SNDR enhancement in noisy sinusoidal signals by non-linear processing elements

Ferran Martorell[a], Mark D. McDonnell[b], Derek Abbott[b] and Antonio Rubio[a]

[a]High Performance IC Design Group, Univ. Politècnica de Catalunya (DEE), Ed.C-4 Jordi Girona 1-3 08034, Barcelona, Spain;

[b]Centre for Biomedical Engineering (CBME) and School of Electrical & Electronic Engineering, The University of Adelaide, SA 5005, Adelaide, Australia

ABSTRACT

We investigate the possibility of building linear amplifiers capable of enhancing the Signal-to-Noise and Distortion Ratio (SNDR) of sinusoidal input signals using simple non-linear elements. Other works have proven that it is possible to enhance the Signal-to-Noise Ratio (SNR) by using limiters. In this work we study a soft limiter non-linear element with and without hysteresis. We show that the SNDR of sinusoidal signals can be enhanced by 0.94 dB using a wideband soft limiter and up to 9.68 dB using a wideband soft limiter with hysteresis. These results indicate that linear amplifiers could be constructed using non-linear circuits with hysteresis. This paper presents mathematical descriptions for the non-linear elements using statistical parameters. Using these models, the input-output SNDR enhancement is obtained by optimizing the non-linear transfer function parameters to maximize the output SNDR.

Keywords: SNDR, non-linear functions, noise, linear amplification

1. INTRODUCTION

The interaction between non-linear components and noisy signals in the areas of information theory and communication is an intriguing area of research that has the potential to provide novel techniques for improving amplifier design. Several interesting phenomena have been found. In recent years both Stochastic Resonance (SR) and Suprathreshold Stochastic Resonance (SSR) have received much attention. These phenomena appear in suboptimal systems or in systems with large noise sources. SR occurs when a signal that is not able to excite a non-linear element (e.g., the signal is subthreshold) can excite the system when a certain amount of noise is added. Therefore, the information transferred from input to output is maximized for a non-zero noise value.[1,2] SSR occurs in an array of N identical non-linear elements excited by the same signal and with independent internal noise sources. The SSR effect transmits more information to the system output[3,4] than conventional SR can. This is because the independent noise in the array allows the output to become an $N+1$ state representation of the input signal. In the absence of noise, only two output state are available. SSR effects occur for signals that are not sub-threshold, and also if the noise is very small compared to the signal. This phenomenon has been demonstrated to be the optimal set-up for systems with large noise.[5] Both phenomena indicate that noise is capable of improving system performance under certain conditions and several systems that are capable of making use of it have been reported either in biology[6–8] or in engineering.[9,10] Generally these applications take advantage of noise to improve the global sensitivity or extend the input range.

SR and SSR require the presence of a certain amount of noise in order to maximize system performance. This fact limits their applicability since, by introducing noise to a system, while it is possible to improve a certain circuit, the noise can also disturb other adjacent elements. A third phenomena, more fitting for engineering applications, is the possibility of designing circuits capable of enhancing the output Signal-to-Noise Ratio (SNR)

Further author information: (Send correspondence to F.M.)
F.M.: E-mail: ferranm@eel.upc.es, Telephone: +34-93-4017-485
M.D.M.: E-mail: mmcdonne@eleceng.adelaide.edu.au, Telephone: +61-8-8303-3165
D.A.: E-mail: dabbott@eleceng.adelaide.edu.au, Telephone: +61-8-8303-5748
A.R: E-mail: rubio@eel.upc.es, Telephone: +34-93-4017-485

Noise and Fluctuations in Circuits, Devices, and Materials,
edited by Massimo Macucci, Lode K.J. Vandamme, Carmine Ciofi, Michael B. Weissman,
Proc. of SPIE Vol. 6600, 660011, (2007) · 0277-786X/07/$18 · doi: 10.1117/12.724688

using noisy non-linear elements (i.e. systems with a positive noise figure). This topic has been addressed by many groups through the years, mainly considering limiting elements (e.g. hard-limiters) and sinusoidal signals.[11–17] These works proved that it is possible to enhance the input-output SNR from -1 to 3 dB when the input signal is a sinusoid. In these works the non-linear transfer function is optimized for the given input signal and noise to achieve the maximum SNR enhancement. Following this example, we investigate the possibility of building linear amplifiers with a positive noise figure using simple non-linear elements. Such an amplifier is interesting because in future novel nanoscale electronic technology it will be easy to build simple non-linear elements, but quite complex to obtain high quality linear elements to process analogue signals.[18]

In the present work we analyze soft limiters with and without hysteresis driven by sinusoidal signals. We consider any amplitude for the sinusoidal signal and noise and optimize the non-linear element parameters (gain in the linear region and hysteresis width) to achieve the maximum output Signal-to-Noise and Distortion Ratio (SNDR). The use of this measure provides a better representation of the problem than the classic SNR definition. Our results indicate that the soft limiter is able to enhance the SNDR by 0.94 dB. More importantly, we show that the soft limiter with hysteresis is capable of reaching 9.68 dB.

