Heating Effects of a Laser Beam on a GaAs Substrate

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Abstract

A laser beam is frequently used to characterise the optical response of GaAs devices. It can alter the local temperature and, in order to assess the magnitude of this problem, we have solved the steady-state heat equation with the aid of Kirchhoff's transformation. We find that the most coupled variable to any temperature increase is the power of the laser beam and conclude that, for low power applications, heating effects can be considered negligible.

Introduction

Gallium Arsenide (GaAs) has many important optoelectronic applications. A useful method in characterising its optical response is with the aid of a scanning laser beam, which moves along the surface of a GaAs device or chip. This technique, for example, has been applied to thermal emission measurements [1] evaluation of lattice damage in semiconductors [2], or to study new photogain mechanisms in GaAs MESFETs [3]. Consequently, it is important to determine if any significant local temperature increase occurs in the GaAs substrate, due to laser-induced heating. Only a few degrees increase in temperature creates a significant shift in measured electrical parameters.

Our main objective is to determine the extreme value of steady-state local temperature increase of a GaAs chip, when a focused laser beam impinges its surface (see Fig. 1). This paper offers a worst-case analysis, in which the scanning CW laser beam is, in fact, considered stationary.

We assume that pure GaAs is used. For the worst case, we assume perfect transmission of photons through any passivation layers and zero reflection at the surface layers of the chip. The physical parameters [4] are shown in Table 1. The indicated absorption coefficient, α, corresponds to a 670 nm wavelength of the laser beam. The GaAs substrate has a radius, b, of 1 cm, and a depth, h, of 150 μm. The power of

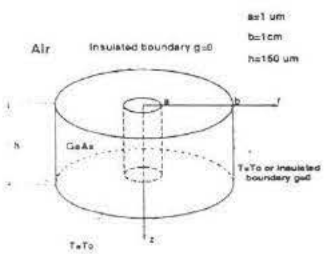

Figure 1: *Diagram defining the dimensions of the laser beam and the wafer.*

c ($Jkg^{-1}K^{-1}$)	325
ρ (kgm^{-3})	5317.4
T_{melt} (K)	1513
α (m^{-1})	31.1×10^5
k_o ($Wm^{-1}K^{-1}$)	57.95

Table 1: *GaAs physical parameters.*

the laser beam was 1.4 μW with a spot radius of 1 μm, as described in our previous work [3]. The ambient temperature is T_o=300 K.

The differential steady-state heat equation was solved with the aid of Kirchhoff's Transformation [5] and modified Bessel functions. Different boundary conditions (b.c.'s) and two possible power dissipation densities were considered. A solution for practical dimensions was obtained.

Theory

The steady-state heat equation and Kirchhoff's transformation.

Taking into account the dependence of the semiconductor thermal conductivity k on the temperature T, the nonlinear steady-state heat equation to be solved is:

$$\nabla(k(T)\nabla T) = -g(z) \tag{1}$$

On the top of the chip there is negligible heat loss to the air, so we assume an insulated b.c. On the other hand, the bottom of the chip acts as a good enough heat sink, therefore we assume it to be at constant room temperature. For the edge of the chip, we distinguish two different b.c.'s: (1) constant temperature, for a good heat sink, and (2) an insulated b.c. Then the b.c.'s of the heat equation are :

(a) $T = T_o = 300\ K$ for $z = h$.
(b) $\partial T/\partial z = 0$ for $z = 0$.
(c.1) $T = T_o$ for $r = b$.
(c.2) $\partial T/\partial r = 0$ for $r = b$.

Kirchhoff's transformation defines a transformed temperature U as:

$$U(T) = \int_{T_0}^{T} k(T')/k_o dT' \tag{2}$$

where $k_o = k(T_o)$. The Kirchhoff's transformation converts the nonlinear heat equation into a linear one, with linear b.c.'s [6]. Due to the fact that heat transfer is radial and symmetrical, the new equation in cylindrical coordinates and the associated bc's are:

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = -g^* \tag{3}$$

(a) $U = 0$ for $z = h$.
(b) $\partial U/\partial z = 0$ for $z = 0$.
(c.1) $U = 0$ for $r = b$.
(c.2) $\partial U/\partial r = 0$ for $r = b$.

$g^* = g/k_o$, is the transformed power density. We considered two cases (appendix 1):

1) Constant dissipated power density, g_1: we consider the average power in the cylinder limited by the laser beam and the substrate (other approximations were attempted with the depth of the heat cylinder equal to $1/\alpha$, and with the average of g or $g = g(1/\alpha)$ in that cylinder, but less realistic predictions for the temperature were obtained). In this case ξ=0 and $G = P/(\pi a^2 h k_o)$, then:

$$g_1^* = P/(\pi a^2 h k_o) \tag{4}$$

2) Exponential dissipated power density, g_2: that is ξ=α, and $G = \alpha P/(\pi a^2 k_o)$.

$$g_2^* = \frac{\alpha P}{\pi a^2 k_o} e^{-\alpha z} \tag{5}$$

After the solution for U has been obtained, the temperature can be determined with the appropriate inverse transformation (appendix 2).

