Neural Information Transfer in a Noisy Environment

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ABSTRACT

For an array of N summing comparators, each with the same internal noise, how should the set of thresholds, θ_i, be arranged to maximise the information at the output, given the input signal, x, has an arbitrary probability density, $P(x)$? This problem is easy to solve when there is no internal noise. In this case, the transmitted information is equal to the entropy of the output signal, y. For N comparators there are $N+1$ possible output states and hence y can take on $N+1$ values. The transmitted information is maximised when all output states have the same probability of occupation, that is, $1/(N+1)$. In this paper we address some preliminary considerations relating to the maximisation of the transmitted information $I = H(y) - H(y|x)$ when there is finite internal noise.

Keywords: Threshold devices, optimal quantization, transmitted information, stochastic resonance, noisy sensory neuron models

1. INTRODUCTION

Consider an array of N threshold devices or comparators, subject to the same continuously valued input signal, x, as shown in Figure 1. The i–th device is subject to independent continuously valued additive noise, η_i $(i = 1, .., N)$. The output from each device is 1 if the input signal plus the noise is greater than the threshold θ_i of that device and 0 otherwise. The outputs from the devices are summed to give the output signal y. Hence y is a discrete signal taking on integer values from 0 to N and can be considered as the number of devices that are currently "on".

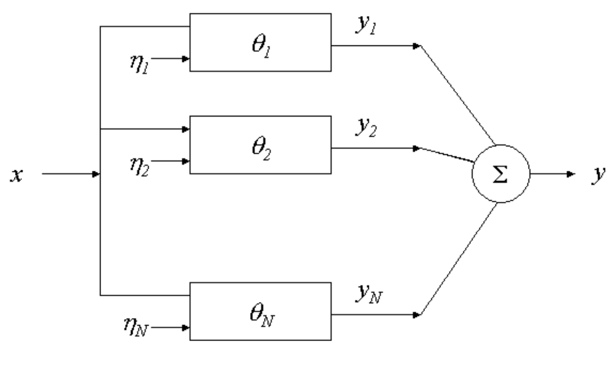

Figure 1: Array of N summing comparators

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Such an array has many similarities to neuron configurations, such as a summing network of N FitzHugh-Nagumo neurons.[1] It is known that neurons can be very noisy, with some papers reporting signal to noise ratios of 0 decibels,[2] yet still effectively transfer information. It is of interest to investigate this phenomenon to see if it is possible to utilize it to improve non-linear electronic devices such as motion detection systems. Such arrays are also good models of FLASH analog to digital converters (ADCs)[3] (when the thresholds are uniformly distributed across the signal space) and DIMUS (Digital Multibeam Steering) sonar arrays, in the "on target" position.[4,5]

Our aim is to investigate the conditions under which the performance of such an array can be optimised by a certain non-zero noise setting. The phenomenon of a non-linear system performing optimally for non-zero noise levels is known as stochastic resonance.[6–8] Stochastic resonance was first reported as an explanation of the periodicity of ice ages.[9] Since then, it has been shown to occur in many non-linear systems, such as electronic devices,[10] ring lasers,[11] SQUIDS (super conducting quantum interference devices)[12] and in biological sensory neurons[13] and ion channels.[14]

A number of methods have been used to quantify "optimal performance" in the literature. Originally, stochastic resonance was defined as an increase in signal to noise ratio for a weak periodic signal in a non-linear system. However, it was later shown that stochastic resonant phenomena could occur for broadband signals. This is known as Aperiodic Stochastic Resonance (ASR).[15,16] For such broadband signals, cross-correlation measures,[17] transmitted information[15,18] (Shannon information), Fisher information,[19] Kullback entropy,[20] ϕ–divergences[21,22] and channel capacity[23,24] have all been shown to possess maxima for non-zero noise values in various systems.

In this paper, we consider only the (Shannon) transmitted information.[25] For general conditions, such a quantity is more robust for broadband excitation signals than quantities such as signal to noise ratio and cross-correlation coefficient for non-linear systems where the signal is large compared to the noise.[26] A brief summary of this paper follows. Section 2 gives mathematical descriptions of the transmitted information for the array of threshold devices, and of various probability densities on which the transmitted information depends. Three different probabilistic distributions of the signal and noise amplitude are considered. In particular, a general formula is presented for calculating the probability that n thresholds are "on" for a given signal x.

Section 3 presents results obtained showing how the transmitted information varies with the noise intensity for two different threshold configurations and the three probability distributions. An alternative approach to analysing the array is given in Section 4. Here we consider the output to be an estimator of the input, and derive a formula for the variance of the error between the output and the input. It is shown that this variance has a minimum for non-zero noise intensity when $N > 1$ and hence displays stochastic resonant–like behaviour. Finally, Section 5 summarises the paper and presents some conclusions and future directions for this work.