This paper is organized into five sections: Section 2 defines the performance measure we use to evaluate the performance of non-linear elements. In Section 3 the non-linear elements are modeled, and expressions for their expected output value and variance are derived. These expressions allow us to calculate the SNDR. Then, using these models, Section 4 shows optimization results for the maximization of SNDR enhancement for each set-up. Finally, Section 5 draws some conclusions.

2. MEASURING THE NON-LINEAR ELEMENT PERFORMANCE

The measure used to evaluate the system performance is crucial for analyzing the response of non-linear noisy elements. Using performance measures designed to evaluate linear signals is not always a good solution. In fact several wrong results have been reported in this area, due to inappropriate performance measures.[19,20] Usually the most robust performance measures for non-linear systems –at least, for aperiodic signals – are those based on information theory such as mutual information.[21] However, these measures often require a complete statistical description which is not always possible or practical to calculate. In general, a sufficient performance measure is one that is able to capture all the important effects arising in the system. In the examples considered in this paper, there are three signal components that determine the non-linear element response: *i*) the information transferred from input to output; *ii*) the degradation of the output signal (distortion) due to the non-linear element; and *iii*) the amount of random noise present at the element output. A simple measure capable of evaluating these three signal components is signal-to-noise and distortion ratio. Besides, SNDR is closely related to information theory measures which further supports its selection.[22,23]

SNDR is defined as the ratio between the signal power and the sum of the noise and the distortion powers. Expressed in dB, its mathematical expression is:

$$\mathrm{SNDR} = 10 \log_{10} \left(\frac{P_s}{P_n + P_d} \right), \tag{1}$$

where P_s, P_n, and P_d stand for the signal, noise and distortion powers respectively. If a signal is not distorted ($P_d = 0$), the measure is equivalent to the classical SNR.

2.1. Calculation of the Power Components

There are several ways to calculate the three power components. We choose to use only time domain information to avoid the numerical errors derived from the time-frequency conversion. Besides, it allows us to easily evaluate the power of any input signal (i.e., whether periodic or aperiodic) and simplifies our method of separating the signal power from the distortion power as we show in this subsection. This method may seem trivial for sinusoidal signals, but it is very useful when more complex signals are considered.

Using the signal's time domain information, it is possible to evaluate the power of a signal using the first two conditional statistical moments (expected value, $E\{y|x\}$, and variance, $\sigma^2_{y|x}$) and the input signal pdf, $f_x(x)$. The calculation of these parameters is relatively simple and permits a quick evaluation of the non-linear element response. Using these two parameters we calculate the three power components. Noise power has a straightforward relation to its variance,

$$P_n = \sigma_y^2 = \int_x \sigma_{y|x}^2 f_x(x)dx. \tag{2}$$

Therefore, integrating the conditional output variance along the time (or along the input signal as $x = x(t)$) we obtain the output noise power. Signal and distortion powers are obtained from the output expected value. We need to introduce some consideration of the system response in order to separate both parameters. As our goal is to investigate the possibility of implementing linear amplifiers, the output signal is expected to be a linear combination of the input signal, x. Then, we can express the output expected value as a linear combination of the input signal and the output distortion, $D(x)$, as:

$$E\{y|x\} = g_{\text{lin}}x + D(x). \tag{3}$$

The key parameter to correctly separate both power components is the equivalent linear gain, g_{lin}. In fact, we consider the system output to be the best linear approximation of the input signal. Then, a simple method to calculate g_{lin} is by projecting the output signal into the input signal[24] as:

$$g_{\text{lin}} = \frac{\overline{(x-\overline{x})\cdot(E\{y|x\}-\overline{E\{y|x\}})}}{\overline{(x-\overline{x})\cdot(x-\overline{x})}}, \tag{4}$$

where $\bar{\cdot}$ denotes time average, x is the input signal, $\overline{x}$ is the constant offset of x (or input DC value), $E\{y|x\}$ is the expected value of the output conditioned on the input signal x, and $\overline{E\{y|x\}} = E\{y\}$ is its DC component. Using g_{lin} we can separate signal and distortion powers as:

$$P_s = \int_x (g_{\text{lin}} \cdot (x-\overline{x}))^2 f_x(x)dx \tag{5}$$

$$P_d = \int_x (E\{y|x\} - \overline{E\{y|x\}} - g_{\text{lin}} \cdot (x-\overline{x}))^2 f_x(x)dx \tag{6}$$

3. MODELING THE NON-LINEAR ELEMENT RESPONSE

This section mathematically describes the non-linear components response. We consider a simple set-up for our soft limiters. As depicted in Figure 1 left, the non-linear element receives an input signal with an added noise, η, and produce an output signal, y. We assume η to be white and Gaussian distributed (with standard deviation σ_η and mean μ) as it is the most fitted noise model for internal and multiple external noise sources – the main situation for electronic systems. The non-linear circuit is powered with symmetrical sources ($V/2$ and $-V/2$). It simplifies the model expressions without limiting the result validity as a constant offset does not modify the obtained results.

