Measurement of linearity in THz-TDS

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Abstract—**This article presents an approach to the measurement of the amplitude linearity in terahertz time-domain spectroscopy (THz-TDS) systems. The approach exploits a single wafer of high-purity float-zone silicon to produce multiple Fabry-Pérot reflections, which are stepwise attenuated and delayed. Comparison between the theoretical and experimental results can indicate a deviation in linearity.**

I. Introduction and Background

Terahertz time-domain spectroscopy (THz-TDS) has become a predominant technique for accessing the *terahertz gap* of the electromagnetic spectrum, loosely defined between 0.1 and 10 THz. The prevalence of THz-TDS is underpinned by its unparalleled SNR in this frequency range as a result of the coherent generation/detection scheme.

As well as other measurement systems, the measurable data should be able to represent the measurand—here the terahertz field amplitude—in a linear fashion. However, nonlinearity in the THz-TDS system may be introduced by electronic and optical noise [1], limited dynamic range of the system [2], misalignment of optical components, or even saturation in electronic parts. A significant factor amongst these is the optical misalignment, as the system is extremely sensitive to the positions of those delicate optical components. Regular calibration of the system for its linearity is of prime importance to maintain the measurement accuracy. Despite that, only a few techniques for examining the system linearity have been reported so far.

Recent work by Naftaly & Dudley [3] proposed that the nonlinearity in THz-TDS systems can be characterized using a stack of silicon wafers, which sequentially attenuate the propagating terahertz waves. In that technique, measurement has to be carried out several times with a number of wafers in the stack increased accordingly. For a different number of wafers, the terahertz amplitude (power) reaches the detector with different attenuation. A plot of the amplitude with respect to the number of wafers is expected to hold a linear relation. Any deviation from the prediction indicates the nonlinearity in the system, which perhaps requires calibration. Note that the method does not exploit the coherent nature of THz-TDS, so incoherent detectors such as Golay cells and pyroelectric sensors can be tested for their linearity as well.

As opposed to the method presented in [3], our method proposed in this article demands only a single transmission measurement with a single dielectric slab by exploiting Fabry-Pérot reflections. The multiple reflections that overlap with the transmitted pulse are successively attenuated and delayed, and hence can be used to determine the system's linearity. This method does not require several expensive prime-grade silicon wafers and hence reduces the problem of surface uniformity. The method also eliminates the process of aligning multiple wafers, which is indeed more susceptible to alignment uncertainty [1]. In addition, requiring only one measurement, the method can avoid drift in the system over a course of multiple measurements that may introduce nonlinear artifacts. Furthermore, there is no concern about interference among wafers that may build up standing waves and bandgap [4]. Importantly, this level of simplicity well suits the system calibration that needs to be performed on a regular basis.

II. Theory

The transmission of a flat dielectric slab in response to a pulse incident normal to the surfaces is expressed as

$$E_t/E_i = \tau\tau'\exp\left\{-\kappa\frac{\omega l}{c}\right\}\exp\left\{-j[n-1]\frac{\omega l}{c}\right\}\times$$
$$\sum_{M=0}^{\infty}\left(\rho'^2\exp\left\{-2\kappa\frac{\omega l}{c}\right\}\exp\left\{-2jn\frac{\omega l}{c}\right\}\right)^M, \quad (1)$$

where E_i is the free-space reference, $\tau=2/(n+1)$ and $\tau'=2n/(n+1)$ are the transmission coefficients, $\rho'=(n-1)/(n+1)$ is the reflection coefficient, κ is the extinction coefficient, n is the refractive index, and l is the slab thickness. The summation represents the Fabry-Pérot effect, by which the incident pulse partially undergoes several reflections inside the sample before reaching the detector—hence, temporally overlapping of the transmitted pulses. Provided that the overlapping pulses are well separated in the time domain, the transmission response for each individual pulse can be inferred from Eq. (1) as

$$E_M/E_i = \tau\tau'\rho'^{2M}\exp\left\{-j[(2M+1)n-1]\frac{\omega l}{c}\right\}, \quad (2)$$

where M denotes the index of individual pulses. Here, M=0 is for the directly transmitted pulse, M=1 for the pulse reflected twice, M=2 for the pulse reflected four times, and so on. Note the sample's absorption is assumed to be negligible here.

Taking the logarithm of the absolute value of Eq. (2) yields

$$\log_{10}|E_M| = \log_{10}|E_i| + \log_{10}|\tau\tau'| + M\log_{10}|\rho'^2| \quad . \quad (3)$$

It is clear that plotting the individual pulse's amplitude in a logarithmic scale as a function of M would theoretically result in a linear relation with its slope determined by the reflectivity. For silicon with n=3.418, $\log_{10}|\rho'^2|$ equals -0.5235. Eq. (3) applies both in the time and frequency domains.