2. CALCULATING INFORMATION TRANSMITTED THROUGH THE ARRAY

The array of N comparators in shown in Figure 1. The output of device i is given by

$$y_i = \begin{cases} 1 & \text{if } x + \eta_i > \theta_i, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, the output of the array is $y = \sum_{i=1}^{N} y_i$. We consider the array to be an information channel. The transmitted information I through a channel is given by the entropy $H(y)$ of the output less the conditional entropy $H(y|x)$ of the output given the input as

$$I = H(y) - H(y|x). \tag{1}$$

As noted by Stocks,[18] $H(y|x)$ can be interpreted as the amount of *encoded* information about the input signal lost through the channel. Since the input to the array is continuously valued and the output is discretely valued, we can consider the channel to be semi–continuous.[27] The transmitted information through such a channel is given by

$$I = -\sum_{n=0}^{N} Q(n)\log_2 Q(n) - \left(-\int_{-\infty}^{\infty} P(x)\sum_{n=0}^{N} P(n|x)\log_2 P(n|x)dx\right), \tag{2}$$

where $P(x)$ is the probability density of the input signal x, $Q(n)$ is the probability of the output signal y being equal to n ($n = 0, 1, \ldots, N$) and $P(n|x)$ the conditional probability that the output is n given the input is x.[18, 26, 28]

We have also the equation

$$Q(n) = \int_{-\infty}^{\infty} P(n|x)P(x)dx \tag{3}$$

relating $Q(n)$ and $P(n|x)$. Hence, the transmitted information can be expressed in terms of only $P(x)$ and $P(n|x)$. In turn, $P(n|x)$ is determined by $P(x)$ and the channel characteristics, that is, the number N of threshold devices, the values θ_i of the thresholds and the noise probability density $R(\eta)$. The next subsection describes briefly the method we use to calculate $P(n|x)$ given the channel characteristics. We shall consider three different signal and noise probability distributions: uniform, Gaussian and Rayleigh. Expressions for these probability distributions are given in subsequent subsections.

2.1. Calculating $P(n|x)$

Following the notation of Stocks,[18] let $P_{1|x,i}$ be the probability of device i being "on" (that is, signal plus noise exceeding the threshold θ_i), given the input signal x. Then

$$P_{1|x,i} = \int_{\theta_i - x}^{\infty} R(\eta)d\eta \quad (i = 1, .., N). \tag{4}$$

Given a noise density and threshold value, $P_{1|x,i}$ can be calculated numerically for any value of x from (4). Assuming $P_{1|x,i}$ has been calculated for desired values of x, a convenient way of numerically calculating the probabilities $P(n|x)$ for a given number N of devices is as follows. Let $T^k_{n|x}$ denote the probability that n of the devices ($n = 1, \ldots, k$) are "on", given x. Then $T^1_{0|x} = 1 - P_{1|x,1}$ and $T^1_{1|x} = P_{1|x,1}$ and we have the recursive formulæ

$$\begin{aligned} T^{k+1}_{0|x} &= (1 - P_{1|x,k+1})T^k_{0|x}, \\ T^{k+1}_{n|x} &= P_{1|x,k+1}T^k_{n-1|x} + (1 - P_{1|x,k+1})T^k_{n|x} \quad (n = 1, .., k), \\ T^{k+1}_{k+1|x} &= P_{1|x,k+1}T^k_{k|x}. \end{aligned} \tag{5}$$

We have $P(n|x)$ given by $T^N_{n|x}$. An alternative evaluation is the coefficient of z^n in the power series expansion of

$$\prod_{i=1}^{N}\left[1 - P_{1|x,i} + zP_{1|x,i}\right].$$

In particular, when the thresholds all have the same value, then each $P_{1|x,i}$ has the same value $P_{1|x}$ and we have the binomial distribution

$$P(n|x) = \binom{N}{n}(P_{1|x})^n(1 - P_{1|x})^{N-n} \quad (0 \le n \le N)$$

Thus, for any arbitrary threshold settings and signal and noise probability distributions, $P(n|x)$ can be easily calculated from (4) and (5) and therefore the transmitted information can be calculated by numerical integration of (2). In previous papers, numerical integration of (2) has been verified by digital simulations.[18, 26]

However digital simulation becomes very difficult for large N, so that N was limited to be less than 100.[26] Furthermore, numerical integration was restricted only to the case of the thresholds all set equal to the signal mean. The approach given here has the benefit that $P(n|x)$ can be found even for very large values of N, for any given threshold settings.

