

Optical MTF and Quantum Efficiency Analysis for Smart Sensors

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ABSTRACT

In this paper we critique the first order quantum efficiency and modulation transfer function (MTF) analysis for photocollection in both a finite and semi-infinite slab of semiconducting material. We derive the correct equations from first principles and perform a case study comparing photodetection in silicon and GaAs. This is of interest in the first order design of GaAs and silicon optical smart sensors and solid-state imagers.

Keywords: Internal quantum efficiency, IQE, modulation transfer function, MTF, optical image sensors, optical smart sensors, photodetection.

1. INTRODUCTION

An analysis that compares the optical absorption coefficients of GaAs and silicon, shows that GaAs is optically the superior material for use in photodetection and imaging. The governing equation for the penetration of light into the material is given by Lambert's law of absorption (also known as Bouguer's or Beer's law),

$$\Phi(x) = \Phi_0 e^{-\alpha x}$$

– where α is called the absorption coefficient. Comparison of the absorption coefficients of GaAs and silicon against wavelength (λ), clearly highlights the superiority of GaAs.

Absorption Coefficients		
$\lambda(\mu\text{m})$	GaAs $\alpha^{-1}(\mu\text{m})$	Si $\alpha^{-1}(\mu\text{m})$
0.4	0.01	0.1
0.6	0.20	2.0
0.8	0.82	12.0

Table 1. Absorption coefficients

The absorption length or penetration is given by α^{-1} . So for instance, the percentage of light absorbed within two absorption lengths, ie. $2\alpha^{-1}$, is $100(1 - e^{-2}) = 86\%$. Lambert's law is graphically illustrated in Fig. 1 for GaAs.

To understand what this means, from Table 1 we deduce that, at $\lambda = 0.6 \mu\text{m}$, 86% is absorbed within $0.4 \mu\text{m}$ for GaAs & within $4 \mu\text{m}$ for Si. The implication is that GaAs is a more efficient photocollector as most of the light is detected near the surface, where transistor action collects the photocharge. Improved spatial resolution is expected, as a result, and fewer wasted carriers imply improved quantum efficiency and hence greater responsivity.

The absorption length dependence on wavelength, in GaAs, is displayed in Fig. 2. The positions of the cut-off wavelengths in GaAs and Si, are indicated showing that GaAs is closer to the peak response of the human eye and is better placed with respect to the visible spectrum. Indeed, at $5.5 \mu\text{m}$, the peak eye response, the absorption length for GaAs is precisely in the channel region – whereas for silicon, absorption at this wavelength, occurs well into the substrate where many carriers recombine and are thus not collected.

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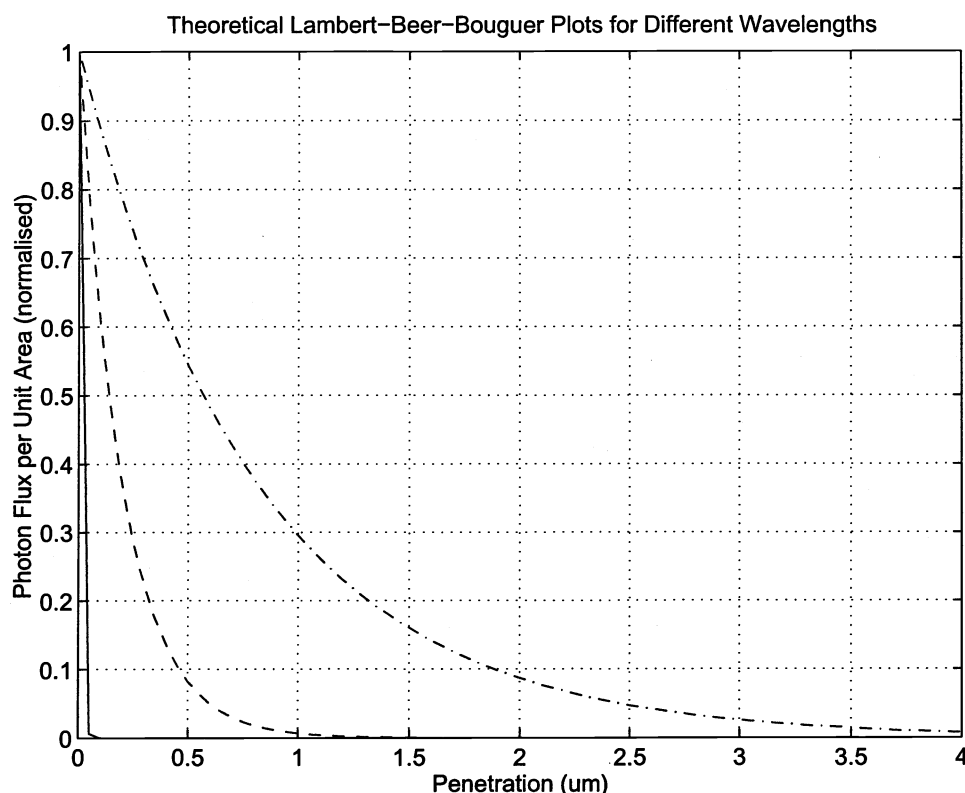


Figure 1. Lambert's Law of Absorption for GaAs. Solid line: $\lambda = 0.4 \mu\text{m}$. Dashed line: $\lambda = 0.6 \mu\text{m}$. Chained line: $\lambda = 0.8 \mu\text{m}$.

The relative positions of GaAlAs and red He-Ne lasers are indicated on the curve showing that GaAlAs emitters are suited for investigations at channel-substrate interface depths, whereas red He-Ne lends itself for observing substrate effects. Fig. 2, clearly shows that neither GaAs nor silicon can intrinsically achieve sensitivity in the standard 3-5 μm MWIR or 8-12 μm LWIR infrared bands.

However, a GaAs imager can be extended to operate in the Near IR waveband, as indicated in Fig. 2. The introduction of states in the forbidden gap, introduced intentionally by impurities such as chromium, provide 'stepping stones' for electrons to traverse the bandgap via excitation from these lower energy wavelengths. This technique is well known in silicon infrared imagers, where the impurity used is typically indium. Such imagers are called *extrinsic* detectors.