As discussed in Section 2.1, in order to evaluate the three power components necessary to calculate the output SNDR, we need to calculate the output expected value and variance conditioned on the input signal. The first step to calculate these statistical parameters is to obtain the probability density function (pdf) for the element output signal conditioned on the element input signal which we denote by $f_y(y|x)$. Then, using this pdf we calculate the expected value and variance in each case using the following expressions:

$$E\{y|x\} = \int_y y f_y(y|x)dy. \tag{7}$$

$$\sigma_{y|x}^2 = \int_y (y - E\{y|x\})^2 f_y(y|x)dy = \int_y y^2 f_y(y|x)dy - E^2\{y|x\}. \tag{8}$$

From a practical point of view we can understand the output expected value conditioned on the input as the non-linear element transfer function, and the standard deviation, which is defined as the square root of the variance,

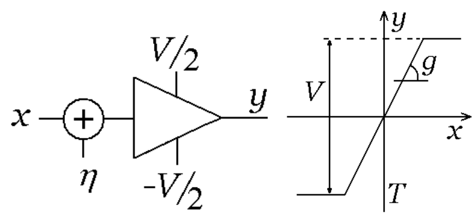

Figure 1. Left: Set-up of the non-linear element under study. Right: Soft limiter transfer function.

as the output noise amplitude for each input value (opposed to linear systems, the output noise is not constant for the whole input range). We use the standard deviation because it provides information about the noise amplitude, while the variance indicates the noise power. Therefore, we present a straightforward comparison between noise and signal amplitudes.

The obtained expressions are checked by comparing the model predictions with numerical simulations of the elements. These simulations compute the response of the non-linear transfer function for an input range between -0.5 and 0.5 V and several input noise amplitudes, σ_η, and diverse values of the element parameters. In the case of the soft limiter with hysteresis the simulation consider an input signal starting at -0.5 V, rising to 0.5 V and returning to -0.5 V. This double path is necessary to illustrate the effects of the element's hysteresis. The first region (-0.5 to 0.5 V) shows the low to high transition while the returning path (0.5 to -0.5 V) demonstrates the high to low transition.

3.1. Soft Limiter

One of the simplest electronic non-linear circuits is the soft limiter. It is a first order model with a linear section linking two saturation regions, as depicted in Figure 1. This element is widely used in electronics in several configurations, either for digital processing (inversion and/or buffering) or in analogue processing (amplification). The complete circuit transfer function is the same in both modes, but the working region is different. In analogue processing the goal is to obtain a linear amplifier, so only the central linear region is used. In contrast, digital circuits use the function in a saturated mode to obtain either high or low voltages and avoid the central linear section that produces undetermined digital values. We consider the use of the entire transfer function to achieve our objective. The mathematical definition for this element response is:

$$y = \begin{cases} \frac{V}{2} & x+\eta > T + \frac{V}{2g} \\ g(x+\eta-T) & \text{otherwise} \\ -\frac{V}{2} & x+\eta < T - \frac{V}{2g}, \end{cases} \tag{9}$$

where V defines the distance between the saturated regions (source voltage in electronic circuits). For this analysis we consider a symmetrical function – it simplifies the expressions and adding a constant offset at the output does not modify the obtained results. The central linear section has a gain g and is centered at the threshold value T (i.e., the input value that forces the output to 0 V).

Starting from Equation (9) we calculate the probability density function of the element output conditioned on the input signal as:

$$\begin{aligned} f_y(y|x) = \frac{1}{\sigma g\sqrt{2\pi}}\, \mathrm{e}^{-\dfrac{(y/g+T-x-\mu)^2}{2\sigma^2}} \left(U(y+\frac{V}{2}) - U(y-\frac{V}{2})\right) \\ +\frac{1}{2}\left[1+\ \mathrm{erf}\left(\frac{T-\frac{V}{2g}-x-\mu}{\sigma\sqrt{2}}\right)\right]\delta(y+\frac{V}{2}) + \frac{1}{2}\left[1-\ \mathrm{erf}\left(\frac{T+\frac{V}{2g}-x-\mu}{\sigma\sqrt{2}}\right)\right]\delta(y-\frac{V}{2}). \end{aligned} \tag{10}$$

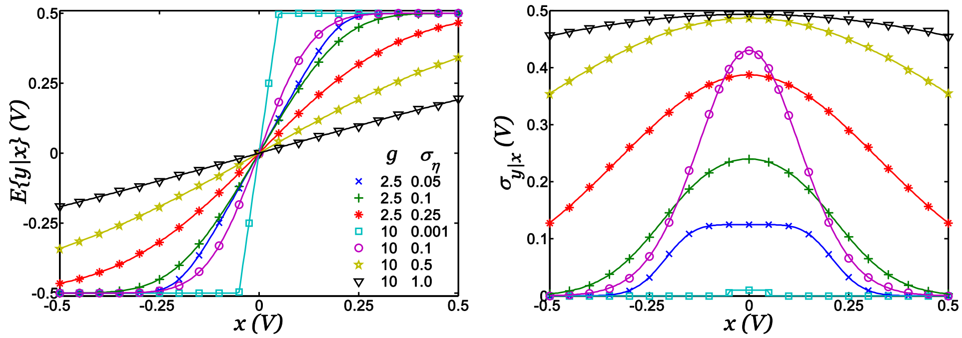

Figure 2. Expected value and standard deviation for several set-ups of a soft limiter. Continuous lines correspond to model predictions and symbols indicate simulated data.