The following subsections give expressions for three different probability densities, uniform, Gaussian and Rayleigh, and for each of these gives expressions for $P_{1|x,i}$ derived by direct integration of (4). We have arbitrarily chosen the mean of the uniform and the Gaussian signal to be zero and the Rayleigh distributed signal to be zero for $x < 0$, so that its mean is $\sigma_p\sqrt{\pi/2}$. The results in this paper are independent of the value of the mean, so we have chosen these values for convenience.

2.2. Uniformly distributed signal and noise

If the input signal, x is uniformly distributed between $-\sigma_p/2$ and $\sigma_p/2$, then

$$P(x) = \begin{cases} 1/\sigma_p & \text{for } -\sigma_p/2 \leq x \leq \sigma_p/2, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

If the independent noise η in each device is uniformly distributed between $-\sigma_r/2$ and $\sigma_r/2$, then

$$R(\eta) = \begin{cases} 1/\sigma_r & \text{for } -\sigma_r/2 \leq \eta \leq \sigma_r/2, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Substituting (7) into (4) gives

$$P_{1|x,i} = \begin{cases} 0 & \text{for } x < \theta_i - \sigma_r/2, \\ x/\sigma_r + 1/2 - \theta_i/\sigma_n & \text{for } \theta_i - \sigma_r/2 \leq x \leq \theta_i + \sigma_r/2, \\ 1 & \text{for } x > \theta_i + \sigma_r/2. \end{cases} \tag{8}$$

2.3. Gaussian signal and noise

If the input signal has a Gaussian distribution with zero mean and variance σ_p^2, then

$$P(x) = \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left(-\frac{x^2}{2\sigma_p^2}\right). \tag{9}$$

If the independent noise in each device is Gaussian with zero mean and variance σ_r^2, then

$$R(\eta) = \frac{1}{\sqrt{2\pi\sigma_r^2}} \exp\left(-\frac{\eta^2}{2\sigma_r^2}\right). \tag{10}$$

Substituting (10) into (4) gives

$$P_{1|x,i} = 0.5\text{erfc}\left(\frac{\theta_i - x}{\sqrt{2\sigma_r^2}}\right),$$

where erfc is the complementary error function.[29]

2.4. Rayleigh signal and noise

If the input signal x (≥ 0) has a Rayleigh distribution with mean $\sigma_p\sqrt{\pi/2}$, then

$$P(x) = \frac{x}{\sigma_p^2} \exp\left(-\frac{x^2}{2\sigma_p^2}\right). \tag{11}$$

If the independent noise η (≥ 0) in each device has a Rayleigh distribution with mean $\sigma_r\sqrt{\pi/2}$, then

$$R(\eta) = \frac{\eta}{\sigma_r^2} \exp\left(-\frac{\eta^2}{2\sigma_r^2}\right). \tag{12}$$

Substituting (12) into (4) gives

$$P_{1|x,i} = \begin{cases} \exp\left(-\frac{(\theta_i - x)^2}{2\sigma_r^2}\right) & \text{for } x < \theta_i, \\ 1 & \text{for } x \geq \theta_i. \end{cases}$$

3. COMPARISON OF TRANSMITTED INFORMATION FOR DIFFERENT THRESHOLD SETTINGS AND NOISE DISTRIBUTIONS

3.1. Thresholds distributed optimally for zero noise

For the case where all comparators are noiseless $H(y|x)$ is zero, since the output of the array is completely deterministic given the input. Therefore, from (1), the transmitted information is simply the entropy $H(y)$ of the output signal. Maximizing the output entropy is achieved by ensuring all output states are equally probable, that is, $Q(n) = 1/(N+1)$ for all n.[25] In this case, from (2), the transmitted information is given by $\log_2(N+1)$ bits per sample. Since there is no noise, $P_{1|x,i}$ is zero when $x < \theta_i$ and unity otherwise. Therefore $P(n|x)$ is equal to unity when x is between θ_n and θ_{n+1} and zero otherwise. If the sequence $(\theta_n)_{n=1}^N$ is increasing, then from (3)

$$\int_{\theta_i}^{\theta_{i+1}} P_x(x)dx = \frac{1}{N+1} \quad (i = 0, .., N).$$

In the case of a uniformly distributed signal, as in (6), the thresholds are

$$\theta_i = -\frac{\sigma_p}{2} + \frac{i\sigma_p}{N+1} \quad (i = 1, .., N).$$

In the case of a Gaussian signal distribution, as in (9), the thresholds are

$$\theta_i = \sqrt{2}\sigma_p \mathrm{erf}^{-1}\left(\frac{2i}{N+1} - 1\right) \quad (i = 1, .., N),$$

where erf^{-1} is the inverse of the error function.[29]

