1 Question 2(a) from the 2006 Exam

In a broadcast communication system the transmit power is 9 kW, the channel attenuation is 80 dB, the noise power spectral density is $S_{nn}(f) = N_o/2$ with $N_o = 1.5 \times 10^{-10}$ W/Hz and the normalised baseband message signal $m(t)$ has a bandwidth of 15 kHz, $|m(t)| \leq 1$ and a mean square value $\langle m^2(t) \rangle = 0.1$.

(i) If the modulation used is amplitude modulation (AM) with a modulation index $a = 0.90$, calculate the following for a receiver with bandwidth equal to that of the signal:

- the bandwidth of the signal;
- the predetection signal to noise ratio (SNR$_p$) in decibels;
- the output signal to noise ratio (SNR$_o$) in decibels.

(ii) If the modulation used is frequency modulation (FM) with peak frequency deviation 75 kHz, calculate the following for a receiver with a bandwidth given by Carson’s rule:

- the (approximate) bandwidth of the signal;
- the predetection signal to noise ratio (SNR$_p$) in decibels;
- the output signal to noise ratio (SNR$_o$) in decibels.

(iii) What is the maximum channel attenuation (in decibels) allowed if the FM system in (ii) is to be above threshold?

2 Adapted from Exercise 6.26 from Proakis and Salehi

Design a ternary Huffman code for a source with output alphabet probabilities given by

$$\{0.05, 0.1, 0.15, 0.17, 0.13, 0.4\}$$. What is the entropy of the source? What is the average codelength of your Huffman code, and the coding efficiency?

Hint 1: Ternary means the Huffman code has three symbols, instead of two.

Hint 2: You can add a dummy source output, with zero probability.
Exercise 7.1 from Proakis and Salehi

Determine the average energy of a set of $M$ PAM signals of the form

$$s_m(t) = s_m \psi(t), \quad m = 1, 2, \ldots, M \quad 0 \leq t \leq T,$$

where

$$s_m = \sqrt{\xi_g A_m}, \quad m = 1, 2, \ldots, M.$$

The signals are equally probable with amplitudes that are symmetric about zero and are uniformly spaced with distance $d$ between adjacent amplitudes, as shown below.

![Diagram of PAM signals](image)

Hint:

$$\sum_{m=1}^{M} m = \frac{M(M + 1)}{2},$$

and

$$\sum_{m=1}^{M} m^2 = \frac{M(M + 1)(2M + 1)}{6}.$$