Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

Theorems

$x(t)$	$X(f)$
$x(t/T)$	$\|T\|X(fT)$
$x(t-T)$	$X(f)\, e^{-j2\pi f T}$
$x(t)\, e^{j2\pi F t}$	$X(f-F)$
$x(-t)$	$X(-f)$
$\frac{dx(t)}{dt}$	$j2\pi f\, X(f)$
$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\, \delta(f)$
$t\, x(t)$	$-\frac{1}{j2\pi} \frac{dX(f)}{df}$
$X(t)$	$x(-f)$
$rep_T\{x(t)\}$	$\|F\| comb_F(f)\, X(f)$
$\|T\| comb_T(t)\, x(t)$	$rep_F\{X(f)\}$
$x(t)\, y(t)$	$X(f) \otimes Y(f)$
$x(t) \otimes y(t)$	$X(f)\, Y(f)$
$x^*(t)$	$X^*(-f)$

Note that F and T are real constants, with FT = 1.

Transforms

$u(t)\, e^{-at}$	$\frac{1}{a + j2\pi f}$; $a > 0$
$e^{-a\|t\|}$	$\frac{2a}{a^2 + (2\pi f)^2}$; $a > 0$
$\frac{1}{a^2 + t^2}$	$\frac{\pi}{a} e^{-\|2\pi f a\|}$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$sgn(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\, sgn(f)$
$rect(t/T)$	$\|T\| sinc(fT)$
$sinc(t/T)$	$\|T\| rect(fT)$
$\Delta(t/T)$	$\|T\| sinc^2(fT)$
$comb_T(t)$	$\|F\| comb_F(f)$
$e^{-t^2/2T^2}$	$\|T\| \sqrt{2\pi}\, e^{-\frac{1}{2}(2\pi f T)^2}$
$sgn(t)\, rect(t/T)$	$\frac{1 - \cos(\pi f T)}{j\pi f}$

Note that a is a real positive constant.

Definitions

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\mathrm{rep}_P\{f(x)\} = \sum_{n=-\infty}^{\infty} f(x - nP)$$

$$\mathrm{comb}_P(x) = \sum_{n=-\infty}^{\infty} \delta(x - nP)$$

$$u(x) = \begin{cases} 0 & ; x < 0 \\ 1 & ; x > 0 \end{cases}$$

$$\delta(x) = \text{unit impulse (area} = 1)$$

$$\mathrm{sgn}(x) = \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1 & ; |x| < 0.5 \\ 0 & ; |x| > 0.5 \end{cases}$$

$$\Delta(x) = \begin{cases} 1 - |x| & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$$

$$f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(\lambda)\, g(x - \lambda)\, d\lambda$$

Relations

$$x(0) = \int_{-\infty}^{\infty} X(f)\, df = \text{area of } X(f)$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{area of } x(t)$$

$$X(-f) = X^*(f) \quad \text{if } x(t) \text{ is real}$$

$$X(f) = \text{real \& even if } x(t) \text{ real \& even}$$

$$X(f) = \text{imaginary \& odd if } x(t) \text{ real \& odd}$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Unless otherwise stated, these relations are true for x(t) real or complex.