In the case of a Rayleigh signal distribution, as in (11), the thresholds are

$$\theta_i = \sigma_p \sqrt{-2\ln\left(1 - \frac{i}{N+1}\right)} \quad (i = 1, .., N).$$

3.2. Thresholds equal to the signal mean

The second configuration of thresholds we shall consider is the case where all thresholds are set equal to the signal mean. This setting will give a transmitted information of exactly 1 bit when there is no noise, since all of the threshold devices will be simultaneously either "on" or "off" and y can be only 0 or N, each value occuring with probability of 0.5. Setting all of the thresholds to values other than the signal mean will give transmitted information of less than 1 for zero noise.

3.3. Results

Figures 2, 3 and 4 show plots of transmitted information against $\sigma = \sigma_r/\sigma_p$ for the case where the signal and noise are identically distributed. Hence, σ is the ratio of the signal standard deviation to the noise standard deviation for all three probability densities. In all figures σ_p is set equal to one. The value of σ_r is varied between 0 and 1.6 and the transmitted information calculated numerically from (2). It is important to note, however, that since we have plotted the transmitted information against the *ratio* of σ_r to σ_p that the results plotted below are valid for any size (characterised by the variance) of the signal.

It can be seen from the figures that when the thresholds are set so that the transmitted information is maximised for zero noise, the transmitted information in the absence of noise ($\sigma = 0$) is indeed given by

$\log_2(N+1)$ bits per sample. As σ increases, the transmitted information decreases monotonically from this value. However for the case of the thresholds all set to the signal mean, the transmitted information is always 1 bit per sample for zero noise. For $N > 1$, as the noise intensity increases from zero, the transmitted information also increases until it reaches a maximum before decreasing again. As N becomes large, the value of σ that gives the maximum transmitted information approaches unity.

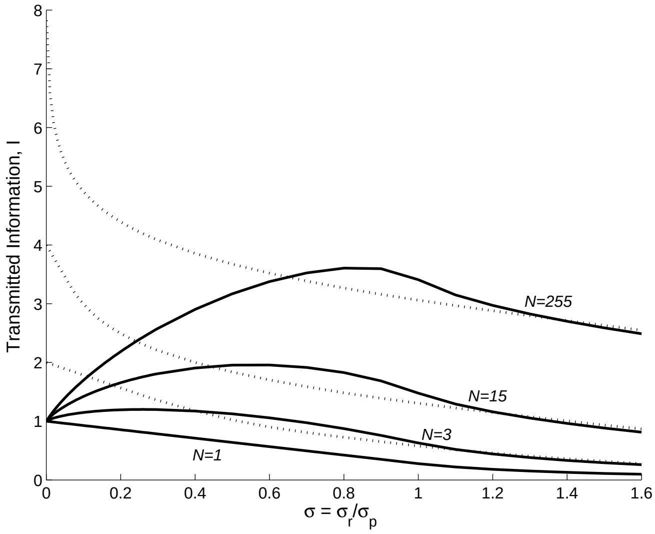

Figure 2. Plot of transmitted information against σ for various values of N and uniformly distributed signal and noise. The solid lines are the case where all thresholds are set to the signal mean. The dotted lines are the case where the thresholds are set to optimise the noiseless transmitted information.

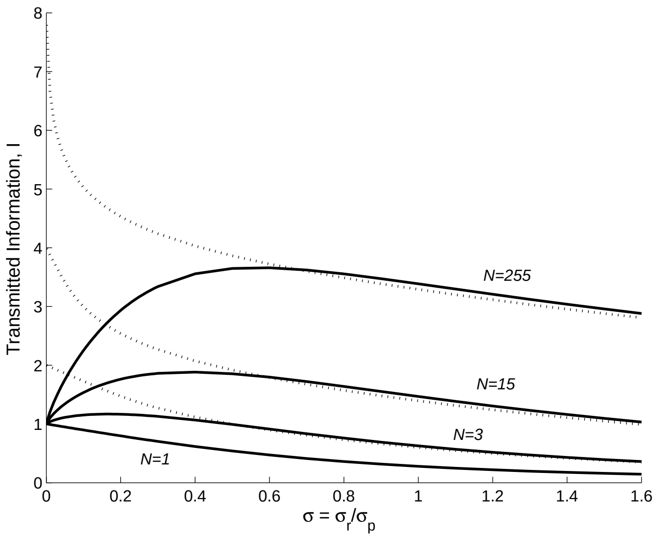

Figure 3. Plot of transmitted information against σ for various values of N and Gaussian signal and noise. The solid lines are the case where all thresholds are set to the signal mean. The dotted lines are the case where the thresholds are set to optimise the noiseless transmitted information.

