# OPTIMAL DESIGN OF AN OFFSET STRIP FIN HEAT SINK USING HARMONY SEARCH

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## ABSTRACT

A method for optimizing the design configuration of an offset strip fin heat sink is proposed with a comparative study of the results. Optimization was conducted through common entropy generation rate minimization using the newly developed Harmony Search Algorithm. For comparison and support for viability of the proposed method, results for both the Harmony Search and Genetic Algorithm methods are given. These demonstrate the merit of Harmony Search optimization in such complex engineering problems.

Keywords: Offset strip fin, Heat sink, Optimization, Entropy generation, Harmony Search, Genetic Algorithm

## INTRODUCTION

The optimum design of heat sinks is an essential component affecting the operation of thermal systems. Performance optimization of fins could be achieved through different approaches according to their performance indicators. In some applications minimum material consumption for a specific heat transfer task is the objective parameter (Ahmadi and Razani, 1973, Bar-Cohen and Iyengar, 2002) whereas, the minimization of thermal contact resistance between the substrate and flow through the fins has been of interest in some other works (Bejan and Morega, 1993). Recently a new criterion has been developed that relates the heat transfer characteristic of a system to the synergy between the velocity and temperature gradient vectors in a flow field called the *synergy number* (Chen and Meng, 2008, Chen et al., 2007, Guo et al., 2009). On the other hand, a combined effect of heat transfer and fluid flow can be evaluated base on thermodynamic second-law analysis proposed by (Bejan, 1982, Bejan, 1996) which has been used extensively by others later (Jha and Chakraborty, 2005, Khan et al., 2006, Ndao et al., 2009, Poulikakos and Bejan, 1982). This method reveals the advantage of enhanced surfaces with regard to the rate of entropy generation associated with them.

A comprehensive review of studies on thermal fluid characteristics of offset strip heat sinks can be found in (Webb and Kim, 2005). A thermal design of OSFs was performed applying Matlab's multi-objective genetic algorithm optimization toolbox (GA) by (Ndao et al., 2009). They optimized the problem by simultaneous minimization of thermal resistance and pumping power under constant pressure drop and a fixed base area of 100 mm$^2$. In another study, optimal dimensions of an array of OSF were achieved based on the minimum entropy generation rate and employing single objective genetic algorithm method (Jha and Chakraborty, 2005). Application of evolutionary algorithms is not restricted to optimization of OSFs and has been used in many other thermal problems (Kahrom et al., 2009, Selbas et al., 2006).

Nevertheless, alongside GAs, other meta-heuristic search techniques such as the recently developed Harmony Search (HS) algorithm are establishing their application in optimization of thermal systems. Fesanghary et. al. (Fesanghary et al., 2009) have used HS for design optimization of shell and tube heat exchangers from the economic point of view. They have also compared the results of HS with those of GA for the same problem and concluded that HS can converge to the optimum solution with higher accuracy. There are several studies that have explored the application of HS in heat transfer systems. In this work we employed HS to find optimal values of geometrical design parameters for an array of OSF based on entropy generations principle adapted from (Jha and Chakraborty, 2005). In addition to compare the performance of HS and demonstrate its applicability we attained another set of results using Matlab's single objective genetic algorithm toolbox for the same problem.

This paper is organized as follows, first an analytical model of entropy generation rate for the given problem, based on the work of (Jha and Chakraborty, 2005) is presented. In the next section a short introduction to the newly developed HS algorithm is given followed by the details of optimization process. Results are discussed in the last section followed by the conclusion.

## MODELING OF ENTROPY GENERATION RATE

To evaluate the performance of a heat sink defined by a given set of design geometries, (Jha and Chakraborty, 2005) developed a fitness function based on the rate of entropy generation. A schematic of staggered fins and their relevant parameters are depicted in Figure 1. $H$ is the fin height, $\delta$ is the fin thickness, $\varepsilon$ is the swept length of each fin, and $D$ represents the spacing between fins. The substrate entrance width is $B$ and $X$ is streamwise length of that. The fins' base is taken to be uniformly heated at the rate of $Q_b$. And the free stream flow velocity is $U$ at initial temperature of $T_\infty$.