Substituting Equation (10) into (7) and (8) we obtain

$$\begin{aligned} E\{y|x\} = &-\frac{\sigma g}{\sqrt{2\pi}}\left[\mathrm{e}^{-\frac{(B_h-x)^2}{2\sigma^2}} - \mathrm{e}^{-\frac{(B_l-x)^2}{2\sigma^2}}\right] \\ &+\frac{g}{2}(x+\mu-T)\left[\mathrm{erf}\left(\frac{B_h-x}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{B_l-x}{\sigma\sqrt{2}}\right)\right] \\ &-\frac{V}{4}\left[\mathrm{erf}\left(\frac{B_h-x}{\sigma\sqrt{2}}\right) + \mathrm{erf}\left(\frac{B_l-x}{\sigma\sqrt{2}}\right)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} \sigma^2_{y|x} = &\frac{V^2}{4} - \frac{\sigma g^2}{\sqrt{2\pi}}\left[-(B_l-x)\,\mathrm{e}^{-\frac{(B_h-x)^2}{2\sigma^2}} + (B_h-x)\,\mathrm{e}^{-\frac{(B_l-x)^2}{2\sigma^2}}\right] \\ &+\frac{g^2}{2}(\sigma^2+(x+\mu-T)^2)\left[\mathrm{erf}\left(\frac{B_h-x}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{B_l-x}{\sigma\sqrt{2}}\right)\right] \\ &-\frac{V^2}{8}\left[\mathrm{erf}\left(\frac{B_h-x}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{B_l-x}{\sigma\sqrt{2}}\right)\right] - E^2\{y|x\} \end{aligned} \tag{12}$$

where we have defined $B_h = T + \frac{V}{2g} - \mu$ and $B_l = T - \frac{V}{2g} - \mu$ to reduce the notation. To check the validity of these expressions we have performed several simulations varying the three parameters of the non-linear transfer function. Some of these results are summarized in Figure 2. In these plots we observe the output expected value and standard deviation obtained from the simulated data and from the model predictions for gain values of 2.5 and 10 V/V and σ_η = 0.001, 0.05, 0.1, 0.25, 0.5, and 1.0 V with $T = 0$ V and $V = 1$ V. The match between both sets of data indicates the model validity.

3.2. Soft Limiter with Hysteresis

The soft limiter with hysteresis is also an elementary electronic circuit. It is used to avoid noisy transitions in threshold detectors when noisy signals are close to the threshold value. The element has two state, high and low. Each state has a limiter response but with a threshold value shifted by $C/2$ (low) and $-C/2$ (high). The element enters the low state when the input signal goes below $T - C/2 - V/2g$ and returns to high state when x goes above $T + C/2 + V/2g$. The region comprised between two limits has two possible output values depending on the component state. For modeling purposes we consider the hysteretic limiter response as two superimposed limiters with threshold values separated by the hysteresis width, C. Figure 3 depicts the element transfer function.

The element transfer function is governed by the response of each state. The expected value and variance are a weighted average of the response of each state according to the probability of being in each state. Then, the expression for the output pdf is:

$$f_y(y|x) = p_L f_L(y|x) + p_H f_H(y|x) \tag{13}$$

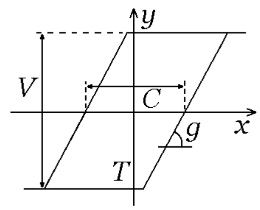

Figure 3. Soft limiter with hysteresis transfer function.

where p_H and p_L are the probabilities of being in each state and $f_{y_L}(y_L|x)$ and $f_{y_H}(y_H|x)$ the pdf for each state independently. The expressions for these pdfs are equivalent to that in Equation (10), but substituting $\mu = C/2$ for the high state and $\mu = -C/2$ for the low state. Introducing the pdf in Equations (7) and (8) and integrating we found the expressions for the expected value and variance

$$E\{y|x\} = p_L E_L\{y|x\} + p_H E_H\{y|x\} \tag{14}$$

$$\sigma^2_{\{y|x\}} = p_L E_L\{y^2|x\} + p_H E_H\{y^2|x\} - E^2\{y|x\} \tag{15}$$

where $E_L\{y|x\}$, $E_H\{y|x\}$ are the expected values of the high and low state outputs and $E_L\{y^2|x\}$, $E_H\{y^2|x\}$ are the expected square values of the output at each state. Again, the expressions are equivalent to those in Equations (11) and (12), but substituting μ by the threshold displacement associated to each state.

