Section 8: Digital Transmission in Bandlimited Channels

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8. Digital Transmission in Bandlimited Channels

So far we have not considered pulse shape or bandwidth. In practice all channels are bandlimited, either because they are physically limited in bandwidth or because of a need to share spectrum space with other users.

We will consider transmission of baseband signals and extend the results to carrier modulated signals.

For an M-ary **baseband** PAM signal s(t) of the form:

$$s(t) = \sum_{k=-\infty}^{\infty} A\, a_k\, p(t - kT)$$

$$S_{ss}(f) = \frac{A^2}{T} E\{a_k^2\} |P(f)|^2$$

where a_k are the symbol amplitudes (k = 1, 2, .. M), $S_{ss}(f)$ is its power spectral density, p(t) is the pulse shape and T is the symbol interval. This result follows from the derivation in Chapter 4.

8.1 Pulse Shape

The optimum pulse to use from bandwidth considerations is a ***sinc pulse***:

$$p(t) = \text{sinc}\left(\frac{t}{T}\right)$$

This has a Fourier transform P(f) = T rect(fT), so the power spectrum of s(t) is:

$$S_{ss}(f) = A^2 T\,\text{rect}(fT) \quad V^2/\text{Hz}$$

This has a bandwidth equal to **1/2T Hz**.

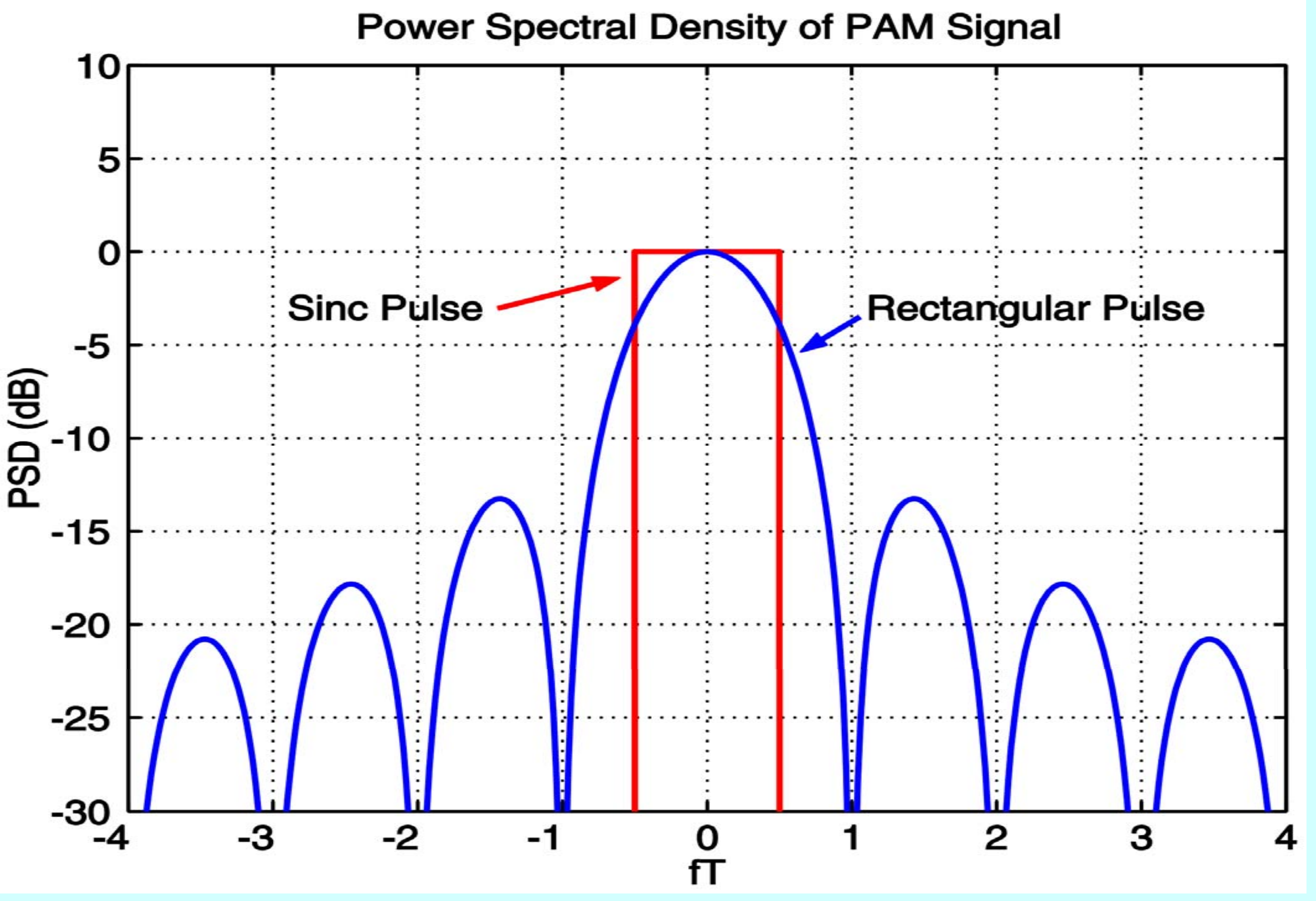
If the pulse has a ***rectangular pulse*** shape:

$$p(t) = \mathrm{rect}\left(\frac{t}{T}\right)$$

This has a Fourier transform P(f) = T sinc(fT), so the power spectral density of s(t) is:

$$S_{ss}(f) = A^2 T \,\mathrm{sinc}^2(fT) \quad V^2/\mathrm{Hz}$$

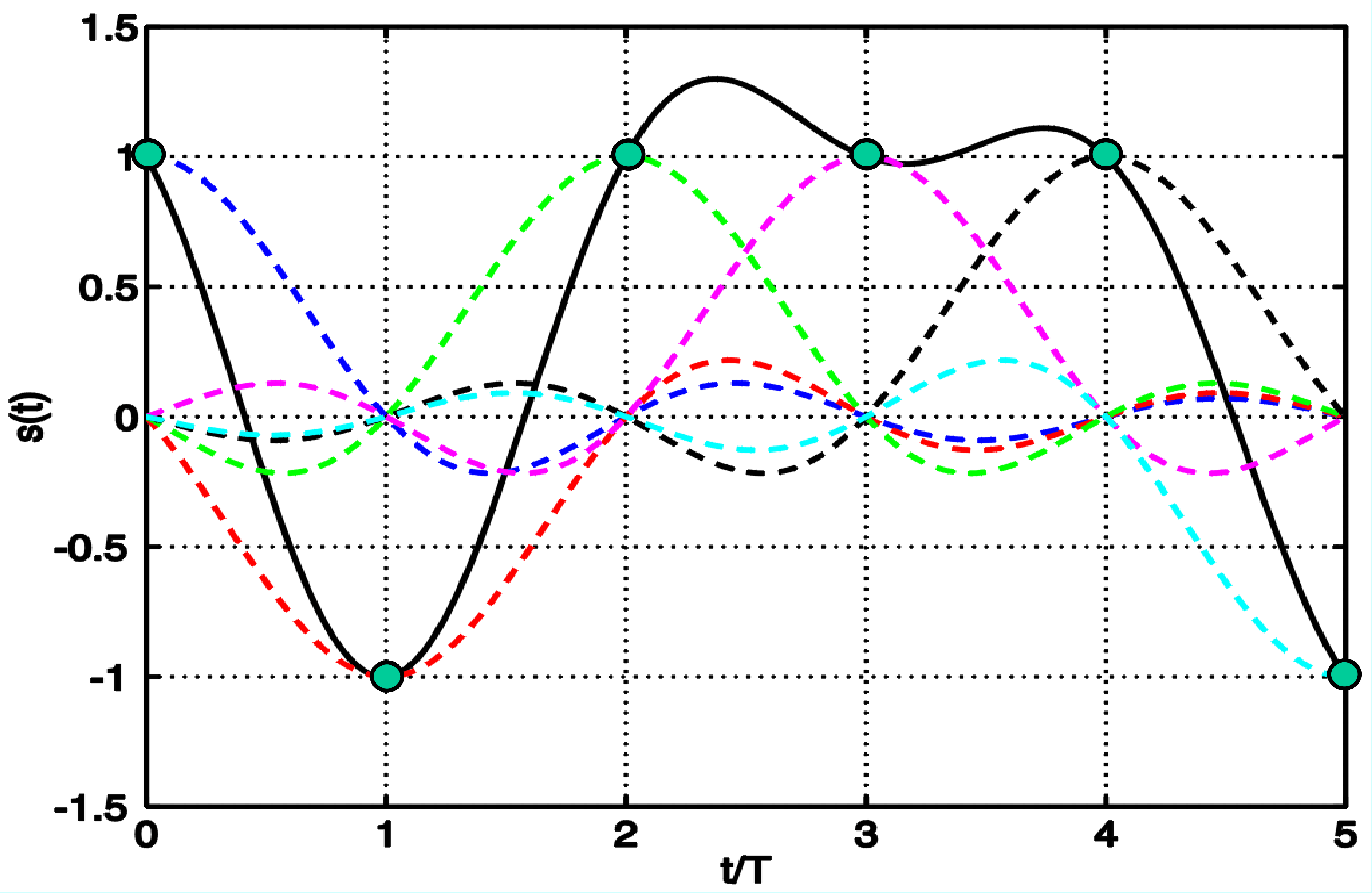
This has a bandwidth in excess of **1/T Hz**, with significant power at frequencies outside this band. Hence rectangular pulses are not used in practice.


Power Spectral Density of PAM Signal
10
5
0
-5
-10
-15
-20
-25
-30
PSD (dB)
Sinc Pulse
Rectangular Pulse
-4
-3
-2
-1
0
1
2
3
4
fT

Any pulse which has the property that p(kT) = 0 (for all k except k = 0) has the property of having zero ***intersymbol interference*** (ISI). This is because when the signal

$$s(t) = \sum_{k=-\infty}^{\infty} A\, a_k\, p(t - kT)$$

is sampled at t = kT, the only contribution is from the symbol a_k of interest. The sinc pulse is the simplest pulse with this property, but there are others.


1.5
1
0.5
0
-0.5
-1
-1.5
s(t)
0
1
2
3
4
5
t/T

With sinc pulses, although the pulses overlap, there is no ***inter-symbol interference*** (ISI). The zero crossings in the sinc function give the signal at the symbol intervals $t = kT$, $s(kT) = A\, a_k$, with no interference from adjacent data values.