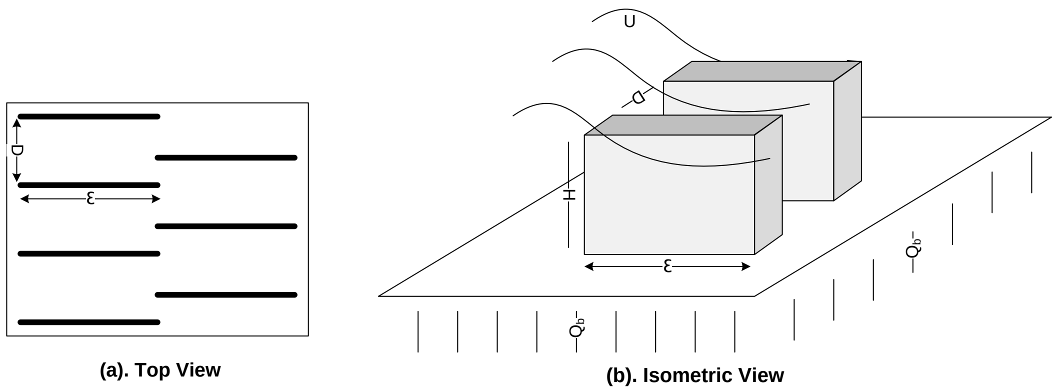


Figure 1. A staggered fin heat sink

The objective is to correlate the total entropy generation rate $\dot{S}_{gen}$ as a function of fin parameters. And consequently find the optimal values of $\varepsilon$, $H$, and $D$ for the given values of $B, X, Q_b, T_\infty$ and $U$ by minimizing the overall entropy generation of the system. The fin thickness assumed to be constant and equal to 1 mm. The assumptions considered in this analysis are: (a) The ratio of $\varepsilon / \delta$ is sufficiently high to assume one dimensional heat transfer from fins, (b) The boundary layers on each plate is distinctive, (c) Negligible heat transfer occurs at the fin tips and (d) The swept length $\varepsilon$, is small so that laminar forced convection is the primary mechanism of heat transfer from the plates.

From the overall energy balance for a single row of OSF the heat rejection rate $q_\infty$, to the passage follow can be written as the sum of the heat provided from the substrate and the viscous heating of the fluid flow,

$$q_\infty = q_b + F_D U \qquad (1)$$

where $F_D$ is the drag force in the row and $q_b$ is the heat dissipation rate from the substrate of that particular row. The number of rows $N_r$, and number of fins in one row $N_c$, can be estimated via equations (2) and (3),

$$N_r = \frac{X}{\varepsilon} \qquad (2)$$

$$N_c = \frac{B+D}{D+\delta} \qquad (3)$$

So, $F_D$ and $q_b$ can be written as $F_D = N_{fr} f_D$, and $q_b = Q_b / N_r$. $f_D$, is the drag force of a follow passage of a particular row. Overall entropy generation of the $i^{th}$ row reads from equation (4).

$$\dot{S}_{gen(i)} = \left(\frac{q_\infty}{T_{a(i)}}\right) - \left(\frac{q_b}{T_{b(i)}}\right) \qquad (4)$$

Substituting equation (1) in (4) gives

$$\dot{S}_{gen(i)} = \left(\frac{q_b\left(T_{b(i)} - T_{a(i)}\right)}{T_{a(i)}T_{b(i)}}\right) + \left(\frac{N_c f_D U}{T_{a(i)}}\right) \qquad (5)$$

In equation (5), $T_{a(i)}$ and $T_{b(i)}$ are the average follow and the substrate temperature of the $i^{th}$ row. The first term on the right hand side of equation (5) represents the irreversibility due to heat transfer $\dot{S}_{gen,T(i)}$, and the second term represents the irreversibility caused by fluid friction $\dot{S}_{gen,f\ (i)}$.