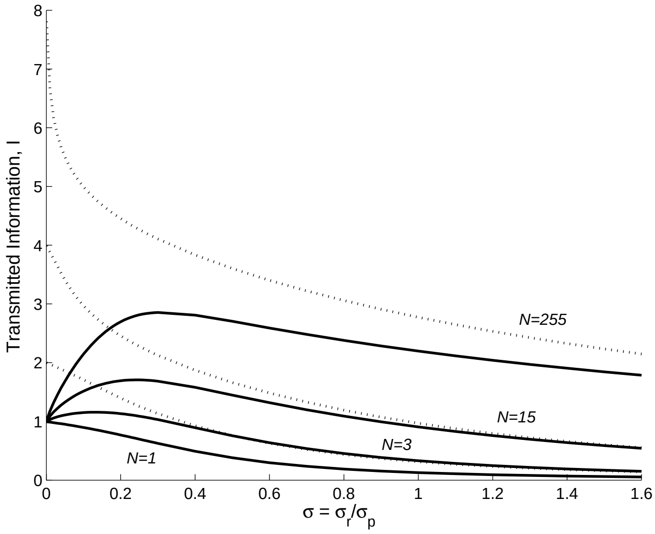

Figure 4. Plot of transmitted information against σ for various values of N and Rayleigh distributed signal and noise. The solid lines are the case where all thresholds are set to the signal mean. The dotted lines are the case where the thresholds are set to optimise the noiseless transmitted information.

These results confirm those reported by Stocks[28] and also extend them to show that a noise–induced maximum occurs in the transmitted information for the Rayleigh distribution. As pointed out by Stocks, since these results are valid for any size of input signal, they contrast with classical stochastic resonance results where the signal has to be subthreshold for stochastic resonance phenomena to occur. Stocks has coined the turn *Supra-Threshold Stochastic Resonance* (SSR) to describe this new result.

3.4. Analysis of Results

For the case of all thresholds set to the signal mean, the reason that only 1 bit is transmitted in the absence of noise is that only the two output states 0 and N are possible. Hence, only 1 bit of information can be transmitted, as this is equivalent to a binary signal. As noise becomes non-zero, the other output states become accessible and hence more than 1 bit of information can be transmitted per sample. This is true for $N > 1$. For $N = 1$, at most 1 bit per sample can be transmitted. This illustrates how using more than one device in parallel can give improvements in signal transfer.

The increase in the transmitted information for non-zero noise can be explained as follows. A probabilistic input signal has a high information content. In the absence of noise, the transmitted information is limited to 1 bit (for all thresholds equal to the signal mean) and much information is lost due to the nature of the channel. As non-zero noise is added independently in each device, all output states become accessible and hence $H(y)$ increases towards a maximum before decreasing again. However $H(y|x)$ increases monotonically with increasing noise. Therefore the transmitted information also contains a maximum.[26] As the number N of threshold devices is increased, the transmitted information also increases, since the number of output states available increases.

In the case where the thresholds are distributed optimally for zero noise, this setting is not optimal for many non-zero values of noise. It is clear from the graphs that for moderate values of σ the transmitted information in the case of all thresholds set to the signal mean can increase above that for the thresholds set optimally for zero noise, or at least is approximately the same. One problem of interest is that of finding the threshold values that maximise the transmitted information for a given number of thresholds and non-zero values of σ. It is anticipated that the recursive formula given by (5) will be of benefit in future work on algorithms that calculate or approximate such maximal threshold values.

Transmitted information has units of bits per sample. This is an average quantity: the average amount of information that can be transmitted through the channel about a single sample of the input distribution. In the configuration we treat we have not considered the bandwidth of the signal and noise and hence have not shown anything regarding the bit rate through the array. A derivation of a formula for the channel capacity of a single threshold device subject to bandlimited Gaussian noise has recently been published.[24] Future work will attempt to extend this formula to the case of N parallel threshold devices, as considered here.