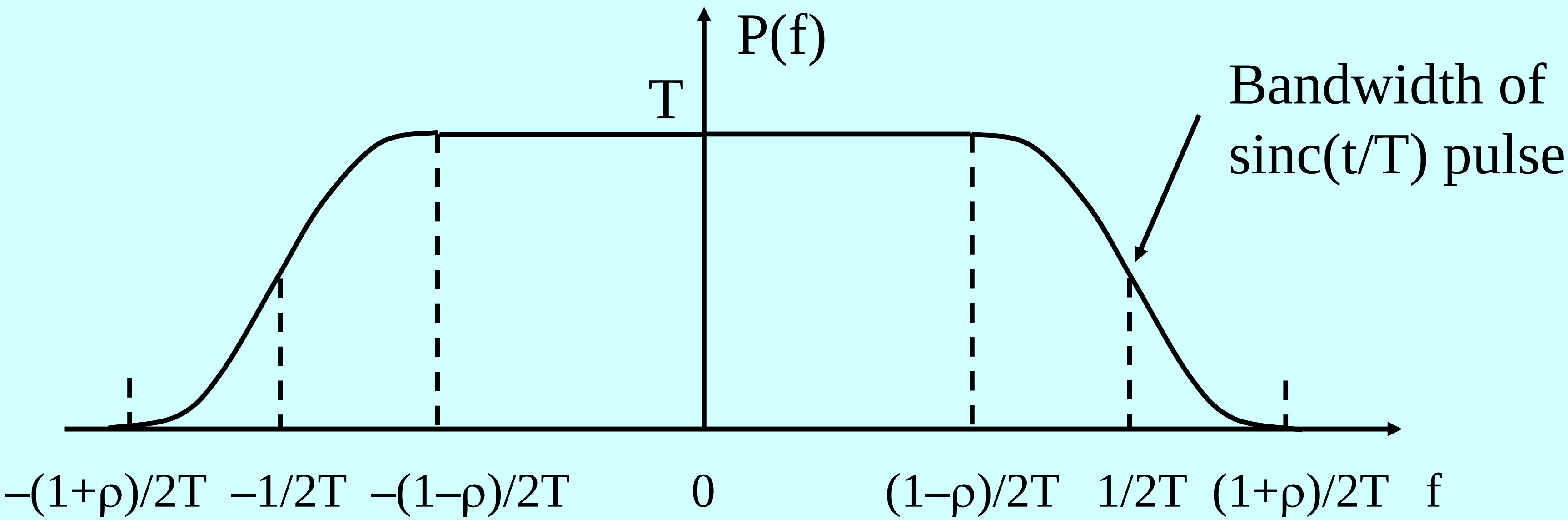
However, as we see shortly, sinc pulses are not suitable for transmitting data in practice. The desirable feature is the ***periodic zero crossings***, and other pulses besides sinc pulses have this property.

8.2 Signal Design and ISI

In practice we do not use sinc pulses because:

- they are hard to generate.
- it is very difficult to maintain zero ISI because the pulses die away slowly and the zero crossings are significantly modified by a non-ideal channel response.
- there are large peak voltages between samples.
- very accurate timing is necessary.

The solution is to use pulses which die away more quickly, but which still have the zero ISI property. This usually requires more bandwidth. Pulses which have this property are ***Nyquist*** pulses, and these have a Fourier transform as shown below.



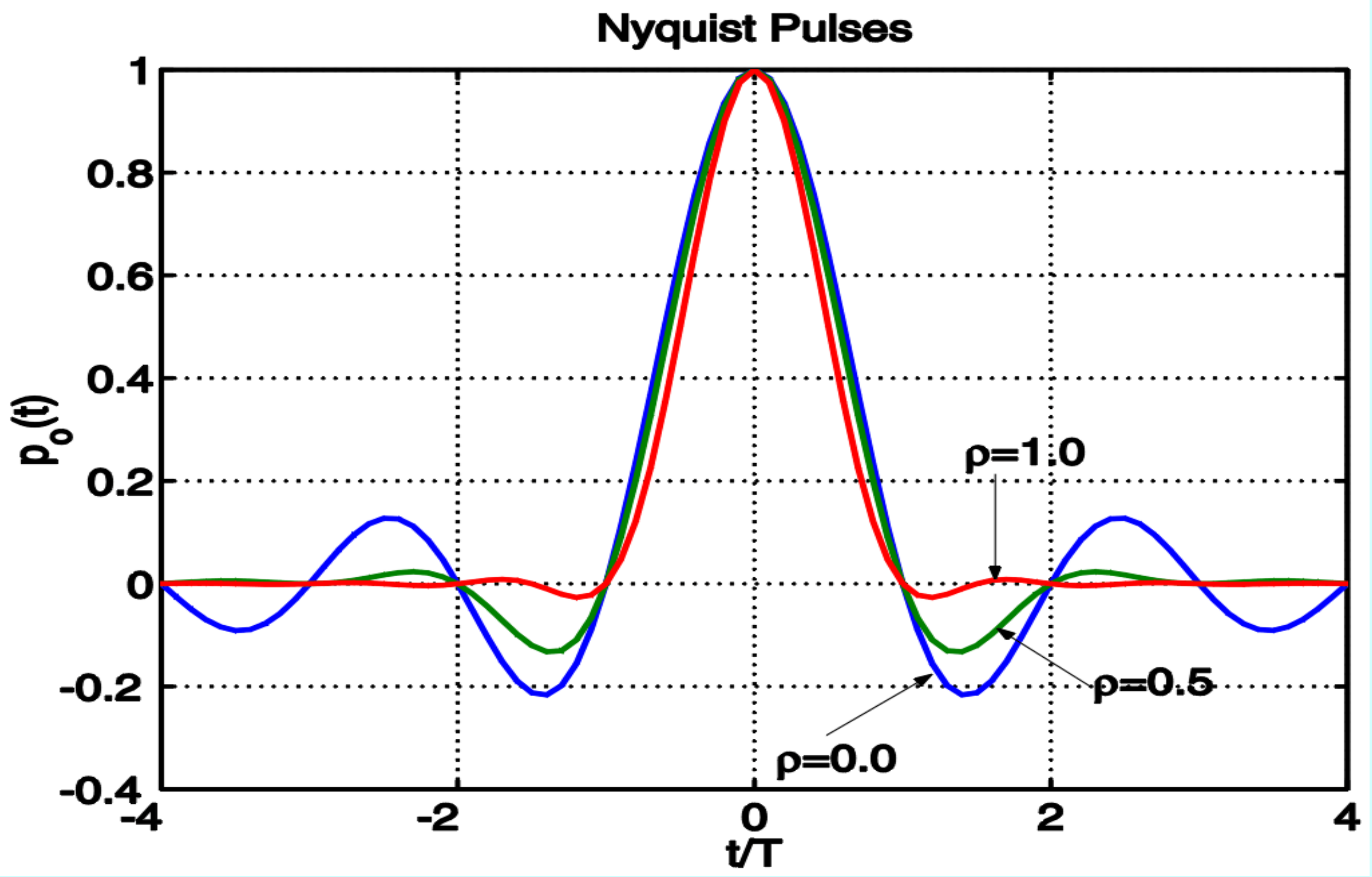
The **bandwidth** required for a **baseband** Nyquist pulse is:

$$B = \frac{(1+\rho)}{2T}\ \text{Hz}$$

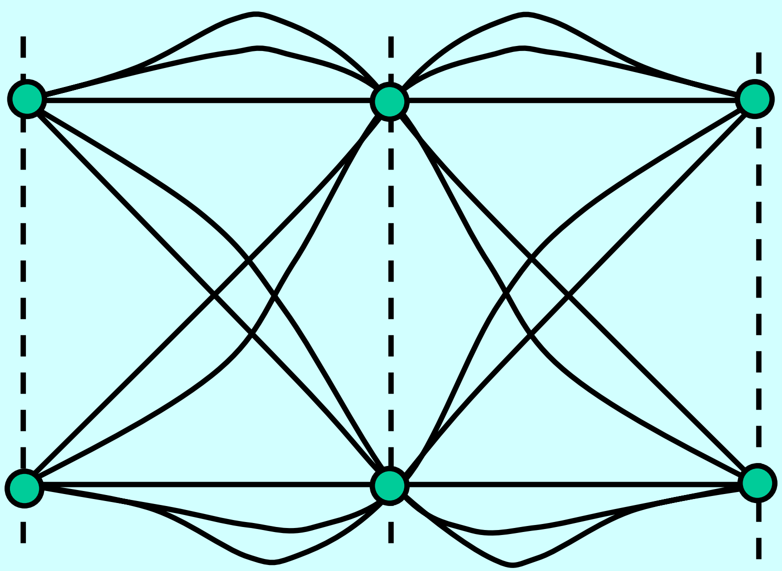
The pulse p(t) can be found by forming the inverse fourier transform of P(f). The expression for p(t) is shown below, and appears on the data sheet

$$p(t) = \frac{\pi}{4}\,\text{sinc}\left(\frac{t}{T}\right)\left[\text{sinc}\left(\frac{\rho t}{T}-\frac{1}{2}\right)+\text{sinc}\left(\frac{\rho t}{T}+\frac{1}{2}\right)\right]$$

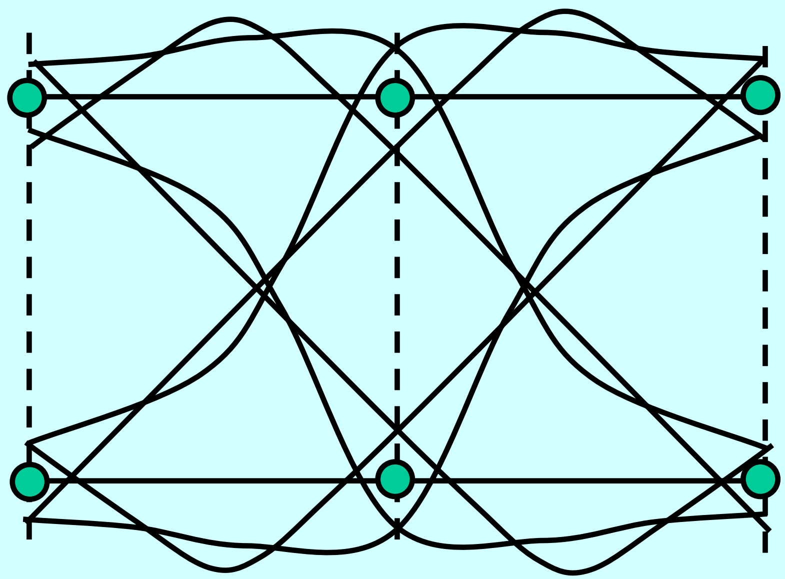
These pulses are characterised by an ***excess bandwidth factor*** ρ, where 0 ≤ ρ ≤ 1.



The ISI properties of a pulse can be illustrated by an eye diagram, which is simply a superposition of all possible signals over a 2T interval.