Now $f_D$ in equation (5) is the sum of drag forces from plate fins and the fin base $f_D = f_{D_{fin}} + f_{D_{base}}$, where,

$$f_{D_{fin}} = 2N_c \tau_{fin}\left(H\varepsilon\right) \qquad (6)$$

And

$$f_{D_{base}} = \left(B - N_c \delta\right)\varepsilon \tau_b \qquad (7)$$

$\tau_{fin}$ in equation (6) is the average fluid shear stress over the swept length of each plate and is given by (Incropera and DeWitt, 2002) as equation (8).

$$\tau_{fin} = 1.328\left(\frac{U\varepsilon}{\vartheta}\right)^{-1/2}\frac{1}{2}\rho U^2 \tag{8}$$

Additionally equation (9) gives $\tau_b$, appears in equation (7) for the $i^{th}$ row (Jha and Chakraborty, 2005),

$$\tau_b = 0.664\left(\frac{U\left(i-0.5\right)\varepsilon}{\vartheta}\right)^{-1/2}\frac{1}{2}\rho U^2 \tag{9}$$

Further, $T_{a(i)}$ in equation (5) can be evaluated by applying the energy balance for the $i^{th}$ row using equations (10) and (11),

$$q_\infty = \dot{m}C_P\left(T_{out(i)} - T_{in(i)}\right), \quad T_{in(1)} = T_\infty \tag{10}$$

$$T_{a(i)} = \frac{T_{out(i)} + T_{in(i)}}{2} \tag{11}$$

Combining equations (10) and (11) yields equation (12) for $T_{a(i)}$ (Jha and Chakraborty, 2005),

$$\dot{m}C_P\left(T_{a(i)} - T_\infty\right) = \left(q_b + F_D U\right)\left(i-0.5\right) \tag{12}$$

Now, average base temperature of the $i^{th}$ row $T_{b(i)}$ in equation (5), can be expressed in terms of $T_{a(i)}$ through equation (13),

$$T_{b(i)} = T_{a(i)} + \frac{q_b}{N_c K_{fin}\varepsilon m\tanh\left(mH\right) + h_{b(i)}(B - N_c\delta)\varepsilon} \tag{13}$$

In the above correlation, $h_{b(i)}$ is the heat transfer coefficient of the $i^{th}$ row base and is given by equation (14), (Jha and Chakraborty, 2005). $K_{fin}$ is the thermal conductivity of the fin material and $m$ is the fin parameter given by $m = \sqrt{Ph_{fin}/K_{fin}A}$. Where $P$ and $A$ are the perimeter and area of the fin cross-section, respectively. And $h_{fin}$ is the average heat transfer coefficient of the plate fin, assumed to be isothermal over the length $\varepsilon$, which is presented in equation (15), (Szargut, 1980),

$$h_{b(i)} = \frac{0.332K_a Pr^{1/3}(U(i-0.5)\varepsilon/\vartheta)^{1/2}}{(i-0.5)\varepsilon} \tag{14}$$

$$h_{fin} = \frac{0.664K_a Pr^{1/3}(U\varepsilon/\vartheta)^{1/2}}{\varepsilon} \tag{15}$$

where $K_a$ in equation (14) is the thermal conductivity of fluid and $(i-0.5)$ refers to the center of the $i^{th}$ row. At the end, the overall entropy generation of the OSF is the summation of that for each individual row, equation (16).

$$\dot{S}_{gen} = \sum_{i=1}^{N_r} \dot{S}_{gen(i)} \tag{16}$$

## OPTIMIZATION TECHNIQUE

Harmony search (HS) is a recent breed of soft computing paradigm, first developed by (Zong Woo Geem et al., 2001) that mimics the improvisation of musicians. Like many other meta-heuristic optimization algorithms HS combines rules and randomness, called intensification and diversification respectively, and tries to find the global optima of a given cost function through stochastic random search rather than a gradient one. This leaves it unnecessary to obtain derivatives of a function making HS suitable for multivariable complex functions of interest (Lee, 2009).

In the last decade genetic algorithms (GA) have been used extensively to solve various mechanical and structural optimization problems and good results have been obtained (Fasanghari, 2009). However, the GA characteristics that make it robust make it computationally intensive, which causes slow convergence compared with other meta-heuristics. In recent years HS has been used in a broad range of problems arising from mechanical, civil, electrical and chemical engineering problems. Results manifest HS as a viable, yet even better alternative to the other complex optimization techniques utilized so far such as GA, simulated annealing (SA) and other stochastic optimization paradigms (Fasanghari, 2009, Yang, 2009).