4. VARIANCE ANALYSIS

The fact that the transmitted information increases with increasing noise for all thresholds set to the signal mean can be seen by considering an output time series of the array when the input is a uniform random signal and the noise is uniform. If the output signal y is normalised so that it takes on values between $-\sigma_p/2$ and $\sigma_p/2$, it becomes a digital approximation to the input signal. We will call this normalised signal

$$\hat{y} = \sigma_p \left(\frac{y}{N} - \frac{1}{2} \right).$$

Let $\varepsilon = \hat{y} - x$. The variance of ε is an indicator of the quantization error incurred by the channel when $\hat{y}$ is taken as an estimate for x. We can derive the variance of ε theoretically for the case of uniform signal and noise. Assuming that $\sigma_r < \sigma_p$, it is straightforward to show that $\mathrm{E}[\hat{y}] = 0$, and that therefore from the definition of variance, $\mathrm{var}[\varepsilon] = \mathrm{E}[\hat{y}^2] + \mathrm{var}[x] - 2\mathrm{E}[x\hat{y}]$. The variance of the signal x is given by $\sigma_p^2/12$. Hence to find the variance of the quantization error, ε, we need only find $\mathrm{E}[\hat{y}^2]$ and $\mathrm{E}[x\hat{y}]$.

First, to derive a formula for $\mathrm{E}[\hat{y}^2]$, note that the possible values of $\hat{y}$ are $\sigma_p(-1/2 + i/N)$ $(i = 0 \ldots N)$. We make use of the theoretical derivation of $Q(n)$ for $\sigma < 1$ given by Stocks[26]:

$$Q(n) = \begin{cases} \frac{\sigma}{N+1} & (n = 1, .., N-1), \\ \frac{\sigma}{N+1} + \frac{1-\sigma}{2} & (n = 0, N). \end{cases}$$

Hence

$$\begin{aligned} \mathrm{E}[\hat{y}^2] &= \sum_{i=0}^{N} \hat{y}_i^2 Q(i) \\ &= \sigma_p^2 \sum_{i=0}^{N} \left(-\frac{1}{2} + \frac{i}{N} \right)^2 Q(i) \\ &= \frac{\sigma_p^2 \sigma}{N+1} \sum_{i=0}^{N} \left(-\frac{1}{2} + \frac{i}{N} \right)^2 + \sigma_p^2 \left(\frac{1-\sigma}{4} \right) \\ &= \sigma_p^2 \left(\frac{1}{4} + \frac{\sigma(1-N)}{6N} \right). \end{aligned} \quad (13)$$

To find $\mathrm{E}[x\hat{y}]$, we first derive a formula for $\mathrm{E}[\hat{y}|x]$. To do this, we write $\hat{y}$ in terms of the sign function as

$$\hat{y} = \frac{\sigma_p}{2N} \sum_{i=1}^{N} \mathrm{sign}(x + \eta_i).$$

Hence

$$\begin{aligned} \mathrm{E}[\hat{y}|x] &= \frac{\sigma_p}{2N} \mathrm{E}\left[\sum_{i=1}^{N} \mathrm{sign}(x + \eta_i) \right] \\ &= \frac{\sigma_p}{2} \mathrm{E}[\mathrm{sign}(x + \eta_i)] \\ &= \frac{\sigma_p}{2} [-1(1 - P_{1|x}) + 1 P_{1|x}] \\ &= \frac{\sigma_p}{2} (2P_{1|x} - 1). \end{aligned}$$

Accordingly

$$\begin{aligned}\mathrm{E}[x\hat{y}] &= \int_{-\infty}^{\infty} xP(x)\mathrm{E}[\hat{y}|x]dx \\ &= \frac{1}{\sigma_p}\int_{-\sigma_p/2}^{\sigma_p/2} x\mathrm{E}[\hat{y}|x]dx \\ &= \frac{1}{\sigma_p}\int_{-\sigma_p/2}^{\sigma_p/2} x\left(\frac{\sigma_p}{2}(2P_{1|x}-1)\right)dx \\ &= -\frac{\sigma_r^2}{24}+\frac{\sigma_p^2}{8}. \end{aligned} \quad (14)$$

Equation (14) is obtained by substitution of (8) (with all $\theta_i = 0$) into the previous line and integrating. Thus from (13) and (14) we get:

$$\begin{aligned}\mathrm{Var}[\varepsilon] &= \mathrm{E}[\hat{y}^2] - 2\mathrm{E}[x\hat{y}] + \sigma_p^2/12 \\ &= \sigma_p^2\left(\frac{\sigma(1-N)}{6N}+\frac{\sigma^2+1}{12}\right). \end{aligned} \quad (15)$$

As N becomes large, the variance of $\hat{y}-x$ approaches the variance

$$\lim_{N\to\infty}\mathrm{Var}[\varepsilon] = \frac{(\sigma_r-\sigma_p)^2}{12} \quad (16)$$

of a uniform distribution. Note from (15) that the variance of the error, for a given non-zero value of σ_p, is a quadratic function of σ and has a minimum of $\sigma_p^2(2N-1)/(12N^2)$ at $\sigma = (N-1)/N$. Hence, the variance of the error is minimized for a non-zero value of σ, and is independent of the size of the signal variance. We saw earlier that as $N \to \infty$, the value of σ that gave the maxima in the transmitted information approaches one. This is also the case here, where from (16) the variance approaches 0 as $\sigma \to 1$.