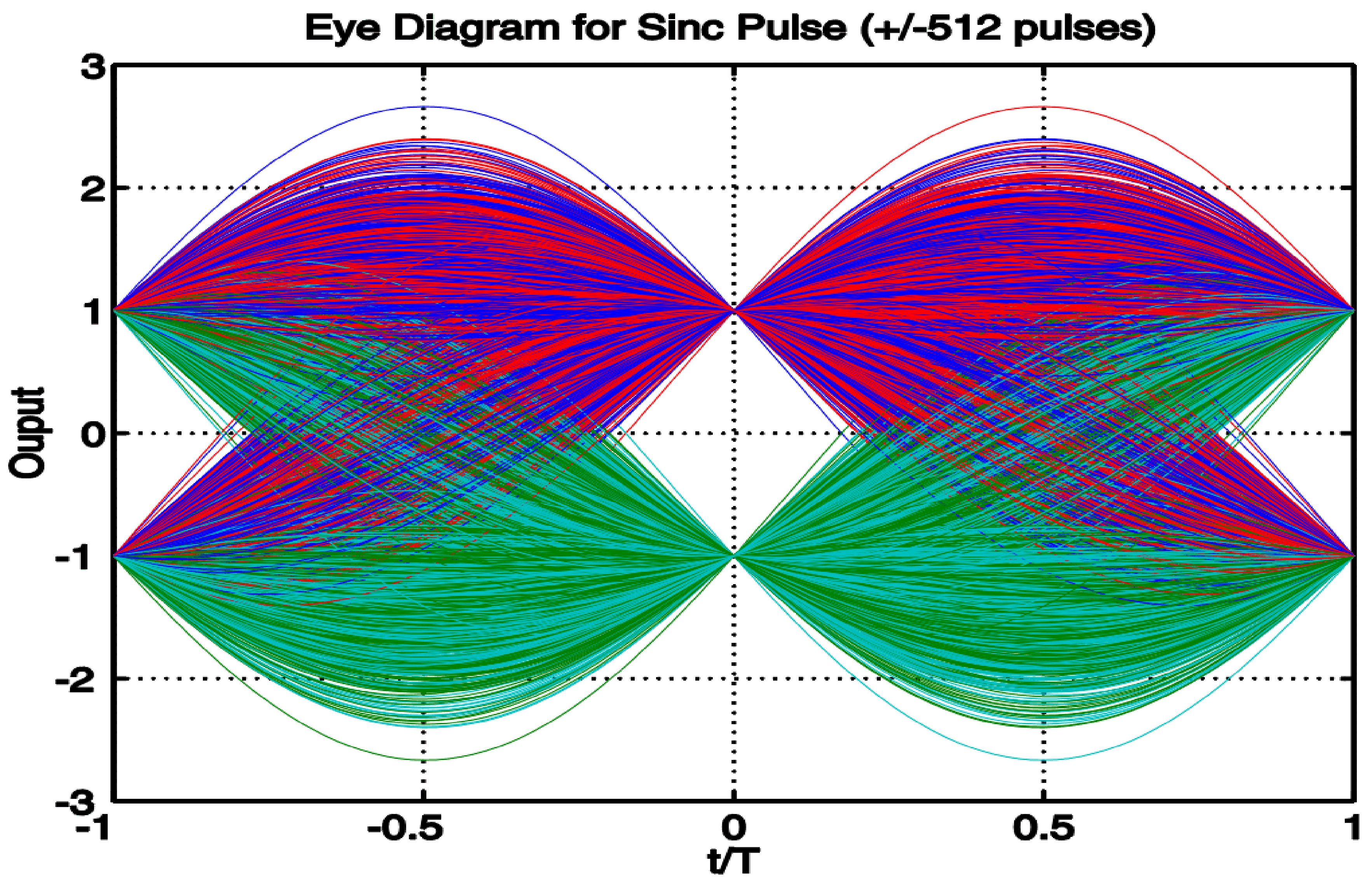
No intersymbol interference

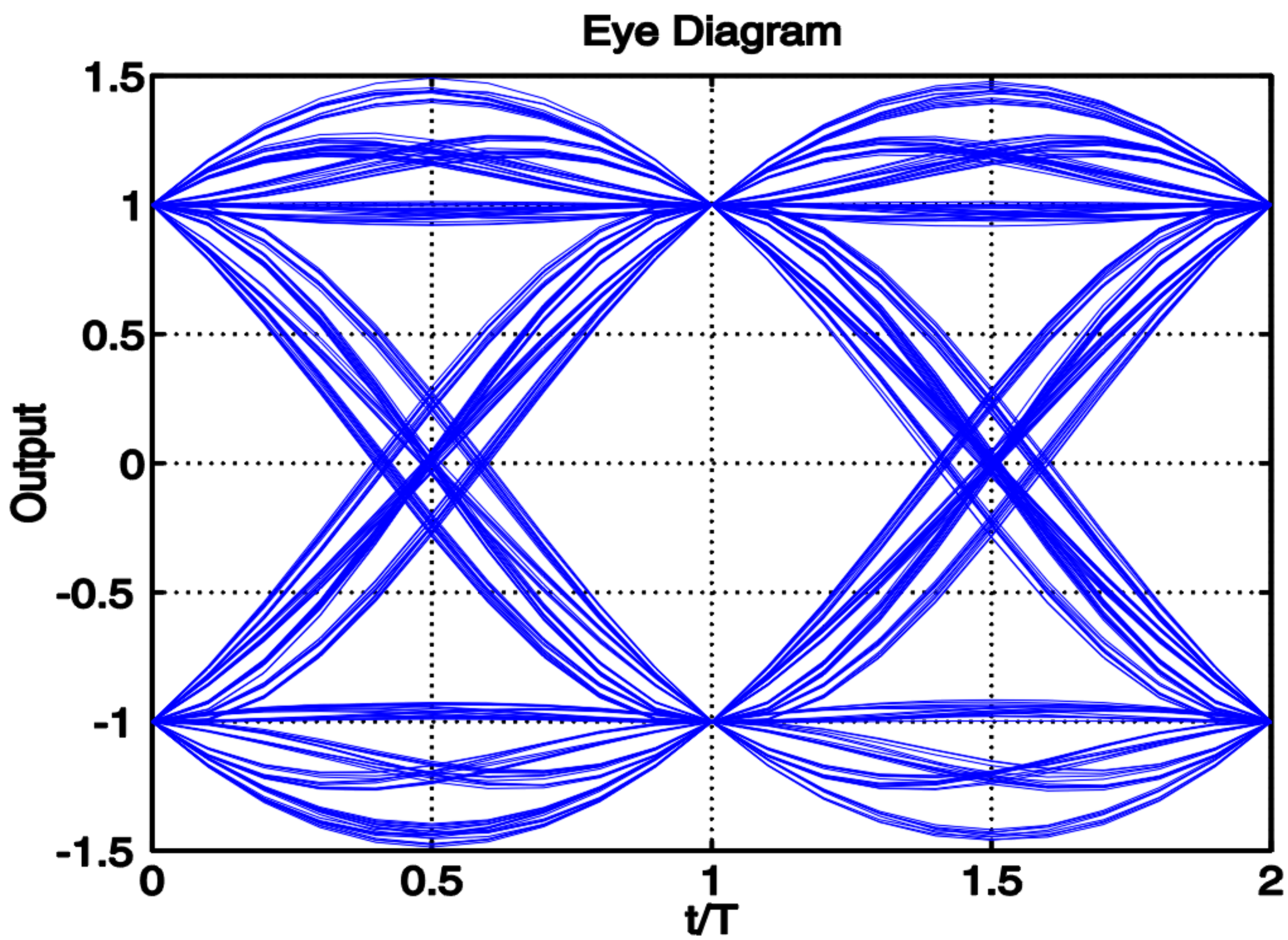


With intersymbol interference