To obtain optimized design parameters the overall entropy generation formula from the previous section is used as the fitness criterion. The purpose is to determine the optimal values of geometrical parameters including fin height (*H*), fin spacing (*D*) and fin length ($\varepsilon$) for constant values of $Q_b$ and *U*. To avoid unrealistic results, design parameters are bounded by the conditions given in Table 1.

Table 1. Design parameters and their ranges considered in this study

| Fin Length | Fin Spacing | Fin height |
|---|---|---|
| $0.01 < \varepsilon < 0.02$ (m) | $0.001 < D < 0.01$ (m) | $0.01 < H < 0.05$ (m) |

Thermal irreversibility diminishes as input heat to the system decreases. So, to verify the results of the optimizer, optimization was also performed by relaxing $Q_b$ and *U* to variables of the fitness function. Input heat to the heat sink varied from 5 to 15 m/s and the intuitive optimal result should achieve when $Q_b$ is at its lowest value. Due to the specifications of current study, HS parameters (Yang, 2009) has been chosen based on Table 2.

Table 2. Selected HS parameters for entropy minimization

| Parameter | Notation | Value |
|---|---|---|
| Number of Harmonies | HM | 25 |
| Harmony Memory Consideration Rate | $R_{\text{accept}}$ | 0.95 |
| Pitch Adjustment Rate | $R_{\text{pa}}$ | 0.5 |
| Pitch Adjustment Bandwidth | $b_{\text{range}}$ | 0.1 |
| Number of Iterations | IT | 10,000 |

HS starts by generating 25 random harmonies then attempts to generate better harmonies through parameter optimization in each iteration. Finally the process stops when the maximum number of iterations is reached. At each iteration parameters are assigned new values while the following constraints are applied to satisfy the assumptions used to develop the cost function:

1. Laminar boundary layer

$$\frac{U\varepsilon}{\nu} < 5\times10^{5} \tag{17}$$

2. Distinct boundary layer

$$D > 10\varepsilon \Big/ \sqrt{U\varepsilon/\nu} \tag{18}$$

## DISCUSSION AND RESULTS

We implemented the HS algorithm in C++ and executed both the HS program and Matlab GA toolbox on an Intel 2.8 GHz Core 2 Quad machine with 4 GB of RAM. To make the HS results comparable we also used GA to do the optimization. The substrate area is a rectangle of $0.1\times0.1\text{m}^2$, $U$ is in the range of 0.5-15 m/s and $Q_b$ is in the range of 5-150 W. The ambient temperature is assumed to be 300 K.