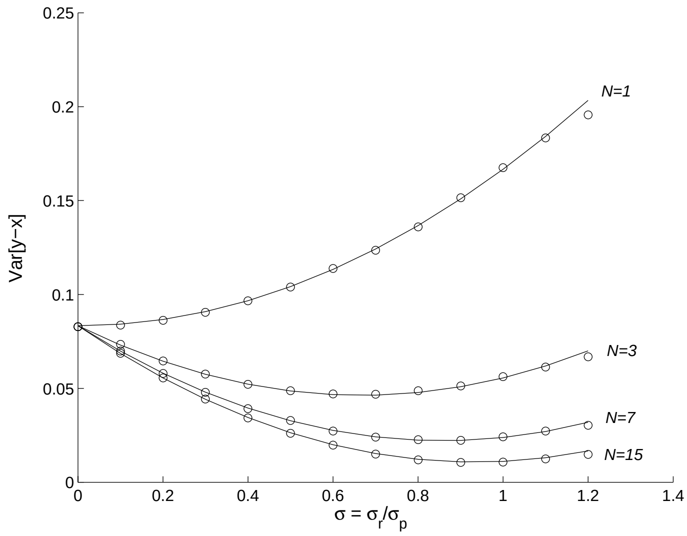

Figure 5. Plot of the variance of $\hat{y}-x$ against σ for various values of N and uniformly distributed signal and noise, with $\sigma_p = 1$. The solid line plots are the exact value of the variance from (15) and the circles are from digital simulation. Note that when $\sigma > 1$, the simulated point start to diverge from the theoretical line, as the theory was derived only for $\sigma < 1$. The minimum value of the variance occurs for non-zero σ for $N > 1$ and approaches 1 as N becomes large.

Interestingly, the value of σ that minimises $\hat{y} - x$ is not the same as that which maximises the transmitted information. However the minimum of the variance of ε has the same qualitative behaviour as the maximum in the transmitted information. Stocks calculated an approximation for the value of σ which maximises I for large N and $\sigma < 1$.[26] This is given by $\sqrt{N+1}/(\sqrt{N+1}+3.297)$ which clearly is not equal to $(N-1)/N$, although both approach unity as N becomes large. Plots of the variance of ε for various N are shown in Figure 5. The theoretical calculation was verified by digital simulation.

It has been pointed out that for threshold devices, stochastic resonance is related to, or indeed equivalent to dithering in ADC's.[30,31] In further work, we intend to investigate the relationship of the results presented here to dithering.

5. CONCLUSIONS AND OPEN QUESTIONS

In this paper we have examined issues relating to the problem of optimising the transmitted information through a summing array of N comparators, where each comparator is subject to additive noise. We have verified the results of Stocks which show that for uniform or Gaussian signal and noise there is a maximum in the transmitted information at a non-zero value of noise when all thresholds are set to the signal mean. This phenomena is known as Supra-threshold Stochastic Resonance. We have also shown that SSR occurs when the signal and noise have a Rayleigh distribution and derived a convenient method of calculating the transmitted information for any value of N, and any given set of threshold values and noise and signal probability densities. Furthermore, we showed that the variance of the difference between the input and output signals is minimised for non-zero values of noise when the signal and noise are uniformly distributed. The direction of future work is aimed towards finding the optimal threshold settings for a given noise variance, and for finding the channel capacity for such an array.

