Communications IV Exercise Solutions

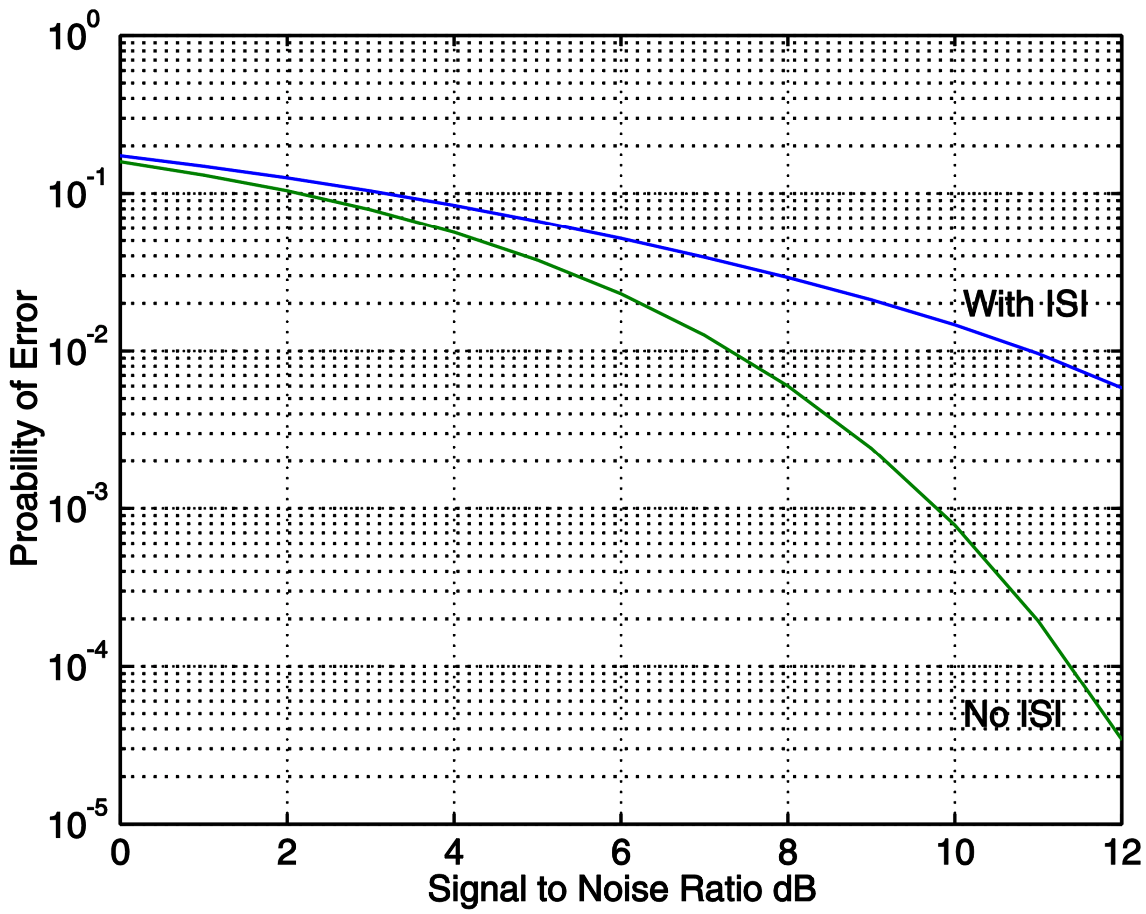
Exercise 8.2

If $a_m = 1$, the possible sample values in the absence of noise are $1 + i_m$. With additive gaussian noise of rms value σ_n, the probability of error for a decision boundary of 0 volt is:

$$P_e(i_m) = Q\left\{\frac{1+i_m}{\sigma_n}\right\}$$

$$P_e(\text{avg}) = 0.25\,Q\left\{\frac{0.5}{\sigma_n}\right\} + 0.5\,Q\left\{\frac{1.0}{\sigma_n}\right\} + 0.25\,Q\left\{\frac{1.5}{\sigma_n}\right\}$$