The transformed steady-state heat equation solutions.

Non radiated region

To obtain the temperature in the GaAs chip outside the laser beam region, we must solve the homogeneous heat equation (i.e. $g^* = 0$). Let us name the transformed temperature, U_o. The solution is achieved by using the separation of variables method, i.e. $U_o(r,z) = A_o(z)B_o(r)$. Two independent ordinary differential equations result:

$$\partial^2 A_o/\partial z^2 + p^2 A_o = 0 \tag{6}$$

$$r^2(\partial^2 B_o/\partial r^2) + r(\partial B_o/\partial r) - r^2 p^2 B_o = 0 \tag{7}$$

where p^2 is a positive constant ($-p^2$ was rejected because it does not satisfy the b.c.'s). Taking into account b.c.'s (a) and (b) the solution for A_o is:

$$A_{on} = F\cos(p_n z) \tag{8}$$

$$p_n = (2n-1)\pi/2h; \; n = 1,2,3,... \tag{9}$$

and F is a constant. On the other hand, the solution for B_o depends on whether b.c. (c.1) or (c.2) is selected. For b.c. (c.1) ($B_o = B_{o1}$):

$$B_{o1n} = C_{o1n} f_{o1n}(r) \tag{10}$$

$$f_{o1n}(r) = I_o(p_n r)K_o(p_n b) - I_o(p_n b)K_o(p_n r) \tag{11}$$

where C_{o1n} are n dependent constants, I_o is the modified Bessel function of the first kind and K_o is of the third kind (order ν=0). For b.c. (c.2) the solution, B_{o2}, is:

$$B_{o2n} = C_{o2n} f_{o2n}(r) \tag{12}$$

$$f_{o2n}(r) = I_o(p_n r)K_1(p_n b) + I_1(p_n b)K_o(p_n r) \tag{13}$$

where I_1 and K_1 are the same modified Bessel functions mentioned above but with order ν= 1, and C_{o2n} are n dependent constants.

Therefore we have two different global solutions for U_o depending on which of the b.c.'s is selected. For b.c.'s (c.1) and (c.2) we have respectively ($i = 1,2$):

$$U_{oi} = \sum_{n=1}^{\infty} D_{in} f_{oin}(r) \cos(p_n z) \tag{14}$$

where D_{in} are n dependent constants, to be determined for every (c) b.c.

Radiated region

The solution U_l, for the region under the laser beam, can be obtained as the sum of the homogeneous equation solution, U_{lh}, and a particular solution U_{lp}. Then $U_l = U_{lh} + U_{lp}$. The homogeneous solution is solved again using the separation of variables method: $U_{lh} = A_{lh}(z)B_{lh}(r)$. For A_{lh} we have the same b.c. as for the non-radiated region. Then $A_{lh}(z, n) = A_{lhn} = A_{on}$ for Eqn. 8. For B_{lh} the solution is:

$$B_{lhn} = C_n I_o(p_n r) + F_n K_o(p_n r) \tag{15}$$

Due to $K_o(p_n r)$ diverging when $r \to 0$, then F_n must be equal to 0. Therefore we have for the global homogeneous solution:

$$U_{lhi} = \sum_{n=1}^{\infty} C_{in} I_o(p_n r) \cos(p_n z) \tag{16}$$

where $C_{in}(i = 1, 2)$ are constants to be determined and dependent on the (c) b.c.'s.

If we express the transformed dissipated power density g^* as:

$$g^*(z) = \sum_{n=1}^{\infty} g_n \cos(p_n z) \tag{17}$$

from Eqn. 3 we can obtain the particular solution U_{lp} as:

$$U_{lp} = \sum_{n=1}^{\infty} \frac{g_n}{p_n^2} \cos(p_n z) \tag{18}$$

We have for the global solution in the region exposed to the laser beam, $U_{li} = U_{lhi} + U_{lp}$:

$$U_{li} = \sum_{n=1}^{\infty} [C_{in} I_o(p_n r) + \frac{g_n}{p_n^2}] \cos(p_n z) \tag{19}$$

where i=1,2 corresponds with the selected (c) b.c.'s.

