Trigonometric Identities

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$$
$$2\cos(A)\sin(B) = \sin(A+B) - \sin(A-B)$$
$$2\sin(A)\cos(B) = \sin(A+B) + \sin(A-B)$$
$$2\sin(A)\sin(B) = \cos(A-B) - \cos(A+B)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$
$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos 2\theta = 2\cos^2\theta - 1 \quad = \quad 1 - 2\sin^2\theta \quad = \quad \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$
$$\sin\ 4\theta = 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$$

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$
$$\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$$

$$x\cos\omega t - y\sin\omega t = \sqrt{x^2 + y^2}\ \cos\{\omega t + \arg(x,y)\}$$

$$\arg(x,y) = \begin{cases} \arctan(y/x) & ; x > 0 \\ \arctan(y/x) + \pi\,\mathrm{sgn}(y) & ; x < 0 \\ \frac{\pi}{2}\mathrm{sgn}(y) & ; x = 0 \end{cases}$$

Complex Numbers

$j = \sqrt{-1}$

$z = x + jy = re^{j\theta}$ (cartesian and polar forms)

$z^* = x - jy = re^{-j\theta}$ (complex conjugate)

$x = \mathrm{Re}\{z\} = \frac{1}{2}\{z + z^*\}$ (real part)

$y = \mathrm{Im}\{z\} = \frac{1}{2j}\{z - z^*\}$ (imaginary part)

$|z| = r = \sqrt{x^2 + y^2}$ (magnitude)

$\arg(z) = \theta = \arg(x,y)$ (angle or argument)

$$\arg(x,y) = \begin{cases} \arctan(y/x) & ;x > 0 \\ \arctan(y/x) + \pi\,\mathrm{sgn}(y) & ;x < 0 \\ \frac{\pi}{2}\mathrm{sgn}(y) & ;x = 0 \end{cases}$$

$z z^* = r^2 = x^2 + y^2$

$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$ (addition)

$z_1 z_2 = (x_1 + jy_1)(x_2 + jy_2)$ (multiplication)

$= (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$

$= r_1 r_2\, e^{j(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{x_1 + jy_1}{x_2 + jy_2} = \frac{(x_1 + jy_1)(x_2 - jy_2)}{x^2 + y^2}$ (division)

$= \frac{x_1 x_2 + y_1 y_2}{x^2 + y^2} + j\frac{x_2 y_1 - x_1 y_2}{x^2 + y^2}$

$= \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$

$\ln(z) = \ln(r) + j\theta$

Phasors

$x + jy \Leftrightarrow x\cos(\omega t) - y\sin(\omega t)$ (cartesian phasor)

$r e^{j\theta} \Leftrightarrow r\cos(\omega t + \theta)$ (polar phasor)

$v(t) = \mathrm{Re}\left\{ \text{phasor} \times e^{j\omega t} \right\}$ (peak phasor)

For RMS phasors, multiply the time function by $\sqrt{2}$