Optimal quantization in neural coding

Mark McDonnell[1] and Derek Abbott
School of Electrical and Electronic Engineering and
Centre for Biomedical Engineering
The University of Adelaide
Adelaide SA 5005 AUSTRALIA
e-mail: mmcdonne@eleceng.adelaide.edu.au, dabbott@eleceng.adelaide.edu.au

I. Introduction

Recently, there has been much interest in the use of information theory for the analysis of neural coding. It is also known that despite often operating with very low signal to noise ratios, the brain is able to efficiently encode and process information. One of the possible mechanisms behind this is known as stochastic resonance, which is the name for a broad class of phenomena in which a nonlinear system's performance is optimized by the presence of noise. In particular, it has been shown that the mutual information between the input and output of a population of N identical neurons subject to uncorrelated noise is optimized by a certain non zero noise intensity [1]. The reason for this is that all the neurons in the population are taken to be identical threshold devices that receive the same signal, with the encoding given by the number of neurons, y, that fire for a given input sample, x. In the absence of noise, all neurons either fire or do not fire and the output is binary. When *iid* additive noise is present at the input of each neuron the result is the effective randomization of the N threshold values and all $N+1$ output states are realizable. The result is a non-deterministic analog to digital conversion of the signal, similar to dithering. The optimal SNR is the one for which the mutual information between the input signal and the encoding is a maximum.

II. Problem Formulation and Results

In this paper we investigate the optimality of this encoding by relaxing the constraint of identical threshold values for each neuron and determining the optimal encoding for a range of SNR's. The population of neurons can be considered a semi-continuous information channel so that for a given input probability density of $P_x(x)$, the mutual information is

$$I(x,y) = -\sum_{n=0}^{N} P_y(n)\log_2 P_y(n) - -\int_{-\infty}^{\infty} P_x(x)\sum_{n=0}^{N} P(n|x)\log_2 P(n|x)dx,$$

where $P_y(n) = \int_{-\infty}^{\infty} P(n|x)P_x(x)dx$ is the probability of the output being $y=n$. $P(n|x)$ gives the probability of encoding input value x with output state y. We have previously [2] described an efficient means of calculating these transition probabilities numerically for given N, $P_x(x)$ and threshold values, and using this technique we are in a position to solve our stated objective, which is to solve the noisy optimal quantization problem

$$\text{Find:} \quad \max_{\{\theta_n\}} I(x,y), \qquad \text{subject to:} \quad \theta_n \in \mathbf{R},\ (n=1,..,N), \tag{1}$$

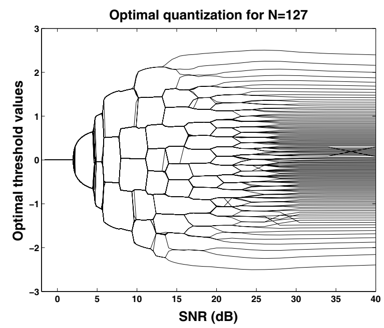

Figure 1: Plot of optimal thresholds against signal to noise ratio for $N=127$ and Gaussian signal and noise.

Our numerical solutions to (1) show several interesting features, including a bifurcational structure where as the signal to noise ratio decreases, more and more thresholds coincide to the same values. This effect is shown in Fig. 1 which was produced for a Gaussian input signal, and *iid* additive Gaussian noise at the input to each of $N=127$ neurons. However, the most important result is that we have found for very low signal to noise ratios, the optimal solution is the most biologically plausible one, that of all thresholds to be equal to the signal mean, as in the original work in [1]. Furthermore, we have shown using Fisher information that the value of SNR at which this last bifurcation occurs asymptotically approaches a fixed value of SNR of about 0 dB, for large N. This result indicates that in the presence of low SNR's, populations of neurons may be able to effectively encode information in a manner similar to a flash analog to digital converter, despite possessing identical thresholds.

Acknowledgments

The authors would like to thank Nigel Stocks of the University of Warwick for useful discussions.

References

[1] N. G. Stocks and R. Mannella, "Generic noise enhanced coding in neuronal arrays," *Physical Review E*, vol. 64, 030902(R), 2001.

[2] M. D. McDonnell, C. E. M. Pearce, and D. Abbott, "An analysis of noise enhanced information transmission in an array of comparators," *Microelectronics Journal*, vol. 33, pp. 1079–1089, 2002.

[1]This work was supported by the Australian Research Council and The Australian Cooperative Research Centre for Sensor, Signal and Information Processing.