C_{in}, D_{in}, and g_n constants

The temperature must be continuous at the sides of the cylinder limited by the laser beam. We assume that the temperature does not change abruptly at this surface. Then T and $\partial T/\partial r$ for $r = a$ are continuous. These conditions for the transformed temperature are:

$$U_{oi} = U_{li} \qquad \text{for } r = a;\ i = 1, 2 \tag{20}$$

$$\partial U_{oi}/\partial r = \partial U_{li}/\partial r \quad \text{for } r = a;\ i = 1, 2 \tag{21}$$

Taking into account the Modified Bessel functions properties [7] we find that for the heat sinking b.c. (c.1):

$$C_{1n} = -\frac{h_{na}[I_1(p_n a)K_o(p_n b) + I_o(p_n b)K_1(p_n a)]}{I_o(p_n b)} \tag{22}$$

$$D_{1n} = -\frac{h_{na} I_1(p_n a)}{I_o(p_n b)} \tag{23}$$

where:

$$h_{na} = g_n / \{p_n^2 [I_1(p_n a)K_o(p_n a) + I_o(p_n a)K_1(p_n a)]\} \tag{24}$$

and p_n as was defined in Eqn. 9. For the case of the insulated b.c. (c.2):

$$C_{2n} = \frac{h_{na}[I_1(p_n a)k_1(p_n b) - I_1(p_n b)K_1(p_n a)]}{I_1(p_n b)} \tag{25}$$

$$D_{2n} = \frac{h_{na} I_1(p_n a)}{I_1(p_n b)} \tag{26}$$

We know that the transformed power dissipation density is given by g^* from Eqn. 17:

$$g^* = Ge^{-\xi z} = \sum_{m=1}^{\infty} g_m \cos(p_m z) \tag{27}$$

For solving g_n Eqn.27 is multiplied by $\cos(p_n z)$ and integrated with respect to the depth, z, between 0 and h; g_n gives:

$$g_n = \frac{2G}{h} \frac{p_n e^{-\xi h}(-1)^{n+1} + \xi}{\xi^2 + p_n^2} \tag{28}$$

Results and Discussion

Solution for practical dimensions.

In practical cases we have that the radius of the GaAs chip, b, can be ten thousand times the radius of the laser beam a ($b \gg a$). A correct solution can be obtained assuming that temperature has asymptotic behaviour ($b \to \infty$). Then [7]:

$$K_o(p_n b), K_1(p_n b) \to 0; \; I_o(p_n b), I_1(p_n b) \to \infty \tag{29}$$

Then both solutions for the (c) b.c.'s are the same:

$$U = \begin{cases} \sum_{n=1}^{\infty} [\frac{g_n}{p_n^2} - h_{na} K_1(p_n a) I_o(p_n r)] \cos(p_n z) & \text{if } r \leq a \\ \sum_{n=1}^{\infty} h_{na} K_o(p_n r) I_1(p_{na} a) \cos(p_n z) & \text{if } r > a \end{cases} \tag{30}$$

with p_n and h_{na} defined in Eqn. 9 and Eqn. 24.

The solutions for the transformed temperature in all cases mentioned above are in terms of series that converge. In order to compute the result, we must chose the necessary number of steps of the series for a correct prediction.

Temperature distribution.

The exponential power dissipation density g_2^*, gave rise to a more realistic temperature dependence with the parameters, as will be explained in next section. For this case we can see in Fig. 2 the temperature distribution in the substrate under the laser beam.

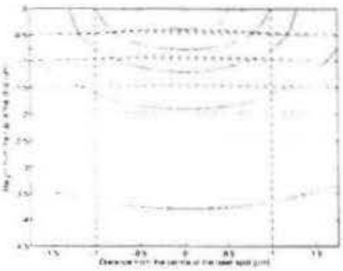

Figure 2: *Constant temperature contours with a quiver plot of the temperature gradient for g_2^*. (P=1.4 μW, α=31.1×10^5 m^{-1}, a = 1 μm, b = 1 cm, h = 150 μm).*

for practical dimensions (see Tab. 1). The vertical lines indicate the region where the laser beam is applied. Solid lines are constant temperature contours. The maximum temperature is produced at the top of the substrate and the middle of the beam, $T_{max} = T(0,0) = 300.006$ K, ie. only a 6 mK increase over room temperature.

The temperature gradient has been also represented in Fig. 2. This shows that the temperature decreases quickly in both directions, radially and with the depth. In fact, only for a 4 μm depth, or from the middle of the laser at the top of the substrate, the increase of the temperature over T_o is reduced to 1 mK. Then all the heating is practically at the surface, and located under the laser beam.

Temperature dependence with the parameters.

In all cases, we refer to maximum temperature at (0,0). We can see in Fig. 3 the temperature dependence with the laser beam radius. When the radius is reduced, the temperature increases lightly, and is higher for the exponential power density case. This is due to the power dissipation density increase when the radius of the laser is reduced and then higher temperatures are expected. In spite of that, the temperature increase is negligible for the radius range of interest. The worst case was for a=0.1 μm with 300.03 K.