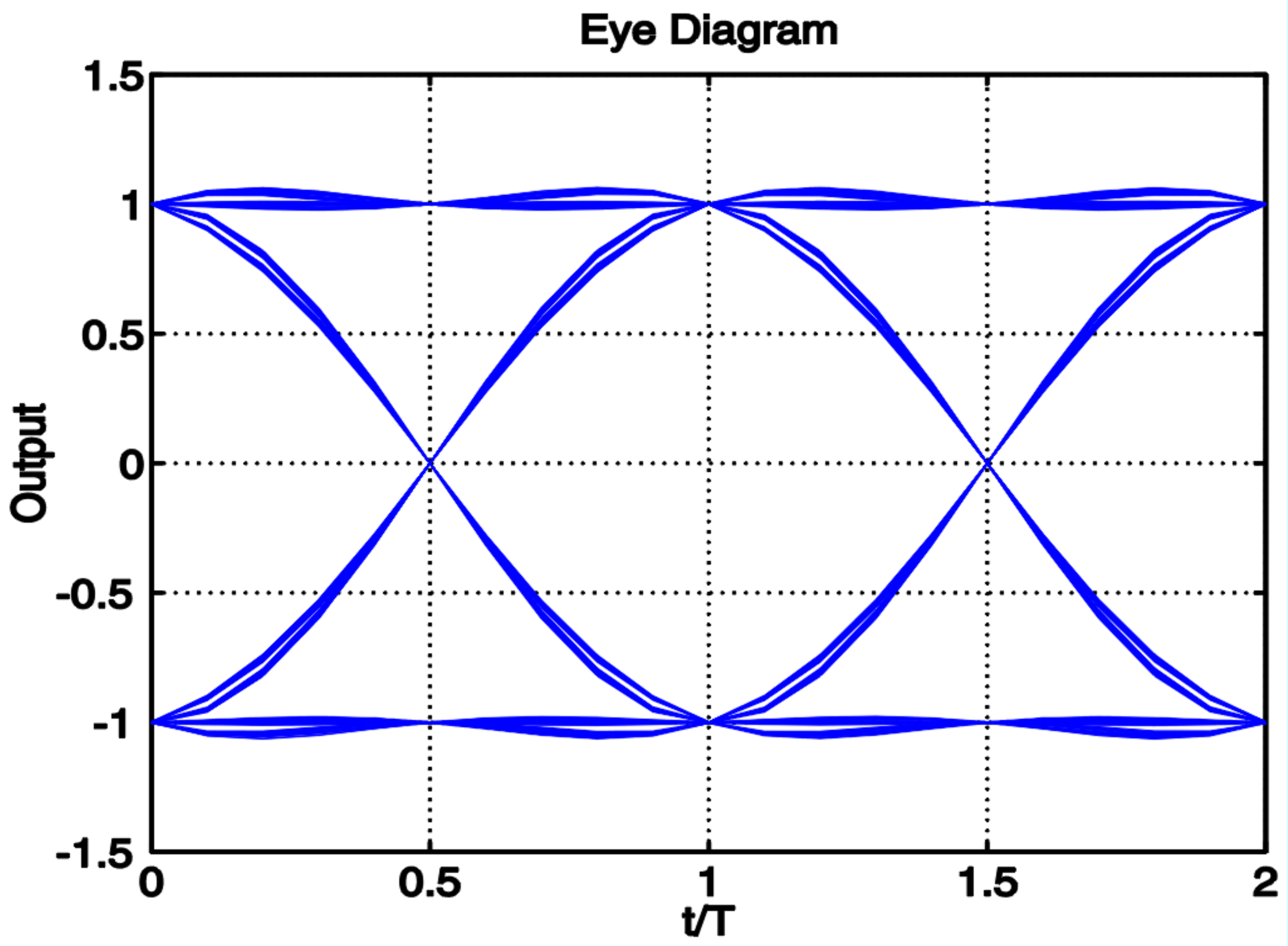
The height of the eye tells us the margin against noise, whereas the width of the eye tells us how accurate the timing has to be.