As indicated, an alternative scheme to achieve Near IR sensitivity, is to utilize the *internal photoemission* effect. Internal photoemission occurs when the incident light frees electrons from the surface of the gate metal, which are then collected if they have enough energy to jump the gate Schottky barrier. Now, the Schottky barrier height for a MESFET is about 0.8 eV, whereas the GaAs and Si bandgaps are 1.4 eV and 1.1 eV, respectively – therefore less energy is required for an electron to jump the Schottky barrier than to jump a bandgap. Thus internal photoemission provides sensitivity to longer wavelengths of lower energy, namely in the Near IR band. Commercial Si imagers, using internal photoemission, are available on the market – translating this scheme over to GaAs would be an interesting open question.

Fig. 3 highlights the superiority of GaAs over Si, for visible band detection, showing clearly that in GaAs the entire visible spectrum is absorbed close to where transistor action occurs, whereas for silicon the longer visible wavelengths are absorbed well into the substrate. Electrons that are generated by this absorption, in the silicon substrate, either

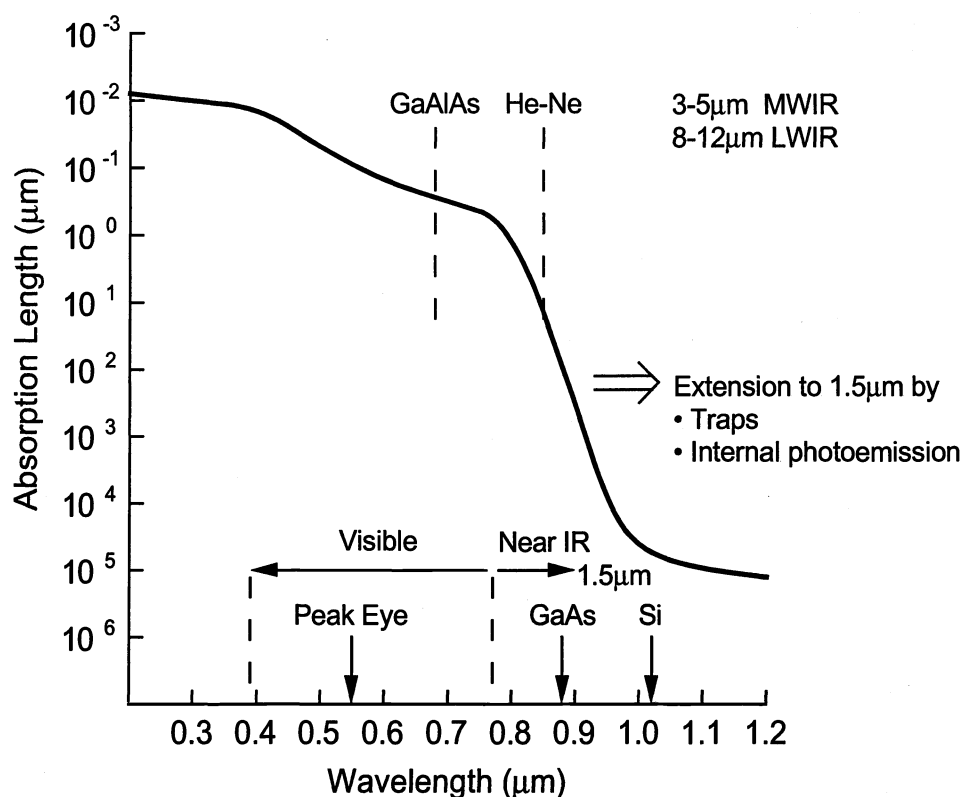


Figure 2. GaAs Absorption Length Dependence on Wavelength

recombine (and are therefore wasted) or diffuse to the channel region causing an unfortunate degradation in spatial resolution.

Note that size and position of the channel/substrate depletion region, in Fig. 3, is for a GaAs MESFET. For a typical silicon MOSFET, the depletion region will be a little smaller – this further highlights the superiority of the GaAs MESFET for photodetection.

2. QUANTUM EFFICIENCY FORMULAE

Before we can compare the quantum efficiencies of GaAs and silicon imagers we must first carefully discuss a few esoteric subtleties.

Firstly, it should be noted that minority carriers in silicon substrates have diffusion lengths much larger than the pixel size. Hence, in order to reduce pixel crosstalk, silicon imagers are all exclusively fabricated on epitaxial substrates. The substrate under the epi-layer is heavily doped so that stray carriers quickly recombine and do not find their way to neighboring pixels. Unfortunately, the quantum efficiency is thereby reduced, as a large number of carriers are forced to recombine in this way.

Due to the unusual hi-lo junction* (n+/SI) situation present in the GaAs imager, it is the *majority* carriers that are being collected – hence a bulk substrate is appropriate for a GaAs imager. Therefore, for our GaAs case, it is appropriate to use the formula for quantum efficiency that approximates the bulk substrate to a semi-infinite slab giving (see Appendix 5).

*This assumes devices with no p-buffer layer and that the SI substrate is slightly n-type and hence we have a hi-lo junction situation.