The state probabilities are calculated by modeling the system as a two states Markov chain. The future state is calculated as a function of the present state and the current element input (x, η). Using the Markov theory we can define a transition matrix as:

$$M_{x,\eta} = \frac{1}{2}\begin{pmatrix} 1 + \operatorname{erf}\left(\frac{B_H - x}{\sigma\sqrt{2}}\right) & 1 - \operatorname{erf}\left(\frac{B_H - x}{\sigma\sqrt{2}}\right) \\ 1 + \operatorname{erf}\left(\frac{B_L - x}{\sigma\sqrt{2}}\right) & 1 - \operatorname{erf}\left(\frac{B_L - x}{\sigma\sqrt{2}}\right) \end{pmatrix}$$

and the state probabilities as:

$$p_L = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{B_H - x}{\sigma\sqrt{2}}\right) p_{L-1} + \operatorname{erf}\left(\frac{B_L - x}{\sigma\sqrt{2}}\right) p_{H-1}\right) \tag{16}$$

$$p_H = \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{B_H - x}{\sigma\sqrt{2}}\right) p_{L-1} - \operatorname{erf}\left(\frac{B_L - x}{\sigma\sqrt{2}}\right) p_{H-1}\right) \tag{17}$$

where $B_L = T - C/2 - V/2g$ and $B_H = T + C/2 + V/2g$. This process is not a homogeneous Markov chain as the matrix varies continuously with x. For each simulation point, the state probabilities are calculated by multiplying the transition matrix by the current state probabilities in each sampling value of the input signal. Therefore, the simulation step slightly changes the the results. Some tests have been performed and once the sampling ratio is about 3 orders of magnitude higher (1:1000) than the input signal maximum frequency there is no significant variation due to this approximation.

To check the model validity we have performed several simulations varying the four parameters of the non-linear function. Figure 4 shows the comparison between the expected value and the standard deviation predicted by the model and calculated from the simulation data for gain values of 2.5 and 10 V/V and $\sigma_\eta = 0.001$, 0.05, 0.1, 0.25, 0.5, and 1.0 V with $T = 0$ V, $V = 1$ V, and $C = 0.2$ V. The different response for each element state is clearly observed in both curve sets. The agreement between both sets of data validates the model.

4. SNDR ENHANCEMENT IN SINUSOIDAL SIGNALS

Using the models presented in Section 3 it is possible to calculate the expected value and variance easily for any input signal. Then, substituting these expressions into Equations (2), (5) and (6) we calculate the three

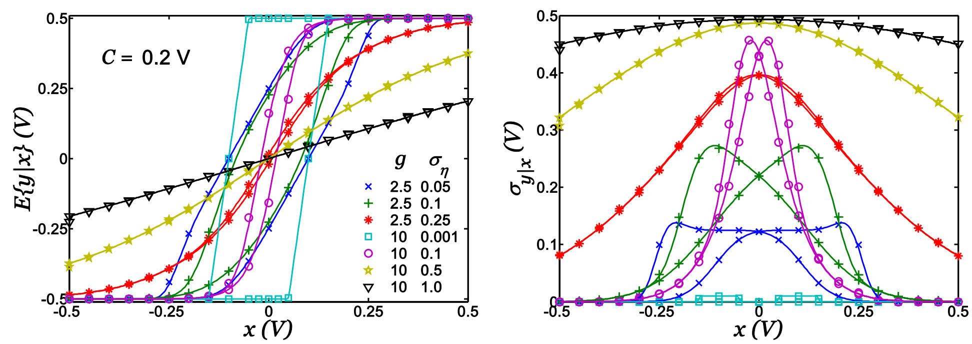

Figure 4. Output expected value and standard deviation for several set-ups of a soft limiter with hysteresis. Continuous lines correspond to the model predictions and symbols indicate the simulated data.

power components and with them the output SNDR. In this section, given an input SNR, the parameters of the non-linear transfer functions are optimized to maximize the output SNDR and consequently the input-output SNDR gain. The optimization is done numerically using a steepest-descent nonlinear optimization algorithm.[25]

In the following subsections we present the results of the optimization process for both non-linear elements. The maximum achievable input-output SNDR gain is given along with the optimal non-linear transfer function parameter values (linear section gain and hysteresis width). The system response depends on the ratio between the signal and noise amplitudes. Therefore, all the plots are referenced to the input SNR. The optimal parameters are bounded to the input signal amplitude and we present them normalized by the input signal range (the actual simulations for the optimization process consider a sinusoidal amplitude of 0.5 V, so the input range is 1 V). Following the same principles for generalizing the results we present a normalized value for the input-output linear gain. The linear gain presented is normalized by the ratio between the source voltage, V, and the linear region width ($b = 1/g$). The input-output voltage gain is calculated as $g_{io} = g_{lin}V/b$. Finally, we include the evolution of the three power components and the expected value and standard deviation as a function of the input signal values for several values of the input SNR. These plots are used to describe the mechanisms by which the non-linear elements enhance the SNDR.