The pulse shape of importance is actually the pulse shape $p_o(t)$ at the ***output*** of the receiver filter, so the actual transmitted pulse shape p(t) may be somewhat different. If we know the receiver filter transfer function H(f), then we have $P_o(f) = P(f)\,H(f)$, so we can find P(f) and hence p(t).


Eye Diagram for Sinc Pulse (+/-512 pulses)
Ouput
t/T
3
2
1
0
-1
-2
-3
-1
-0.5
0
0.5
1



Eye Diagram of Nyquist Pulse ($\rho = 0.5$)



Eye Diagram of Nyquist Pulse ($\rho = 1.0$)

8.3 Bandwidth in Carrier Systems

For carrier systems the bandwidth is usually double that of a baseband system. If T is the symbol period:

$$B = \frac{(1+\rho)}{T} \text{ Hz} \quad \text{for M-PSK and M-QAM}$$

$$B = \frac{(2+\rho)}{T} \text{ Hz} \quad \text{for binary FSK with } h = 1$$

For binary FSK, the bandwidth is greater by an amount equal to the separation of the two carriers, which for h = 1 will be 1/T.

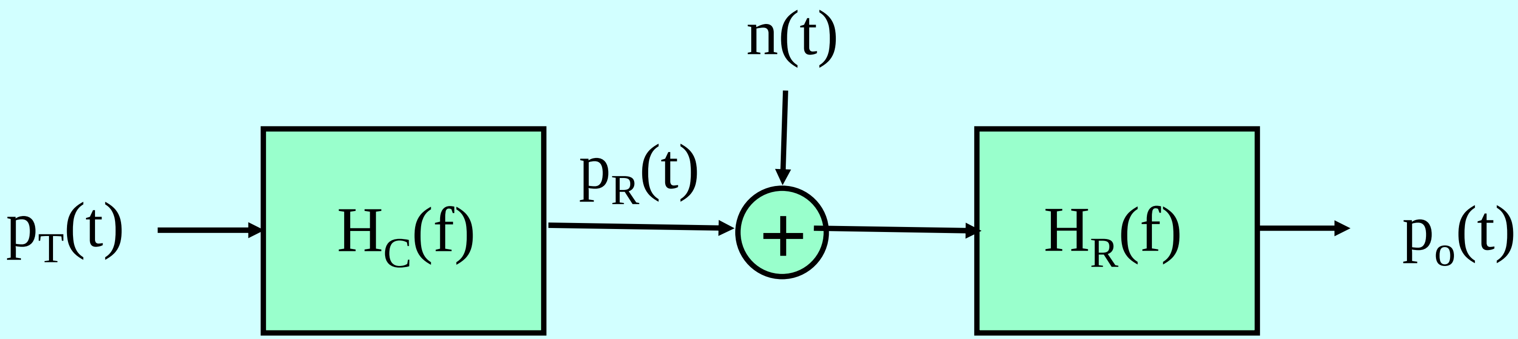
A carrier system is used when we wish to modulate the digital signal onto an RF carrier. However it may also be used for baseband channels when the frequency response does not go down to DC.

For example, for a telephone line the bandwidth is 300 Hz to 3300 Hz, so data modems usually use M-QAM with a sub-carrier frequency of 1800 Hz.

The doubling of the bandwidth is not a problem, because we can use both the cosine and sine subcarrier frequencies to carry data.

8.4 Overall System Design

The overall binary baseband PAM system can be considered to be as shown. The transmitted pulse is $p_T(t)$ with Fourier transform $P_T(f)$, $H_C(f)$ is the channel response and $H_R(f)$ is the receiver filter. The received pulse is $p_R(t)$ with transform $P_R(f)$ and the output pulse is $p_o(t)$ with transform $P_o(f)$.



We specify the output pulse $p_o(t)$, and require $H_R(f)$ to be matched to $p_R(t)$.