REFERENCES

1. N. Stocks and R. Manella, "Generic noise-enhanced coding in neuronal arrays," *Physical Review E* **64**, pp. 030902(R),1–4, 2001.
2. W. Bialek, M. DeWeese, F. Rieke, and D. Warland, "Bits and brains: Information flow in the nervous system," *Physica A* **200**, pp. 581–593, 1993.
3. V. Madisetti and D. Williams, *The Digital Signal Processing Handbook*, CRC Press, 1999.
4. V. Anderson, "Digital array phasing," *The Journal of the Acoustical Society of America* **32**, pp. 867–870, 1960.
5. P. Rudnick, "Small signal detection in the DIMUS array," *The Journal of the Acoustical Society of America* **32**, pp. 871–877, 1960.
6. L. Gammaitoni, P. Hanggi, and P. Jung, "Stochastic resonance," *Reviews of Modern Physics* **70**, pp. 223–287, 1998.
7. D. Luchinsky, R. Mannella, P. McClintock, and N. Stocks, "Stochastic resonance in electrical circuits - I: Conventional stochastic resonance," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal rocessing* **46**, pp. 1205–1214, 1999.
8. D. Luchinsky, R. Mannella, P. McClintock, and N. Stocks, "Stochastic resonance in electrical circuits - II: Nonconventional stochastic resonance," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing* **46**, pp. 1215–1224, 1999.
9. R. Benzi, A. Sutera, and A. Vulpiani, "The mechanism of stochastic resonance," *Journal of Physics A: Mathematical and General* **14**, pp. L453–L457, 1981.
10. V. Melnikov, "Schmitt trigger: a solvable model of stochastic resonance," *Physical Review E* **48**, pp. 2481–2489, 1993.
11. B. McNamara, K. Wiesenfeld, and R. Roy, "Observation of stochastic resonance in a ring laser," *Physical Review Letters* **60**, pp. 2626–2629, 1988.
12. K. Wiesenfeld and F. Moss, "Stochastic resonance and the benefits of noise: from ice ages to crayfish and squids," *Nature* **373**, pp. 33–36, 1995.

13. J. Douglas, L. Wilkens, E. Pantazelou, and F. Moss, "Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance," *Nature* **365**, pp. 337–339, 1993.
14. I. Goychuk, "Information transfer with rate-modulated poisson processes: A simple model for nonstationary stochastic resonance," *Physical Review E* **64**, pp. 021909,1–6, 2001.
15. C. Heneghan, C. Chow, J. Collins, T. Imhoff, S. Lowen, and M. Teich, "Information measures quantifying aperiodic stochastic resonance," *Physical Review E* **54**, pp. R2228–R2231, 1996.
16. J. Collins, C. Chow, and T. Imhoff, "Aperiodic stochastic resonance in excitable systems," *Physical Review E* **52**, pp. R3321–R3324, 1995.
17. J. Collins, C. Chow, A. Capela, and T. Imhoff, "Aperiodic stochastic resonance," *Physical Review E* **54**, pp. 5575–5583, 1996.
18. N. Stocks, "Suprathreshold stochastic resonance in mutlilevel threshold systems," *Physical Review Letters* **84**, pp. 2310–2313, 2000.
19. P. Greenwood, L. Ward, and W. Wefelmeyer, "Statistical analysis of stochastic resonance in a simple setting," *Physical Review E* **60**, pp. 4687–4695, 1999.
20. A. Neiman, D. Shulgin, V. Anishchenko, W. Ebeling, L. Schimansky-Geier, and J. Freund, "Dynamical entropies applied to stochastic resonance," *Physical Review Letters* **76**, pp. 4299–4302, 1996.
21. J. Robinson, J. Rung, A. Bulsara, and M. Inchiosa, "General measures for signal-noise separation in nonlinear dynamical systems," *Physical Review E* **63**, pp. 011107,1–11, 2000.
22. M. Inchiosa, J. Robinson, and A. Bulsara, "Information-theoretic stochastic resonance in noise-floor limited systems: The case for adding noise," *Physical Review Letters* **85**, pp. 3369–3372, 2000.
23. F. Chapeau-Blondeau, "Noise-enhanced capacity via stochastic resonance in an asymmetric binary channel," *Physical Review E* **55**, pp. 2016–2019, 1997.
24. L. Kish, G. Harmer, and D. Abbott, "Information transfer rate of neurons: Stochastic resonance of shannon's information channel capacity," *Fluctuation and Noise Letters* **1**, pp. L13–L19, 2001.
25. T. Cover and J. Thomas, *Elements of Information Theory*, John Wiley and Sons, 1991.
26. N. Stocks, "Information transmission in parallel threshold arrays: Suprathreshold stochastic resonance," *Physical Review E* **63**, pp. 041114,1–9, 2001.
27. A. Feinstein, *Foundations of Information Theory*, McGraw-Hill, 1958.
28. N. Stocks, "Suprathreshold stochastic resonance: An exact results for uniformly distributed signal and noise," *Physics Letters A* **279**, pp. 308–312, 2001.
29. M. Spiegel and J. Liu, *Mathematical Handbook of Formulas and Tables*, McGraw-Hill, 1999.
30. L. Gammaitoni, "Stochastic resonance and the dithering effect in threshold physical systems," *Physical Review E* **52**, pp. 4691–4698, 1995.
31. R. Wannamaker, S. Lipshitz, and J. Vanderkooy, "Stochastic resonance as dithering," *Physical Review E* **61**, pp. 233–236, 2000.