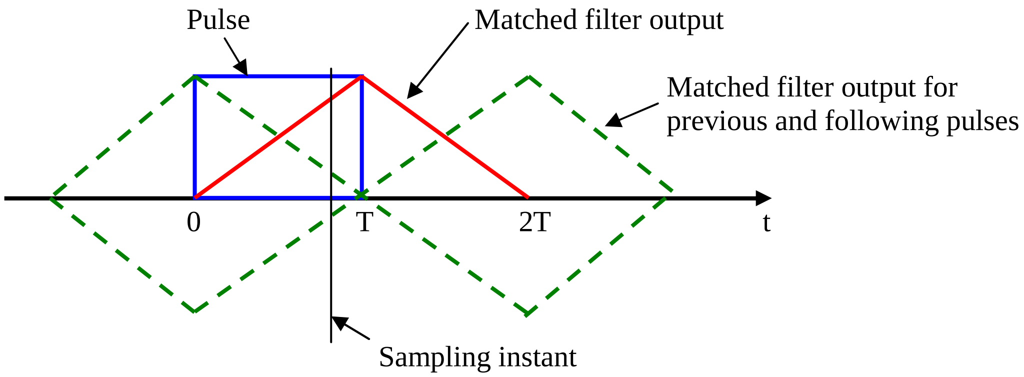
For $a_m = -1$, the result is the same. I used Matlab to generate the plot below.



Exercise 8.3

If the signal pulse is p(t) = u(t) – u(t-T), the matched filter output is Δ[(t–T)/T) which reaches its maximum value at time t = T. If the sample occurs 10% early at t = 0.9T, then the amplitude will be 90% of the ideal value. This will reduce the signal to noise ratio by 0.81 which is –0.92 dB. If the error rate was originally $Q(3.09) = 1.00 \times 10^{-3}$, this would become $Q(0.90 \times 3.09) = 2.71 \times 10^{-3}$. However, the situation is actually worse than that, as there is intersymbol interference from the previous data pulse, and this contributes ±0.1 to the sample value. Hence the possible sample values would be 0.8 giving an error of $Q(0.8 \times 3.09) = 6.72 \times 10^{-3}$, or $Q(1.0 \times 3.09) = 1.00 \times 10^{-3}$ for an average error probability of $\mathbf{3.86 \times 10^{-3}}$.

The same occurs if the timing is 10% late.



Exercise 8.10

1. The bandwidth of 1200 Hz would allow a maximum rate of 2400 bits/sec, but more realistically might be 1600 bits/sec if Nyquist pulses with $\rho = 0.5$ were used.

2. To achieve $P_e = 10^{-7}$, with a matched filter we a signal to noise ratio of $5.199^2 = 2E_b/N_o$, giving
$E_b/N_o = 13.51$ (11.3 dB).

3. If $N_o = 4.1\times10^{-21}$ W/Hz, then $E_b = 5.539\times10^{-20}$ J, which gives $P = E_b \times$ bit rate $= 1.329\times10^{-16}$ W for 2400 bits/sec. Allowing for the loss of 50 dB, this gives $P = 1.329\times10^{-11}$ W (–108.8 dBW).

Exercise 8.15

Use $M = 2^4$ giving a symbol rate of 14400/4 =3600 symbols/sec which is achievable with $\rho = 1/3$ Nyquist pulse shape. To achieve a symbol error probability of 10^{-6}, we require $d^2/2N_o=4.753^2$, giving d = 0.09506 since $N_o/2 = 1\times10^{-4}$.

The symbol levels should be ±(2k+1)d/2 for k = 0, 1, …, 7. The average symbol energy E_s is:

$$E_s = \frac{d^2}{32}\left(1^2 + 3^2 + 5^2 + \cdots + 15^2\right) = 21.25\,d^2 = 0.1920 \text{ J}$$

$$E_b = \frac{0.1920}{4} = 0.04800 \text{ J}$$

$$\frac{E_b}{N_o} = 240.0 \text{ (23.8 dB)}$$

Exercise 8.17

1. If the noise spectral density is $S_{nn}(f)$, we require a filter which satisfies $S_{nn}(f)|H_p(f)|^2 = 1$. This filter is required to be causal. We need not consider the details of its realisation.

2. The signal output from the whitening filter has a Fourier transform $\tilde{S}(f) = S(f)\,H_p(f)$, so the matched filter is $H_m(f) = \tilde{S}^*(f)\,e^{-j2\pi fT} = S^*(f)\,H_p^*(f)\,e^{-j2\pi fT}$, where T is the sampling instant.

3. The overall response is therefore $H(f) = H_p(f)\,H_m(f) = S^*(f)\,|\,H_p(f)\,|^2\,e^{-j2\pi fT} = S^*(f)\,e^{-j2\pi fT}\,/\,S_{nn}(f)$.

4. The signal to noise ratio is:

$$\Gamma = \frac{\text{Energy in }\tilde{s}(t)}{S_{\tilde{n}\tilde{n}}(f)} = \frac{\int_{-\infty}^{\infty}|S(f)|^2\,|H_p(f)|^2\,df}{S_{nn}(f)\,|H_p(f)|^2} = \int_{-\infty}^{\infty}|S(f)|^2\,/\,S_{nn}(f)\,df$$