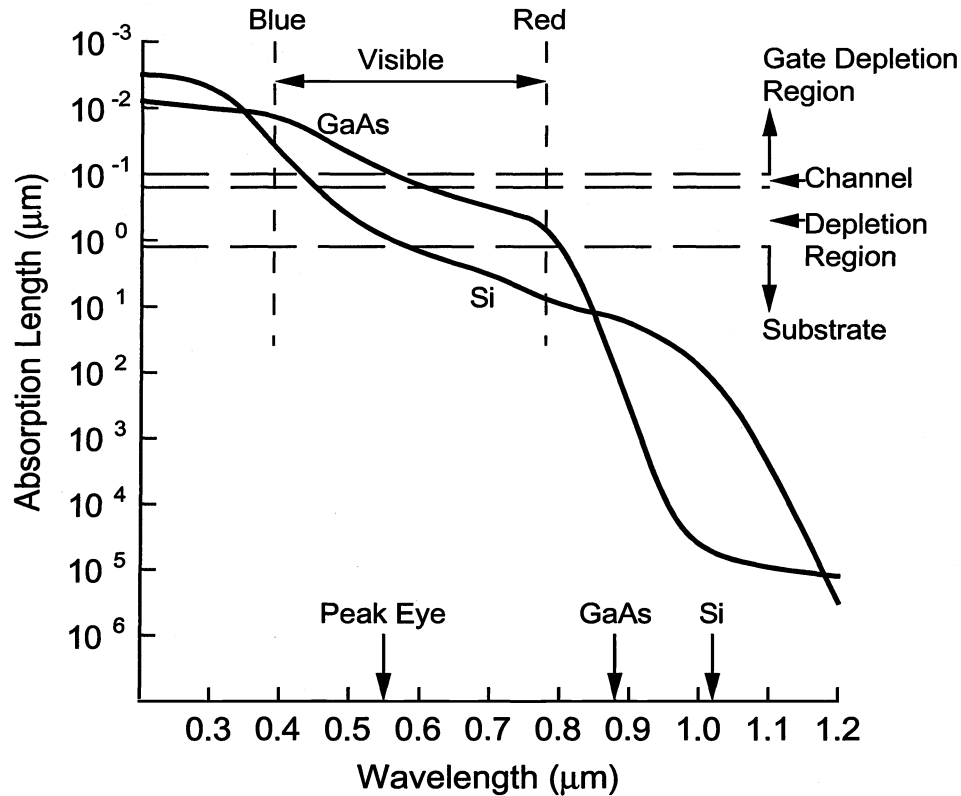


Figure 3. GaAs v. Si Absorption Length Comparison

$$\eta_{\text{bulk}} = T(\lambda) \left(1 - \frac{e^{-\alpha W}}{1 + \alpha L_o} \right). \quad (1)$$

By contrast, in a typical silicon imager it is the *minority* carriers that are collected and there is a quantum efficiency degradation due to minority carriers recombining before they are collected. As this situation does not occur in our GaAs scenario, we may simulate this by setting the minority carrier diffusion length $L_o = 0$ in Eqn. 1. [†]

For the epitaxial silicon case, the semi-infinite slab assumption of Eqn. 1 is no longer valid as the epi-layer is thin and of the order of $h = 15\text{--}25 \mu\text{m}$. In Appendix 5, we show that if we consider a finite slab, the appropriate quantum efficiency formula for epi is

$$\eta_{\text{epi}} = T(\lambda) \left(1 - \frac{e^{-\alpha W}}{1 + \alpha L_o} - \frac{\alpha L_o e^{-\alpha W}}{\alpha^2 L_o^2 - 1} \frac{e^{-h/L_o} - e^{-h\alpha}}{\sinh(h/L_o)} \right). \quad (2)$$

If we let

$$\eta_+ = T(\lambda) \frac{\alpha L_o e^{-\alpha W}}{\alpha^2 L_o^2 - 1} \frac{e^{-h/L_o} - e^{-h\alpha}}{\sinh(h/L_o)}$$

[†]This equation should not be confused with the expressions for quantum efficiency which consider collection of holes by the gate, producing a gate photocurrent. We are considering the more general case of electron collection producing channel photocurrent.

we see the remarkable result that $\eta_{\text{epi}} = \eta_{\text{bulk}} - \eta_+$, where η_+ is the quantum efficiency of the highly doped substrate beneath the epitaxial layer. Notice that this formula assumes that recombination in the highly-doped substrate is instantaneous – this approximation is accurate for most practical cases. The ability to separate η_{epi} into two clearly identifiable terms is significant, as the modulation transfer function (MTF) can then also be separated and the two terms can be analyzed separately in a physically meaningful way.

Both Eqn. 2 and the realization that it can be separated into two physically identifiable terms has not been reported in the open literature. The quantum efficiency formulae are generally poorly presented and ill-conceived in the literature. For instance, a recent attempt at a formula¹ for η_{epi} is clearly dimensionally incorrect, does not have separable terms and under some conditions produces values greater than unity! Note that our expression (Eqn. 2) has all the expected features. Each term is dimensionless – ie. each argument is a ratio of two lengths or an absorption coefficient times a length. This is an important feature which is missing in erroneous attempts found in the literature. Also for $h \rightarrow \infty$ our expression reduces to the semi-infinite case. Our function is also well-behaved in that quantum efficiency goes down for increasing absorption length, as expected, and goes up for increasing diffusion length. Another important feature is that, by inspection, Eqn. 2 always stays below unity.

There is also some confusion in the literature over Eqn. 1. Our equation agrees with the derivation in the seminal work of Seib² and with a fair body of literature. However, there is an erroneous formula due to Barbe³ that appears to have insidiously propagated, unchallenged, into some of the literature, such as in as in McCaughan.⁴ Barbe's expression is in terms of MTF and therefore we plot the MTF expressions for both Barbe and Seib in Fig. 4, showing that Barbe's expression is clearly too optimistic.

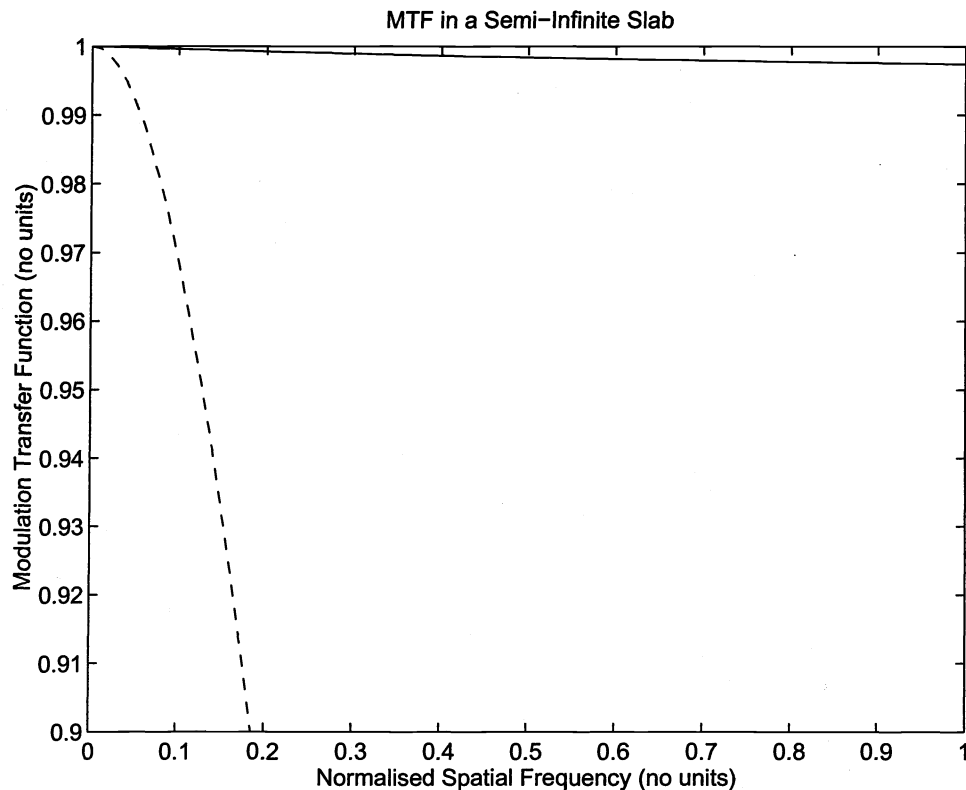


Figure 4. MTF due to diffusion. A comparison of the traditional formula due to Seib (dashed line) and the formula due to Barbe (solid line).

