4063/7080 Communications

Assignment #2

The assignment is to determine the probability of error for a binary FSK system using Monte Carlo simulation. The binary FSK signal will be generated in complex baseband form (ie. as a phasor), noise added and then filtered by a matched filter. The filter output will be sampled and a decision made as to whether the digital data is +1 or −1. By comparing the decision with the original data we can count errors. If the probability is estimated from a count nerrs, the 90% confidence limits for the estimated probability of error are approximately exp[$\pm$ 1.645/$\sqrt{}$(nerrs)] times the estimated probability. To get a reasonably accurate estimate we therefore need nerrs $\geq$ 10, which for a probability of error of 10^{-4} means a data length of 10^5 symbols is required. For nerrs = 10 the exact 90% confidence limits are 1.68 and 0.59 times the estimate.

To avoid large data arrays in MATLAB, the data is to be processed in segments and once processed the data will be discarded. This will enable the program to run on versions of MATLAB which restrict array sizes. While the simulation can be run in one go using large array sizes, for this assignment <u>you are required to use the data segmentation technique</u>. If you wish to make use of Python this is a possible alternative.

The FSK signal will be sampled with m = 8 samples per symbol and each segment will contain n = 32 symbols, giving a data segment of N = m x n = 256 samples.

The amount of noise added will be determined by a factor $\gamma = 2E_b/N_o$, and we will compare our simulation result with the theoretical result.

1. Specified Parameters

Symbol (bit) rate	f_b	1000 bits/sec
Symbol period	T	1 ms
Modulation index	h	$(f_1-f_2)T = 1$
Sampling frequency	f_s	8000 Hz
Sampling interval	t_s	125 μs
Symbols per segment	n	32
Samples per symbol	m	8
Samples per segment	N	256
Number of segments	R	20 and 2000
"Signal to noise ratio"	γ	0 to 14 dB in steps of 1 dB.
Noise distribution		gaussian

2. FSK Signal

The FSK signal with modulation index h = 1 can be represented as:

$$s(t) = A\cos\left[\omega_c t + \phi(t)\right]$$

where $d\phi/dt = \pm h\pi/T$ and $\phi(t)$ is continuous to avoid discontinuities in the signal (which cause spectral spreading). With a modulation index h = 1, the phase increment is $\pm\pi$ for each symbol. Hence the phasor representation is:

$$\tilde{s}(t) = A\, e^{j\phi(t)}$$

In the interval $kT \le t \le (k+1)T$, the phasor is:

$$\tilde{s}_k(t) = A\left\{\cos(\lambda t) + j\,a(k)\sin(\lambda t)\right\}$$

where $\lambda = \pi/T$ and $a(k) = \pm 1$ is the digital data. This corresponds to a carrier frequency shift of $\pm 1/2T$ Hz.

3. Additive Noise

The in-phase and quadrature noise components $n_c(t)$ and $n_s(t)$ are generated as in Assignment #1. The RF noise is assumed white of power spectral density $N_o/2$ V^2/Hz and hence the phasor components have power spectral density N_o V^2/Hz. In sampled form, the samples should have variance σ^2 such that $\sigma^2 = N_o f_s$, where f_s is the sampling frequency. Use the **randn** function and set the initial seed (once only) to ensure repeatable results.

The received phasor signal is then:

$$\tilde{v}(t) = \tilde{s}(t) + n_c(t) + j\,n_s(t)$$

4. Data Generation

We will generate independent data samples using the random number generator in MATLAB. To generate n samples of a(k) first use **a1 = rand(1,n)** to generate n uniformly distributed random variables in the range 0 to 1, then obtain the desired binary data from **a2 = 2 x (a1>=0.5) – 1**. Set the initial seed (once only) to ensure repeatable results.

The signal segment can be generated by generating a sequence of n values of a2 and replicating it a total of m times for each symbol, giving N = m x n samples. This sequence can then used to generate the FSK signal described above, and the additive noise components added.

5. Demodulation

At the receiver, a clock recovery circuit (not implemented in this exercise) is used to determine where a symbol begins and ends and is used to generate local signals $\pm\cos(\lambda t)$ and $\pm\sin(\lambda t)$ (ie. there is a sign ambiguity, both + or both –). These are used to demodulate the FSK signal by multiplying the received signal phasor by $\pm\cos(\lambda t)$ and $\pm\sin(\lambda t)$ and integrating over one symbol period.

$$v_c(k) = \pm\int_{kT}^{(k+1)T} \tilde{v}(t)\cos(\lambda t)\,dt = \pm\frac{1}{2}AT + n_1(k) + jn_2(k)$$

$$v_s(k) = \pm\int_{kT}^{(k+1)T} \tilde{v}(t)\sin(\lambda t)\,dt = \pm j\frac{1}{2}AT\,a(k) + n_3(k) + jn_4(k)$$

$$n_1(k) = \pm\int_{kT}^{(k+1)T} n_c(t)\cos(\lambda t)\,dt, \qquad n_2(k) = \pm\int_{kT}^{(k+1)T} n_s(t)\cos(\lambda t)\,dt$$

$$n_3(k) = \pm\int_{kT}^{(k+1)T} n_c(t)\sin(\lambda t)\,dt, \qquad n_4(k) = \pm\int_{kT}^{(k+1)T} n_s(t)\sin(\lambda t)\,dt$$

Note that v_c and v_s are complex. The correct sign can be determined from Re{v_c), so demodulation is achieved by comparing the signs of Re{v_c} and Im{v_s}. In practice, the other two demodulation parameters Im(v_c} and Re{v_s} are used for carrier and clock recovery (which we are not doing here).

With a discrete time simulation the integrations are approximated by summations over m samples.

6. Counting Errors

By comparing the original data signal with the demodulated signal, an error count for each segment can be made. This is added to a running count of errors and all other quantities can be discarded and overwritten by values which apply to the next segment.

7. Theoretical Probability of Error

Using the integral forms for v_c and v_s above, prove that the probability of Im{v_s} being the wrong sign is $p = Q\{\sqrt{(\gamma/2)}\}$ and similarly for Re{v_c}, and hence the probability of error for the demodulation method used is $P_e = 2pq$ where $q = (1-p)$.

Tasks to be Performed

Your submission should contain a detailed description of your procedures, theoretical analysis, the graphical plots required and your MATLAB code with plenty of comments. The assignment is expected to be your own work, copies of another person's work will not be favourably received.

1. Plot the real and imaginary parts of the signal phasor (without noise) for one segment.

2. Do steps 3 and 4 for number of segments R = 20 and R = 2000.

3. For each value of γ, determine the error probability as determined by the simulation and also calculate the theoretical value.

4. Plot the measured and theoretical probabilities of error against γ in dB on the one graph. Plot the theoretical values as a line and the measured values as an "*". Probabilities should be plotted on a logarithmic scale in the range 10^{-5} to 10^{0}.

Comment briefly on the significance of your results.