4.1. Soft Limiter

In an electronic circuit implementing a soft limiter we have a single degree of freedom, the width of the linear section which is given by $b = 1/g$. The maximum input-output SNDR enhancement is 0.94 dB and is reached for an input SNR of 9.5 dB. As we observe in Figure 5 the maximum SNDR enhancement appears for input SNR around 10 dB. In this region it is possible to trade a small signal distortion for a noise reduction, which is achieved by clipping the signal. The clipping can be observed in the reduction of the linear section width below the input signal range and in an g_{lin} greater than 1 (Figure 5) and also in the expected value and standard deviation at the optimal input SNR presented in Figure 6. In the optimal point response the system clearly introduces a small non-linearity that distorts the extreme values of the signal but further reduces the output standard deviation. Higher or lower input SNR produces a lower SNDR enhancement as distortion and noise cannot be traded to produce a better output. For high quality signals (large input SNR) the optimal non-linear system is a linear amplifier (optimal b equal to the input signal range). For very low quality signals (very low input SNR) the noise is too large to be clipped without distorting the input signal. Therefore, the optimal system minimizes the noise by reducing the linear gain or increasing the linear section width (see the rapid increase of b in Figure 5). By increasing the linear section width the output signal is reduced (important reduction of g_{lin}) and therefore a trade-off value is reached. However, the SNDR enhancement out of the optimal region rapidly decreases to 0 dB and the system responds mainly linearly (Figure 6).

4.2. Soft Limiter with Hysteresis

A soft limiter with hysteresis has two degrees of freedom that can be used for the optimization: the hysteresis width, C, and the linear section width, b, between the saturation regions. We consider a symmetric non-linear

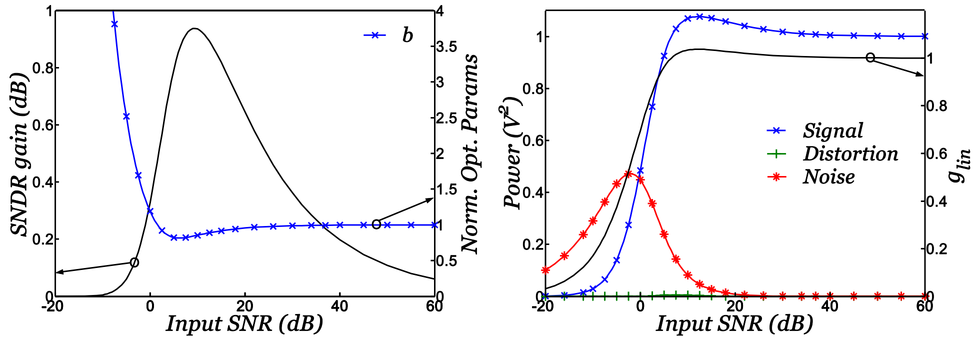

Figure 5. Optimal response of a soft limiter driven by a sinusoidal input signal. Left: Input-output SNDR enhancement and optimal linear width normalized by the input signal range. Right: Signal, distortion and noise power at the system output and input-output linear gain.

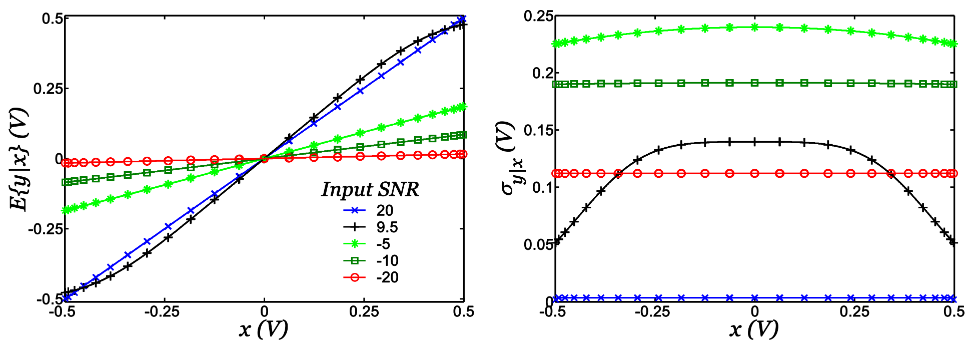

Figure 6. Optimal expected value (left) and standard deviation (right) for a soft limiter at different input SNR values.

transfer function, that is, the high to low and low to high paths are symmetric. As observed in Figure 7, the optimal response is completely different than that observed in the simple soft limiter. The maximum SNDR enhancement reaches a value of 9.68 dB which is attained for an input SNR near -10 dB and is kept nearly constant for lower input SNRs.