3. QUANTUM EFFICIENCY & RESPONSIVITY

Using Eqns. 1 and 2 we can now analyze the quantum efficiency and hence the responsivity in both the silicon and GaAs devices – these formulae were used to plot Fig. 8.

At the red end of the spectrum, ie. 8 μm , we find that the GaAs device has about twice the quantum efficiency of a comparable silicon device. Initially, this advantage will be lost due to the opaque gates in GaAs, but the potential is there for future investigation into transparent ITO gates.

Responsivity is related to the quantum efficiency by,

$$R_\lambda = \frac{q\lambda}{hc} \eta \quad \text{A/W}.$$

For all wavelengths, the total is,

$$R = \frac{\int_0^\infty R_\lambda W_\lambda d\lambda}{\int_0^\infty W_\lambda d\lambda}$$

where, by Planck, radiated power,

$$W_\lambda = \frac{2\pi c^2 h}{\lambda^5 (e^{hc/\lambda kT} - 1)} \quad \text{W/m}^3$$

where this curve is illustrated in Fig. 5.

For reference, a silicon MOS CCD, has typically, $R \approx 50 \text{ mA/W}$ (3000 K tungsten source). We expect to exceed this with a GaAs imager and values are predicted as follows.

Figs. 6 & 7 show measured absorption lengths for silicon and gallium arsenide. From these curves the predicted quantum efficiencies are plotted in Fig. 8.

The superior quantum efficiency of GaAs over silicon, as seen in Fig. 8, is due to the shallower absorption lengths and majority carrier collection process in the GaAs imager. In silicon imagers, minority carriers are collected and so there is a degradation in quantum efficiency due to recombination processes. For the purpose of comparison, we have assumed a transmission coefficient of unity – this simplification does not make a difference to our conclusions. In Fig. 8, a depletion region width of $W = 0.1 \mu\text{m}$ corresponds to that under the gate. The improved quantum efficiency curves with $W = 1\text{--}2 \mu\text{m}$, simulate the expected effect of the substrate/channel depletion region working together with the gate depletion region. As the imager is a low-frequency device, we expect the substrate/channel depletion region to take part in the collection process, leading to an excellent quantum efficiency.

From the quantum efficiency curves we can now apply Planck's law to calculate the responsivities. Fig. 9 & 10 show the weighted responsivities for silicon and gallium arsenide. Total responsivity, over the visible spectrum, is determined by evaluating the areas under the curves in Figs. 9 & 10 and then dividing by the total area under the Planck's Law curve = $46 \times 10^5 \text{ W/m}^2$ (see Fig. 5). A typical tungsten light source can be simulated by inserting a temperature of 3000 K into the Planck equations and the resulting responsivities are displayed in Table 2.

The results for silicon, are a little larger compared to, say, 50 mA/W for a typical silicon XY array imager with $W = 5 \mu\text{m}$. This is because we have omitted the effect of the transmission coefficient and furthermore we have assumed a fixed depletion width. In reality the depletion width diminishes as photocharge collects – however the comparison between GaAs and silicon is still valid if we make the simplification of a fixed depletion width. Table 2 shows that for GaAs operating with just the gate depletion region, $W = 0.1 \mu\text{m}$, the responsivity is inferior. However, if we include the channel/substrate region, with $W = 2 \mu\text{m}$, the responsivity in GaAs is about 20% larger than in silicon. Furthermore, if we take into account that the channel/substrate region has a fixed potential across it, in contrast to the silicon XY array where the depletion region shrinks during photocollection, we can expect that GaAs would have up to a factor of 5 improvement in responsivity. This means that a 'fingered gate' pixel design, may still equal or even out-perform silicon, despite the presence of opaque gates.

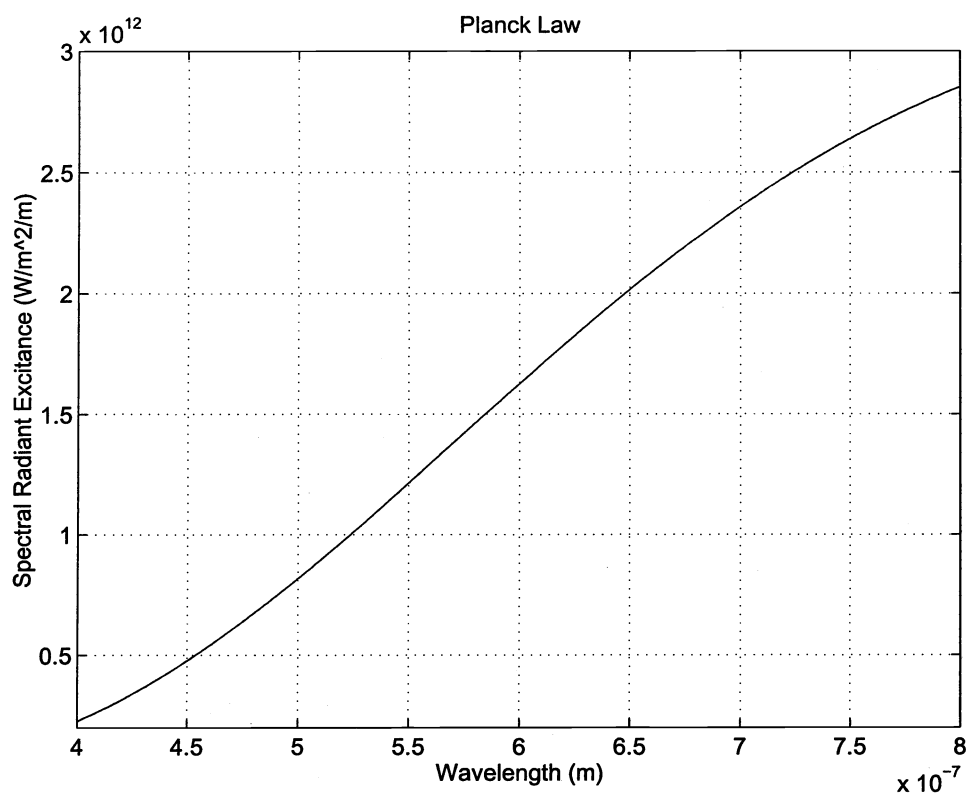


Figure 5. Planck's law curve, W_λ . Area under curve for visible wavelength range is $5.31 \times 10^5 \text{ W/m}^2$. Total area under the curve for all wavelengths, given by Stefan's Law, is $46 \times 10^5 \text{ W/m}^2$.