$$P_R(f) = P_T(f)\,H_C(f)$$

$$H_R(f) = P_R^*(f)\,e^{-j2\pi f t_o} \quad \text{(matched filter)}$$

$$P_o(f) = P_R(f)\,H_R(f) = \left|P_R(f)\right|^2 e^{-j2\pi f t_o}$$

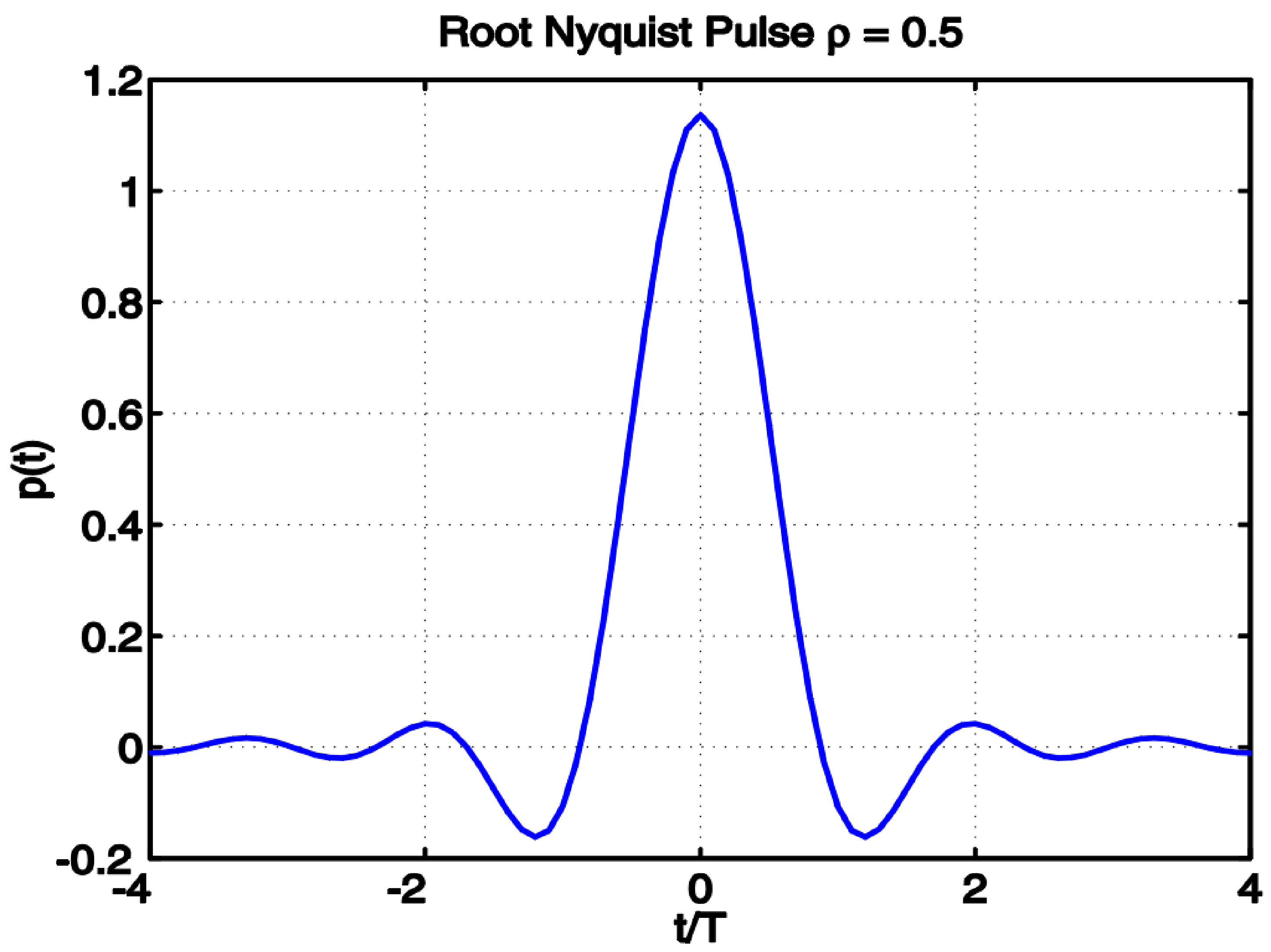
$$P_R(f) = \sqrt{\left|P_o(f)\right|}\,e^{j\theta(f)} \quad [\theta(f) \text{ is arbitrary}]$$

$$H_R(f) = \sqrt{\left|P_o(f)\right|}\;e^{-j[\theta(f)+2\pi f t_o]}$$

$$P_T(f) = \frac{1}{H_C(f)}\sqrt{\left|P_o(f)\right|}\;e^{j\theta(f)}$$

If the channel response is known, we can design a suitable transmitter pulse (if not, we need to use an ***adaptive equaliser*** at the receiver). The phase $\theta(f)$ is arbitrary, but may be chosen to make the transmitted pulse causal (ie. start from t = 0). Note that $p_T(t)$ and $p_R(t)$ may not have the zero ISI properties of $p_o(t)$.

Example: If $H_C(f) = 1$ and $p_o(t)$ is a $\rho = 0.5$ Nyquist pulse, the transmitter pulse is a **Root-Nyquist** pulse as shown.



Root Nyquist Pulse ρ = 0.5
p(t)
t/T
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-4
-2
0
2
4

8.5 Orthogonal Frequency Division Multiplex

When the channel is non-ideal, ISI is a problem. However, if we can increase the symbol time T, the effect of the channel response can be significantly reduced. One such way of doing this is to send a large number of bits in one symbol, and one way of doing this is OFDM.

If we have a channel of bandwidth W, we create M sub-channels of bandwidth W/M and in each sub-channel use a carrier frequency f_i, i = 1,2, … , M.

By selecting the symbol rate T equal to M/W, the carrier signals are orthogonal. Each sub-channel can be modulated using any of the modulation methods discussed earlier, and BPSK and QPSK are common choices.

The main problem with OFDM is that large peak voltages compared to the RMS value can occur, which is undesirable in any power limited system since this may cause ***intermodulation distortion*** if the amplifiers saturate.

OFDM systems are very popular for HF communication channels, because the channels often vary at a rate that makes adaptive equalisation difficult.

Also, by spreading the symbol over a long time, the effects of fading can be reduced, since (hopefully) fades will only occur during a relatively small fraction of the symbol time.

Exercises: You are expected to attempt the following exercises in Proakis & Salehi. Completion of these exercises is part of the course. Solutions will be available later.

8.2 (Use Matlab to plot P_e for SNR = 0:12 dB)
8.3 (Calculate P_e if $P_e = 10^{-3}$ with no error)
8.10
8.15 (use ordinary PAM & Nyquist pulse)
8.17