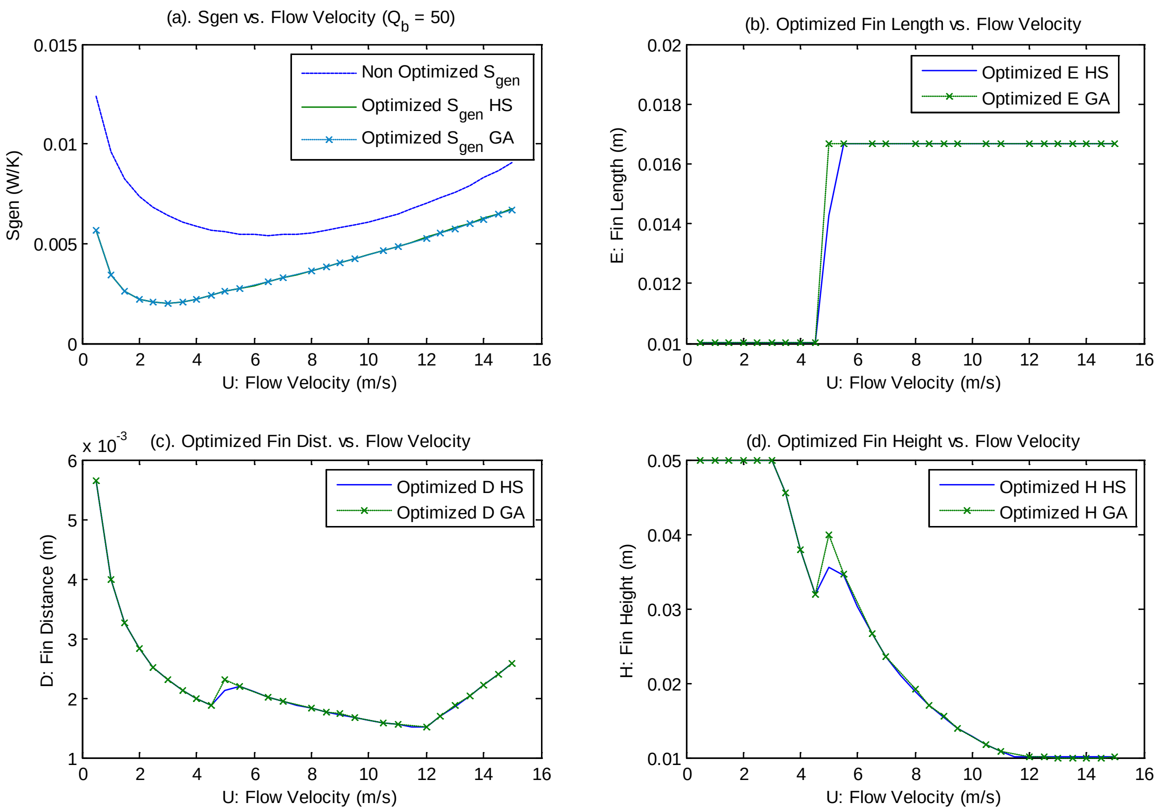


Figure 2. Optimization results with constant $Q_b$ and variable $U$

Figure 2(a) shows the total generated entropy ($\dot{S}_{gen}$), for different values of $U$ and $Q_b$ = 50 W, of optimized and an arbitrary non-optimized case. The values of design geometries in the non-optimized case are $\varepsilon$ = 0.015 m, $D$ = 0.01 m and $H$ = 0.04. In this figure optimization curves of HS and GA are both drawn. Results clearly show how the optimization process has decreased the amount of the overall entropy generation rate. Optimized values of design parameters are shown in Figure 2(b) to (d). As results imply the optimal values of both techniques are close to each other except for a few cases. This proves applicability of HS for such applications. Moreover, considering comparable final results of both algorithms, as the run time of HS is half an order of magnitude less than that of GA, HS presents itself as a worthy alternative to GA.

Figure 3(a) presents the values of the cost function for both optimized and non-optimized cases, where $U$ is fixed at 4 m/s and $Q_b$ varies. Optimized design parameters are also plotted against $Q_b$ in Figure 3(b) to (c).

According to Figure 2(a), there is a particular velocity at which the overall entropy was the least. This is predictable since at very low velocities flow friction is negligible and it becomes significant at higher velocities. However, temperature difference between the base and the flow is high at lower velocities and decreases by increasing the air flow through the heat sink. The optimal values for fin height and fin length become stable at velocities greater than 12 and 5.5 m/s, respectively. Nonetheless, such a trend is not observed for the fin distance in Figure 2(c). On the other hand, at a specific flow velocity, entropy increases monotonically by increasing the thermal load of the heat sink as demonstrated in Figure 3(a). And when $Q_b$ > 30 W optimal values of geometrical parameters become independent of the input heat flux and plateau around a certain value, Figure 3(b) to (d).