Type	Depletion W (μm)	Area Under Curve (W/m^2)	Responsivity (mA/W)
Si	0.5	2.37×10^5	51
Si	2.0	2.95×10^5	56
Si	5.0	2.81×10^5	61
GaAs	0.1	1.00×10^5	22
GaAs	1.0	2.95×10^5	64
GaAs	2.0	3.25×10^5	71

Table 2. GaAs v. Si Responsivity.

4. MTF ANALYSIS – SPATIAL DEGRADATION BY DIFFUSION

According to the Lambertian exponential law for the absorption of light, we see that 63% is absorbed within one absorption length. We have seen that the absorption lengths, for visible light in GaAs, are all of comparable order to the vertical transistor dimensions. The implication of photocharge being collected at these shallow depths is that the GaAs MESFET is an efficient photocollector. Furthermore, if most of the carriers are efficiently collected then the number of stray carriers spreading, thereby causing spatial degradation of the image, is low. Hence we expect the spatial resolution of a GaAs imager to be excellent. The channel/substrate depletion region is expected to assist in this regard, by deflecting minority carriers into the substrate and sweeping stray electrons into the collecting channel.

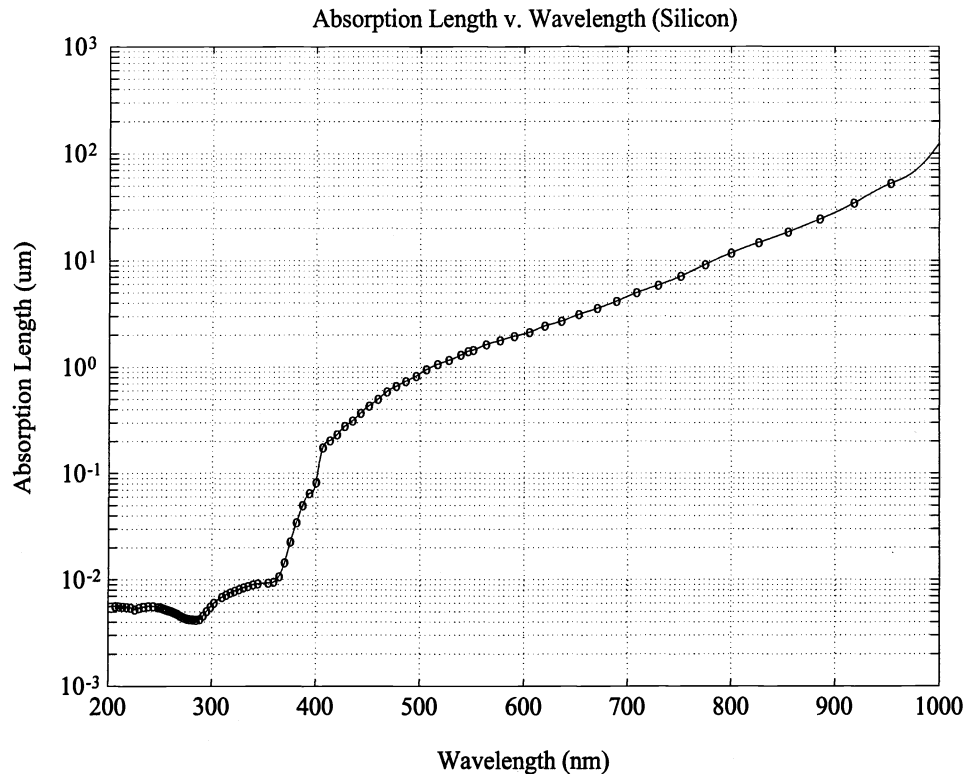


Figure 6. Measured absorption length against wavelength for silicon.

The quantitative comparison of GaAs and silicon, at first sight, appears to be difficult due to the fact that our GaAs example is on a bulk substrate, whereas silicon imagers are on epitaxial substrates. Hence we are not comparing like with like. A thin epi-layer gives rise to a poor quantum efficiency but an excellent spatial resolution, whereas a bulk substrate gives an improved quantum efficiency at the expense of spatial resolution. So how can we make a fair comparison that takes into account this trade-off? The answer is to compare GaAs with silicon, not in terms quantum efficiency or raw spatial resolution *per se*, but in terms of a special figure of merit known as the modulation transfer function (MTF). An ideal MTF has the value of unity and it has the behavior that if either quantum efficiency or spatial resolution decreases, then MTF also decreases below unity. The definition of MTF is simply²

$$\text{MTF} = \frac{\eta_k}{\eta}$$

where η_k is given by the same formula as the quantum efficiency, η , but with each L_o term substituted by $\sqrt{\frac{1}{L_o^2} + (2\pi k)^2}$ and k is the spatial frequency.

This is plotted in Fig. 11, where the importance of epi rather than bulk substrates for silicon is demonstrated. Although the diffusion curve for a wavelength of 550 nm in bulk is acceptable, the MTF for 800 nm is severely degraded in bulk compared to epi. The geometrical-only curve is obtained from the usual sinc function expression and represents the ideal case in absence of diffusion effects. Due to the hi-lo n^+ /SI junction majority carriers are collected, hence the diffusion term vanishes and our GaAs case corresponds to the ideal geometrical curve. This result is attractive for HDTV detector array applications, where high pixel densities are more susceptible to crosstalk by diffusion.

