20} \) J, which gives \( P = E_b \times \text{bit rate} = 1.329 \times 10^{-16} \) W for 2400 bits/sec. Allowing for the loss of 50 dB, this gives \( P = 1.329 \times 10^{-11} \) W (–108.8 dBW).

Exercise 8.15
Use \( M = 2^4 \) giving a symbol rate of \( 14400/4 = 3600 \) symbols/sec which is achievable with \( \rho = 1/3 \) Nyquist pulse shape. To achieve a symbol error probability of \( 10^{-6} \), we require \( d^2/2N_o = 4.753^2 \), giving \( d = 0.09506 \) since \( N_o/2 = 1 \times 10^{-4} \).

The symbol levels should be \( \pm(2k+1)d/2 \) for \( k = 0, 1, \ldots, 7 \). The average symbol energy \( E_s \) is:

\[
E_s = \frac{d^2}{32} \left[ 1^2 + 3^2 + 5^2 + \ldots + 15^2 \right] = 21.25d^2 = 0.1920 \text{ J}
\]

\[
E_b = \frac{0.1920}{4} = 0.04800 \text{ J}
\]

\[
\frac{E_b}{N_o} = 240.0 \text{ (23.8 dB)}
\]
Exercise 8.17

1. If the noise spectral density is $S_{nn}(f)$, we require a filter which satisfies $S_{nn}(f)|H_p(f)|^2 = 1$. This filter is required to be causal. We need not consider the details of its realisation.

2. The signal output from the whitening filter has a Fourier transform $\tilde{S}(f) = S(f)H_p(f)$, so the matched filter is $H_m(f) = S^*(f)e^{-j2\pi fT} = S^*(f)H_p^*(f)e^{-j2\pi fT}$, where $T$ is the sampling instant.

3. The overall response is therefore $H(f) = H_p(f)H_m(f) = S^*(f)|H_p(f)|^2 e^{-j2\pi fT} = S^*(f)e^{-j2\pi fT} / S_{nn}(f)$.

4. The signal to noise ratio is:

$$\Gamma = \frac{\text{Energy in } \tilde{s}(t)}{S_{\tilde{n}\tilde{n}}(f)} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 |H_p(f)|^2 df}{S_{nn}(f)|H_p(f)|^2} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 / S_{nn}(f) df}{S_{nn}(f)|H_p(f)|^2}$$