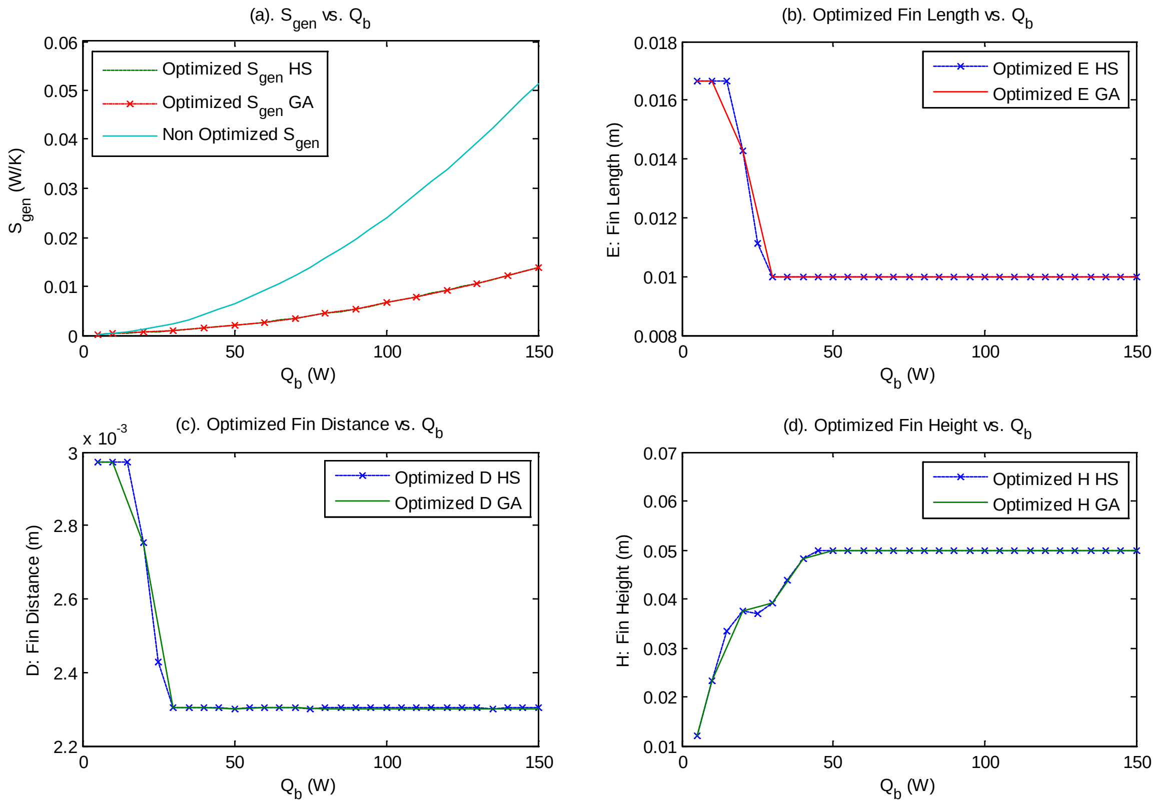


Figure 3. Optimization results with constant $U$ and variable $Q_b$

At each iteration there are 25 harmonies kept in the harmony memory. Also, two pointers to the harmonies with best and worst cost are updated after the generation of a new harmony as shown in Figure 4. Although the HS runs for 10,000 iterations the parameters converge to their final values much earlier than the final iteration. For the sake of brevity in Figure 4(b) to (d) values are not shown for all 10,000 iterations. These results are captured running HS in a case where $Q_b$ = 50 W and $U$ = 3 m/s. Note that HS is faster and simpler to implement than GA, as also suggested in (Yang, 2009). The simplified implementation makes it possible to use HS in embedded applications where computing resources are very limited. This paradigm could be utilized to design dynamically reconfigurable systems such as heat exchangers.