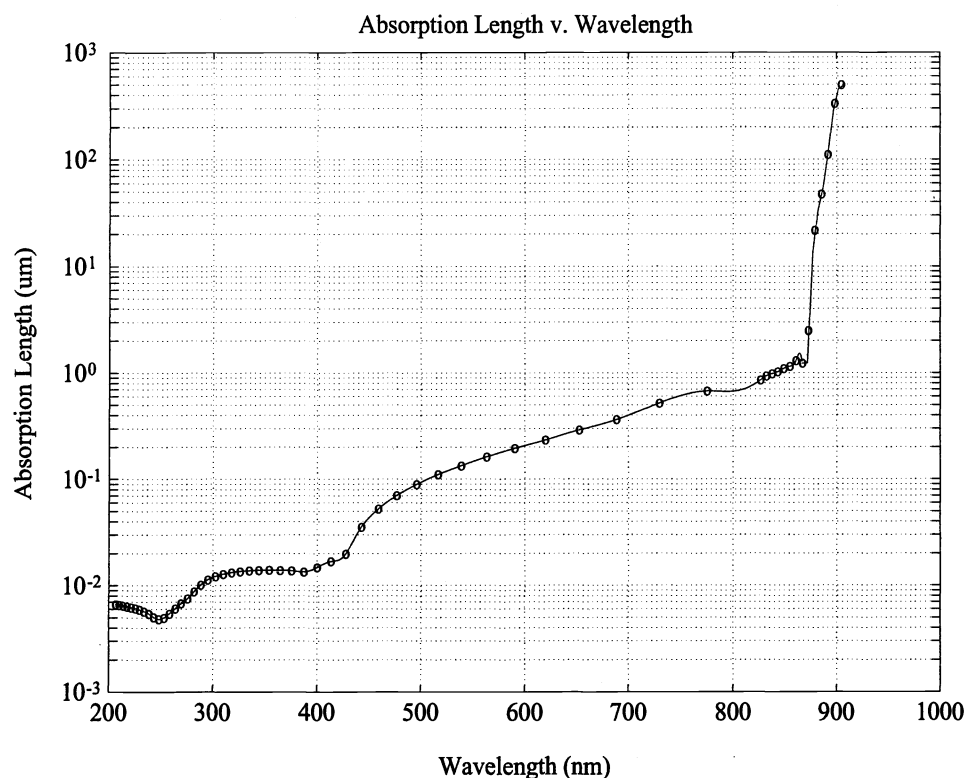


Figure 7. Measured absorption length against wavelength for GaAs.

5. CONCLUSIONS

In the first order analysis of quantum efficiency, we have disputed the equation for quantum efficiency in a frontside illuminated finite slab given in recent literature. From first principles we derive the correct first order equation, which has not been previously reported in the open literature. This result is significant as the equation can be separated into two physically meaningful terms. This new perspective will be useful for simplifying MTF analysis in epi substrates. Furthermore, we have disputed the MTF equation in bulk due to Barbe and have confirmed, from first principles, the validity of Seib's bulk equation. An open question is to extend our analysis to include the effect of surface states and other second order effects.

We have shown that due to the superior absorption coefficients in GaAs, improved quantum efficiency is expected. Furthermore for a hi-lo n/SI junction in GaAs, majority carriers are collected and this leads to improved spatial resolution. This advantage is presented in the form of MTF plots, showing superior overall MTF of GaAs over silicon. Future work could try to exploit this improved spatial resolution for, say, HDTV sensors.

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council and the Sir Ross and Sir Keith Smith Fund (Australia).

REFERENCES

1. A.J.P Theuwissen, *Solid-State Imaging with Charge-Coupled Devices*, Kluwer, 1995, p. 135.
2. D.H. Seib, "Carrier diffusion degradation of modulation transfer function in charge coupled imagers," *IEEE Trans. E. D.*, vol. ED-21, no. 3, pp. 210-217, 1974.

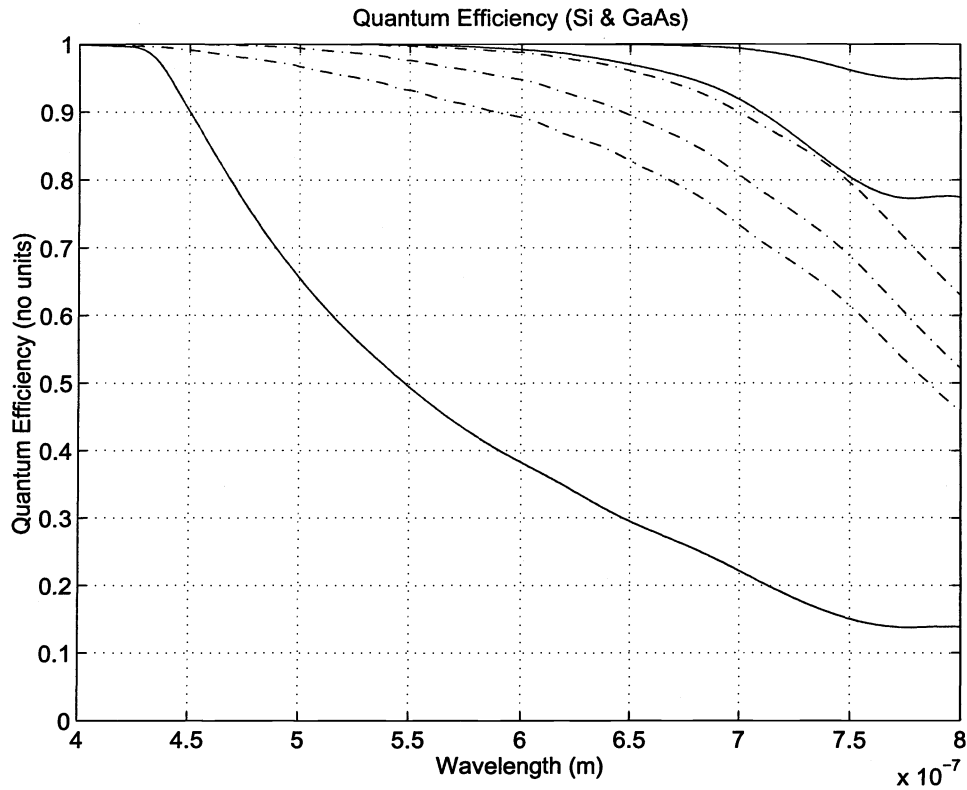


Figure 8. Internal quantum efficiency versus wavelength. Dashed lines, silicon $W=5, 2$ and $0.5 \mu\text{m}$. Solid lines, GaAs $W=2, 1$ or $0.1 \mu\text{m}$. Quantum efficiency increases with depletion width W . Transmission coefficient is set at unity.