There are two different phenomena that the non-linear element uses to improve its input-output SNDR. The first phenomenon is the same arising in the soft limiter where clipping the extreme values of the input signal allows the element to trade noise reduction for a small signal distortion. This effect appears for high to medium quality input SNR (input SNR higher than 10 dB). In this region the optimal hysteresis width is 0 V and the optimal linear width follows the same behavior than for the soft limiter. For lower input SNRs, the optimal system presents a hysteresis width greater than 0 V. The hysteresis width further improves the noise reduction at the cost of increasing the output distortion (see the evolution of the power components in Figure 7 right). The element uses C to reduce the standard deviation for values close to T (see the double peak curves of the standard deviation in Figure 8 right) while the input-output transfer function keeps the signal distortion to a low-mid value (Figure 8 left). At extremely low input SNR values, the non-linear element increases the width of the linear section to reduce the noise power and uses the hysteresis width to maximize the output signal power. In any case, the absolute output SNDR always decreases with decreasing input SNR. Therefore, the region where this phenomenon is of greater practical interest for electronic applications is for input SNR values between 10 to -10 dB.

Clipping and hysteresis effects are applied at both sides of the input signal (above and below the input signal mean value which is considered to be equal to T). This means that input-output SNDR enhancement is maximized when the input signal pdf is symmetrical (related to T). In this situation the effects (clipping and

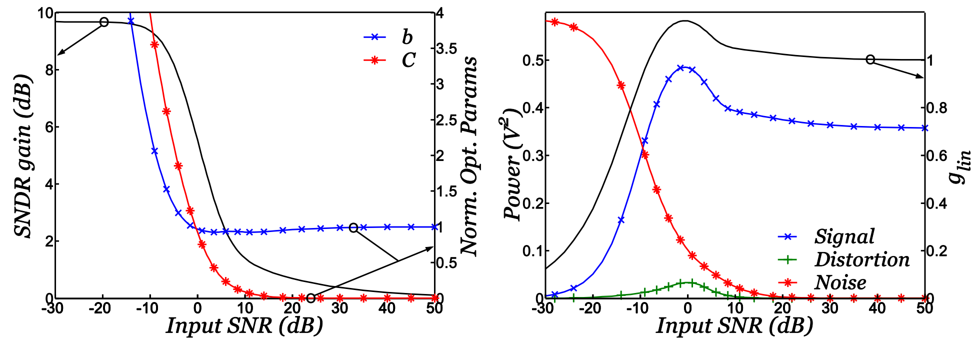

Figure 7. Optimal response of a soft limiter with hysteresis driven by a sinusoidal input signal. Left: Input-output SNDR enhancement and optimal linear width normalized by the input signal range. Right: Signal, distortion and noise power at the system output and input-output linear gain.

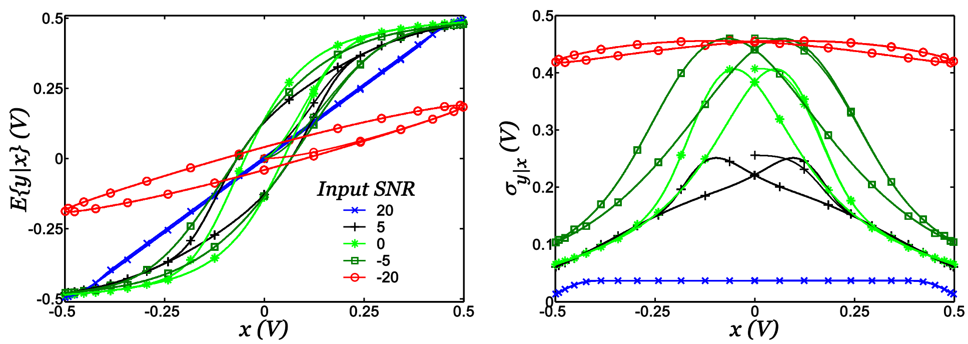

Figure 8. Optimal expected value (left) and standard deviation (right) for a soft limiter with hysteresis at different input SNR.

hysteresis) have a maximum effect. Sinusoidal signals have a symmetrical pdf with a high probability of being in the extreme values (i.e., a high probability of taking advantage of clipping effects). It indicates that the presented results approach an upper bound to the input-output enhancement achievable by such non-linear elements.

5. CONCLUSIONS

We have shown that it is theoretically possible to built linear amplifiers able to enhance the input-output SNDR using non-linear elements. The soft limiter element is capable of improving the SNDR by 0.94 dB. The soft limiter with hysteresis presents a substantially better performance reaching 9.68 dB. The main phenomena in the soft limiter by which the SNDR is enhanced is signal clipping. The soft limiter with hysteresis takes advantage of the hysteresis width to further improve the signal power and reduce a significant part of the noise in the region of practical interest. In general, the SNDR enhancement by such phenomena is best applied to signals with a symmetrical pdf, sinusoidal signals being probably the best suited case of study. Therefore, we expect the current results to be an upper bound to the performance enhancement for general input signals. Nonetheless, the results are important enough to be considered. Such amplifiers may provide an alternative path to build linear functions in future nanotechnologies where circuits will have a limited complexity and devices are expected to be extremely noisy.