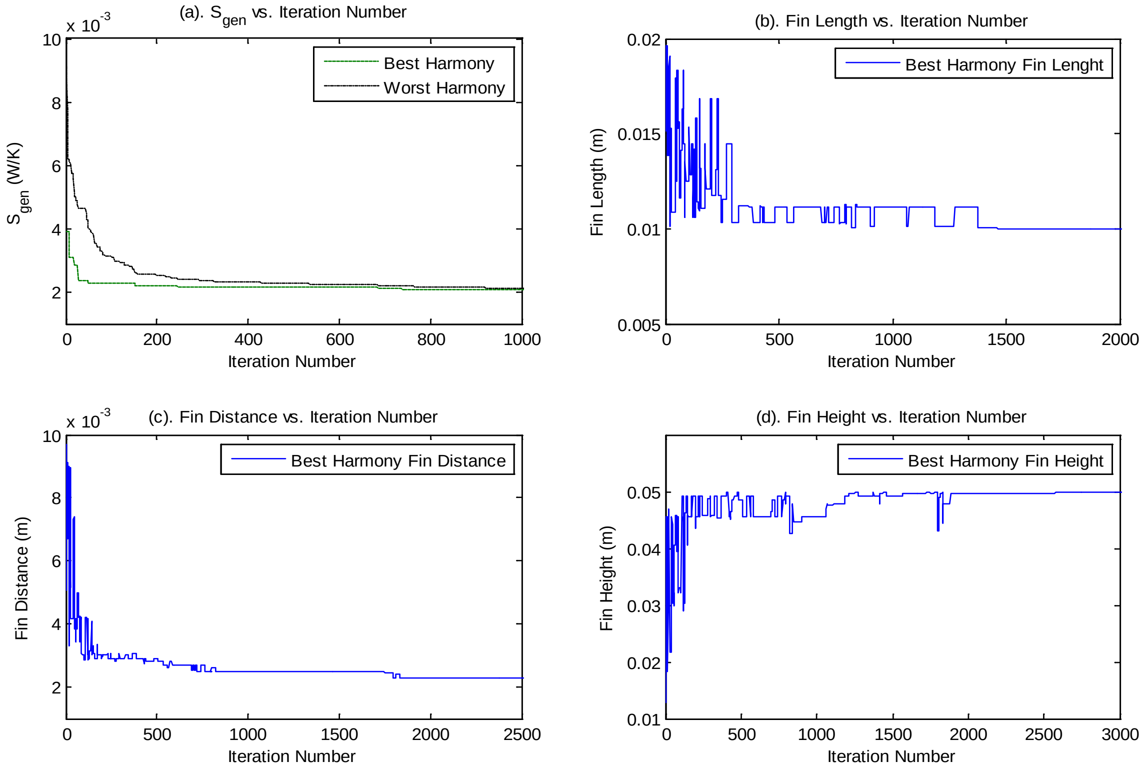


Figure 4. Cost function and parameter convergence pattern during HS execution

Additionally, we conducted the optimization setting all the five design and input parameters as variables. This is to compare the optimal values with the intuitive results of an optimized situation when $Q_b$ is the least. As expected, optimum performance point happens at the minimum $Q_b$. The optimized parameter values are given in Table 3.

Table 3. Results of entropy minimization with five variables

| Objective Function | Decision variables | | | | |
|---|---|---|---|---|---|
| $\dot{S}_{gen}$ (W/K) | $U$ (m/s) | $\varepsilon$ (m) | $D$ (m) | $H$ (m) | $Q_b$ (W) |
| 0.000052 | 0.8 | 0.01 | 0.0043 | 0.05 | 5 |

## CONCLUSION

The optimal layout of an offset strip heat sink was achieved by applying harmony search algorithm as an optimization tool. The cost function is specified with regard to the overall entropy generation rate of the heat sink. Three geometrical parameters including fins height, length and fin to fin distance were chosen as independent variables. Optimal design values of these parameters were attained for different values of base heat flux and flow velocity. Considering comparable results of HS and GA and superior shorter run-time of HS in addition to its simpler implementation, HS introduces itself as a worthy alternative in such complex engineering problems.

## NOMENCLATURE

| | | | |
|---|---|---|---|
| $A$ | Fin cross section area [m$^2$] | *Greek Letters* | |
| $B$ | Substrate width [m] | $\varepsilon$ | Fin swept length [m] |
| $C_p$ | Specific heat [kJ/kg/K] | $\delta$ | Fin thickness [m] |
| $D$ | Fin to fin distance [m] | $\rho$ | Density of air [kg/m$^3$] |
| $F_D$ | Total drag force [N] | $\tau$ | Shear stress [N/m$^2$] |
| $f_D$ | Averaged drag force through each passage [N] | $\vartheta$ | Kinematic viscosity [m$^2$/s] |
| $H$ | Fin height [m] | | |
| $h$ | Average heat transfer coefficient [W/K/m$^2$] | *Subscripts* | |
| $i$ | Rows index | $a$ | Air flow |
| $K$ | Thermal conductivity [W/K/m] | $b$ | Base |
| $m$ | Fin parameter [1/m] | $c$ | Columns |
| $\dot{m}$ | Mass flow rate [kg/s] | $D$ | Drag |
| $N$ | Number | $fin$ | Fins |
| $P$ | Fin cross section perimeter [m] | $in$ | Inlet |
| $Pr$ | Prandtl number | $out$ | Outlet |
| $Q$ | Total heat rejection rate from the heat sink [W] | $r$ | Rows |
| $q$ | Averaged heat transfer from one row [W] | $\infty$ | Free stream |
| $\dot{S}_{gen}$ | Entropy generation rate[W/K] | | |
| $T$ | Temperature [K] | | |
| $U$ | Approach velocity $[\mathrm{m/s}]$ | | |
| $X$ | Substrate length $[\mathrm{m}]$ | | |

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