3. D.F. Barbe, "Imaging devices using the charge-coupled concept," *Proc. IEEE*, vol. 63, pp. 38-67, Jan. 1975.
4. D.V. McCaughan and B.R. Holeman, Chapter 5, pp. 241-295, found in *Charge-Coupled Devices and Systems*, Eds. M.J. Howes and D.V. Morgan, Wiley, 1979.

Appendix A: Quantum Efficiency in a Semi-Infinite Slab

Consider a semi-infinite slab of semiconductor material. Let the surface be at $x = 0$ and the backside be at $x = \infty$. The continuity equation balance for generation and recombination under steady-state is:

$$-D\nabla^2 n + \frac{n}{\tau} = G \quad (3)$$

where D is the diffusion constant, n is the minority carrier concentration in the undepleted region, τ is minority carrier lifetime and G the electron-hole pair generation rate. We shall use n to denote either n or p type carriers as this analysis is independent of whether electrons or holes are the minority carriers, so long as the correct coefficient values for holes or electrons are used in any particular instance. The generation rate G is given by,

$$G = \Phi \alpha e^{-\alpha x} \quad (4)$$

where Φ is the incident photon flux per unit area and α is the absorption coefficient. Combining the above equations with the expression for minority carrier diffusion length, $L_o = \sqrt{D\tau}$, we get,

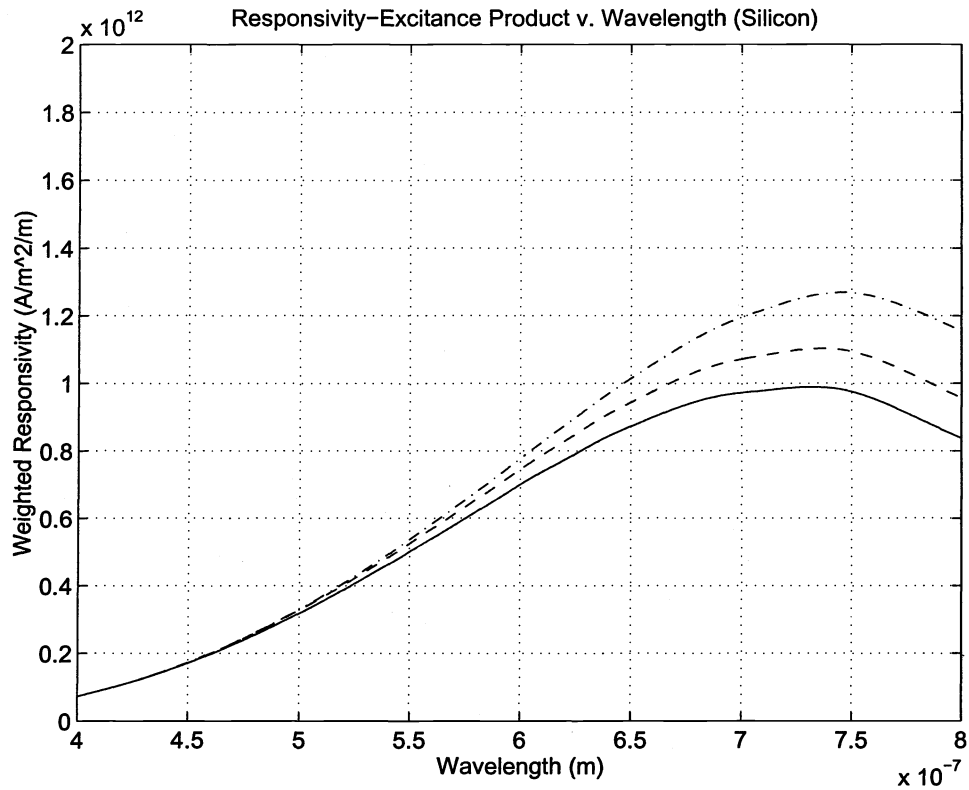


Figure 9. Weighted responsivity versus wavelength for silicon. Solid line, $W = 0.5 \mu\text{m}$. Dashed line, $W = 2 \mu\text{m}$. Chained line, $W = 5 \mu\text{m}$

$$\frac{d^2n}{dx^2} - \frac{n}{L_o^2} = -\frac{1}{D}\Phi\alpha e^{-\alpha x}. \quad (5)$$

Using the solution of the form,

$$n = Ae^{-\alpha x} + Be^{-\frac{x}{L_o}} + Ce^{\frac{x}{L_o}} \quad (6)$$

we assert boundary conditions at the depletion edge $x = W$, $n = 0$ and at the slab backside $x = \infty$, $n = 0$, yielding

$$A = \frac{\alpha L_o^2 \Phi}{(1 - \alpha^2 L_o^2) D}, \quad B = -Ae^{(\frac{1}{L_o} - \alpha)W}, \quad C = 0.$$