ACKNOWLEDGMENTS

This work is funded by the grant AP2002-2600 and the project TEC2005-02739/MIC.

REFERENCES

1. R. Benzi, A. Sutera, and A. Vulpiani, "The mechanism of stochastic resonance," *Jour. Phys. A* **14**, pp. 453–457, 1981.
2. L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, "Stochastic resonance," *Reviews of Modern Physics* **70**, pp. 223–287, 1998.
3. N. G. Stocks, "Suprathreshold stochastic resonance in multilevel threshold systems," *Phys. Rev. Lett.* **84**, pp. 2310–2313, 2000.
4. M. D. McDonnell, C. E. M. Pearce, and D. Abbott, "A characterization of suprathreshold stochastic resonance in an array of comparators by correlation coefficient," *Fluctuation and Noise Letters* **2**, pp. L205–L220, 2002.
5. M. D. McDonnell, N. G. Stocks, C. E. M. Pearce, and D. Abbott, "Optimal information transmission in nonlinear arrays through suprathreshold stochastic resonance," *Physics Letters A* **352**, pp. 183–189, 2006.
6. J. K. Douglass, L. Wilkens, E. Pantazelou, and F. Moss, "Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance," *Nature* **365**, pp. 337–340, 1993.
7. J. E. Levin and J. P. Miller, "Broadband neural encoding in the cricket cercal sensory system enhanced by stochastic resonance," *Nature* **380**, pp. 165–168, 1996.
8. X. Yu and E. R. Lewis, "Studies with spike initiators: linearization by noise allows continuous signal modulation in neural networks," *IEEE Trans. on Biomedical Eng.* **36**, pp. 36–43, 1989.
9. I. Y. Lee, X. Liu, B. Kosko, and C. Zhou, "Nanosignal processing: stochastic resonance in carbon nanotubes that detect subthreshold signals," *Nano Letters* **3**, pp. 1683–1686, 2003.
10. G. G. Karapetyan, "The increase of gravitational-wave interferometer sensitivity by employing stochastic resonance mechanism in nonlinear white-light cavity," *Optics Com.* **238**, pp. 35–43, 2004.
11. N. Blachman, "The output signal-to-noise ratio of a power-law device," *J. App. Phys.* **24**, pp. 783–785, 1953.
12. J. Jones, "Hard-limiting of two signals in random noise," *IEEE Trans. Inf. Theory* **IT-9**, pp. 34–42, 1963.
13. N. Blachman, "The output signal-to-noise ratio in bandpass limiters," *IEEE Trans. Aerosp. Electron. Syst.* **AES-4**, p. 635, 1968.
14. R. Kelly and P. Hariharan, "Ideal limiting of periodic signals in random noise," *IEEE Trans Aerosp Electron Syst* **AES-7**(4), pp. 644–651, 1971.
15. P. Jain, "Limiting of signals in random noise," *IEEE Transactions on Information Theory* **IT-18**(3), pp. 332–340, 1972.
16. A. Geens and Y. Rolain, "Noise figure measurements on nonlinear devices," *IEEE Transactions on Instrumentation and Measurement* **50**(4), pp. 971–975, 2001.
17. F. Chapeau-Blondeau and D. Rousseau, "Noise-aided SNR amplification by parallel arrays of sensors with saturation," *Physics letters A* **351**, pp. 231–237, 2006.
18. "International technology roadmap for semiconductors, ITRS," 2005.
19. I. A. Khovanov and P. V. E. McClintock, "Comment on "signal-to-noise ratio gain in neuronal systems"," *Phys. Rev. E* **67**, p. 043901, Apr 2003.
20. P. Hänggi, M. E. Inchiosa, D. Fogliatti, and A. R. Bulsara, "Nonlinear stochastic resonance: The saga of anomalous output-input gain," *Phys. Rev. E* **62**, pp. 6155–6163, Nov 2000.
21. N. G. Stocks, "Information transmission in parallel threshold arrays: Suprathreshold stochastic resonance," *Phys. Rev. E* **63**, p. 041114, 2001.
22. I. Bar-David and P. Boukris, "Reciprocal mean-square error and signal-to-noise ratio as distinct performance measures in below-threshold communication," *Communications, IEEE Transactions on* **20**, pp. 852–856, Oct 1972.
23. D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean square error in Gaussian channels," *IEEE trans. on inf. theory* **51**, pp. 1261–1282, 2005.
24. P. Lavrador, N. de Carvalho, and J. Pedro, "Evaluation of signal-to-noise and distortion ratio degradation in nonlinear systems," *Microwave Theory and Techniques, IEEE Transactions on* **52**, pp. 813–822, March 2004.
25. P. Gill, W. Murray, and M. Wright, *Practical Optimization*, Academic Press, London, 1981.