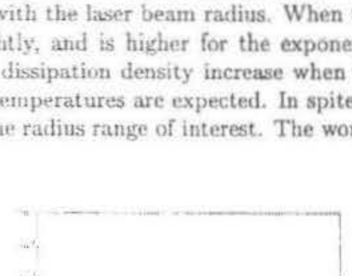

Figure 3: *Temperature dependence with the radius of the laser beam, a. (P = 1.4 μW, α=31.1×10^5 m^{-1}, b = 1 cm, h = 150 μm).*

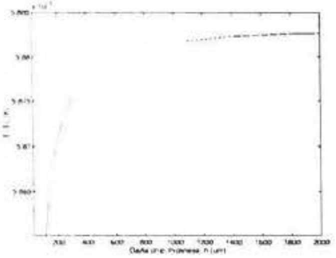

Figure 4: *Temperature dependence with the depth of the substrate h, for* g_2^*. *(P=1.4* μW, $\alpha=31.1\times10^5$ m^{-1}, $a=1$ μm, $b=1$ *cm).*

The case of the temperature dependence with the substrate thickness was different for the two power dissipation densities considered. In Fig. 4 we can see the dependence for the exponential density, g_2^*. The higher the thickness of the substrate, the higher the temperature. This is due to more absorbed photons, contributing to the temperature increase. When the thickness is high enough, practically all photons have been absorbed and then the temperature tends to be constant. Due to photons being absorbed in a very short depth (63% of them are absorbed within a depth of $1/\alpha$=0.32 μm), the temperature dependence with the thickness can be neglected. The maximum temperature was only 0.2 mK higher than for practical dimensions (h = 150 μm) – hence the effect of thicker h can be ignored at low laser powers.

When the power dissipated was assumed constant, using g_1^*, the temperature unrealistically *decreases*, as the thickness is increased – this is because the power is evenly distributed over a greater volume.

The most important temperature dependence is with the power. As the transformed temperature U is proportional to the power, the temperature dependence is independent of the power density selected. We can see in Fig. 5 that when the power of the laser beam is less than 1 mW there is little increase in the temperature (T_{max}=300.1 K), but for higher power there is a quadratic temperature dependence (the linear ap-

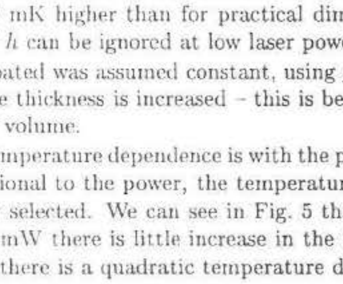

Figure 5: *Temperature dependence with the power of the laser beam.* ($\alpha=31.1\times10^5$ m^{-1}, $a=1$ μm, $b=1$ *cm*, $h=150$ μm).

proximation can be assumed only for low power). The model predicts melting of the GaAs substrate for a power of around 4 W.

In conclusion, an asymptotic behaviour for the temperature ($b \to \infty$), for practical dimensions, can be assumed. Only the power dissipation density with exponential dependence could adequately explain the temperature dependence with all the physical parameters studied. We have shown that the only parameter that affects the temperature, for practical parameters, is the output power of the laser beam. Worst-case CW laser power must be less than 0.1 mW to avoid a shift in the value measured electrical parameters, due to heating.

Appendix 1: Exponential power dissipation density.
In general, the power dissipation varies proportionally with $e^{-\alpha z}$, where z is the depth in the material. Then $g(z) = Ae^{-\alpha z}$, A being a constant to determine. By integrating $g(z)$ over the entire volume of the cylinder which the laser beam is striking, and equating this integral to the total power of the laser beam:

$$P = \int_0^h \int_0^{2\pi} \int_0^a g(z) r dr d\theta dz$$

then $P = (A\pi a^2/\alpha)[1 - e^{-\alpha h}]$. Due to $h \gg 1/\alpha \Rightarrow e^{-\alpha h} \ll 1$, then $A \approx \alpha P/\pi a^2$. The power dissipation can be approximated as:

$$g(z) \approx \frac{\alpha P}{\pi a^2} e^{-\alpha z} \Rightarrow g_2^* \approx \frac{\alpha P}{\pi a^2 k_o} e^{-\alpha z}$$

On the other hand:

$$g_1^* = < g_2^*(z) > = 1/h \int_0^h g_2^*(z) dz = P/\pi a^2 h k_o$$

Appendix 2: Conversion of Kirchhoff's variable U.
The Kirchhoff transformation was defined as in Eqn. 2, where $k(T)$ is the thermal conductivity of GaAs: $k(T) = 54.4 \times 10^3 T^{-1.2}$ [4]. $k_o = k(T_o)$. Then integrating and solving for T:

$$T = \frac{300}{(1-(U/1500))^5} \quad \text{(K)}$$

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