III. Experiment

In the experiment, a dielectric slab used for testing the system's linearity is an intrinsic float-zone silicon wafer polished on both

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sides with a crystal orientation [111] ±0.5°, a diameter of 50.8 mm (2 inches), and a thickness of 1 mm, according to the supplier's specification. Five-point thickness measurement shows that the actual thickness is 1.003 ± 0.002 mm. The wafer resistivity is approximately 11400-18000 ohm·cm, and hence terahertz absorption by free carriers is negligible. Intrinsic float-zone silicon is ideal for studying nonlinearity of the system because of its exceptionally low absorption and dispersion, i.e. α<0.05 cm^{-1} and n=3.4175, in the range between 0.5 and 3.0 THz [5]. Furthermore, its high refractive index results in strong reflections that are critical to validation of the linearity.

The THz-TDS system used in this experiment employs an ultrafast mode-locked laser with a center wavelength of 800 nm and pulse duration of 100 fs, together with two dipole antennas as the emitter and detector. The system yields broadband terahertz performance spanning between 0.2 and 3.0 THz. The measurement is carried out in nitrogen atmosphere to prevent signal ringing induced by ambient water vapor. The focused terahertz beam is directed through the silicon wafer, which is positioned perpendicular to the beam path by observing the maximum transmission during alignment.

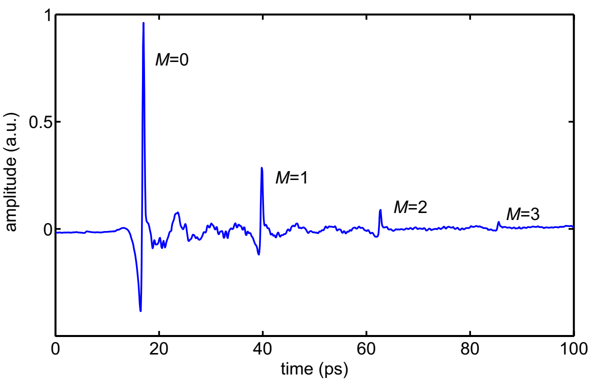

Figure 1. Time-resolved terahertz signal transmitted through the silicon wafer. The smaller pulses, labeled M={1,2,3}, are as a result of multiple reflections at the wafer-air interfaces. The profile is averaged from ten measurements. The time resolution is 0.067 ps.

Fig. 1 shows the time-resolved terahertz signal obtained from transmission measurement with the silicon wafer in place. The measurement was repeated ten times and averaged to find the system's uncertainty. It is obvious that the reflections after the main pulse are from the Fabry-Pérot effect introduced by the wafer because all pulses are separated by 22.8 ps, the time needed for travelling back and forth in the wafer. Temporal windowing of the signal yields individual pulses that can be further analyzed in the frequency domain.

The relations between the magnitude $\log_{10}|E_M|$ and pulse number M at different frequencies extracted from the measurement are illustrated in Fig. 2. The magnitude E_M is relative and can be normalized to the noise level at E_M=1. Obviously, all plots below 1.6 THz incline with a slope of approx. -0.5235, in accordance with the analysis in Eq. (3), indicating the linearity of the system. At frequencies of 1.6 THz and higher, noise sets in and causes divergence from linearity for smaller terahertz magnitude. Nonlinearity in the system introduced by other effects, such as misalignment, tends to be more consistent, i.e., producing lower deviation among several measurements. This kind of nonlinearity is small but observable at some frequencies, urging further investigation.

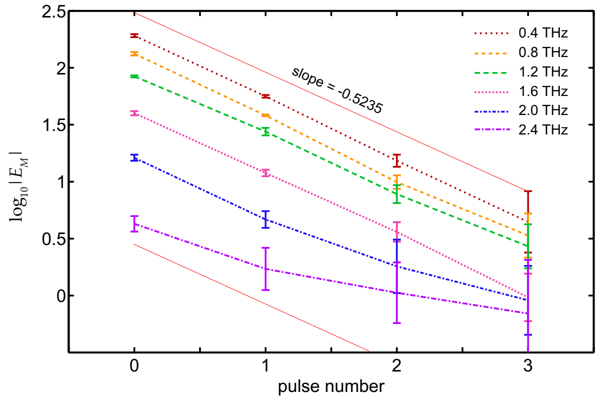

Figure 2. Terahertz magnitude as a function of the pulse number. The slope of -0.5235 represents the case of an ideal system. The magnitude E_M is normalized to unity at noise floor, and hence $\log_{10}|E_M|$ = 0 indicates the noise level. The dotted lines are merely for visual impression.

It is worth noting that in this setting, each successive pulse is rapidly attenuated with a transmission factor of 0.3 determined from the reflection coefficient ρ'^2. In comparison, a series of pulses obtained from several transmission measurements with a different number of wafers experience a transmission factor of 0.7 determined from the transmission coefficient $\tau\tau'$. Hence, for the method proposed here the number of pulses that can be collected is slightly less than in [3]. Despite that, the measurable result with a lower number of pulses proves to be sufficient for investigating linearity in THz-TDS systems. In the case that the refractive index of a slab is greater than 6, the proposed setting has a transmission factor greater than the setting of multiple dielectric slabs, i.e., $\rho'^2 > \tau\tau'$.

IV. Conclusion

The developed method requires only a single silicon wafer for determining linearity in THz-TDS systems. Experimental results demonstrate the feasibility of the method. Qualitative characterization of nonlinearity presenting in the system may be useful in identifying and minimizing the problem.

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