The steady state carrier flux is given by,

$$J = \int_0^W G dx + D \frac{\partial n}{\partial x} \Big|_{x=W} \quad (7)$$

which becomes

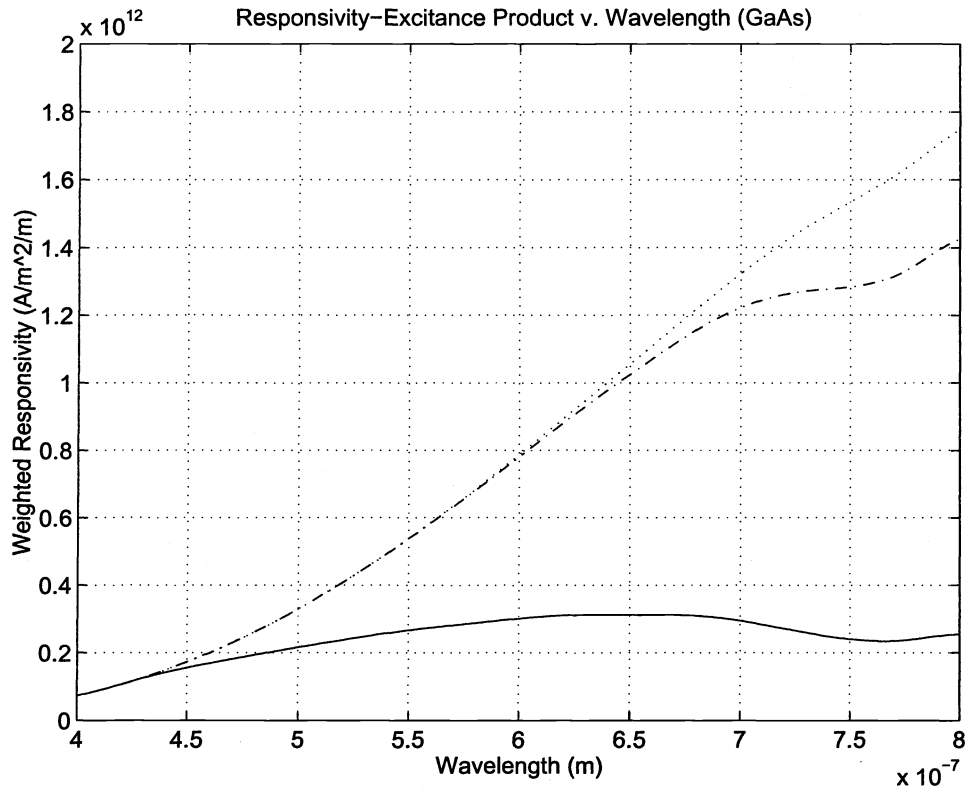


Figure 10. Weighted responsivity versus wavelength for GaAs. Solid line, $W=0.1 \mu\text{m}$. Chained line, $W=1 \mu\text{m}$. Dotted line, $W=2 \mu\text{m}$.

$$J = \Phi(1 - e^{-\alpha W}) - DA\alpha e^{-\alpha W} - \frac{DB}{L_o} e^{-\frac{W}{L_o}}$$

and by substituting in the expressions for A and B ,

$$J = \Phi(1 - e^{-\alpha W}) + \frac{\alpha L_o \Phi e^{-\alpha W}}{1 + \alpha L_o}$$

hence, internal quantum efficiency which is given by, $\eta = J/\Phi$, becomes,

$$\eta = 1 - \frac{e^{-\alpha W}}{1 + \alpha L_o}. \quad (8)$$

Appendix B: Quantum Efficiency in a Finite Slab

Consider a slab of semiconductor material, with finite thickness, h . For mathematical convenience, let the surface be at $x = -W$, the depletion region edge be at $x = 0$ and the backside be at $x = h - W$. We can rewrite the continuity equation, in the last section, as

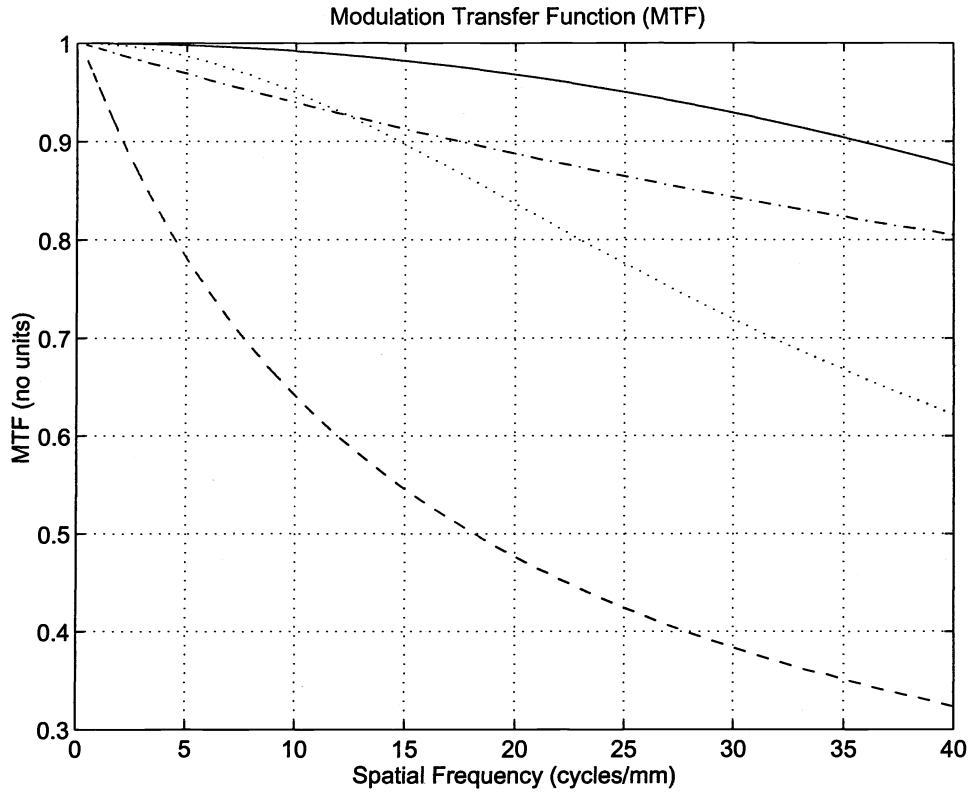


Figure 11. Theoretical MTF versus spatial frequency. Diffusion-only curves are: Si bulk, 550 nm (chained line); Si epi, 800 nm (dotted line); Si bulk, 800 nm (dashed line). Geometrical-only curve is for a 7/20 aperture to pitch ratio and the GaAs case matches this curve (solid line).

$$\frac{d^2 n}{dx^2} - \frac{n}{L_o^2} = -\frac{1}{D} \Phi \alpha e^{-\alpha(x+W)}. \quad (9)$$

Again, using the solution of the form,

$$n = A e^{-\alpha x} + B e^{-\frac{x}{L_o}} + C e^{\frac{x}{L_o}} \quad (10)$$

we assert boundary conditions at the depletion edge $x = 0$, $n = 0$ and at the slab backside $x = h - W$, $n = 0$, yielding

$$A = \frac{\alpha L_o^2 \Phi}{(1 - \alpha^2 L_o^2) D}, \quad B = -A \frac{1 - e^{-(h-W)(\alpha + \frac{1}{L_o})}}{1 - e^{-\frac{2(h-W)}{L_o}}}, \quad C = -A \frac{1 - e^{-(h-W)(\alpha - \frac{1}{L_o})}}{1 - e^{\frac{2(h-W)}{L_o}}}.$$

The steady state carrier flux is given by,

$$J = \int_{-W}^0 \Phi (1 - e^{-\alpha W}) + D \frac{\partial n}{\partial x} \Big|_{x=0} \quad (11)$$

which becomes

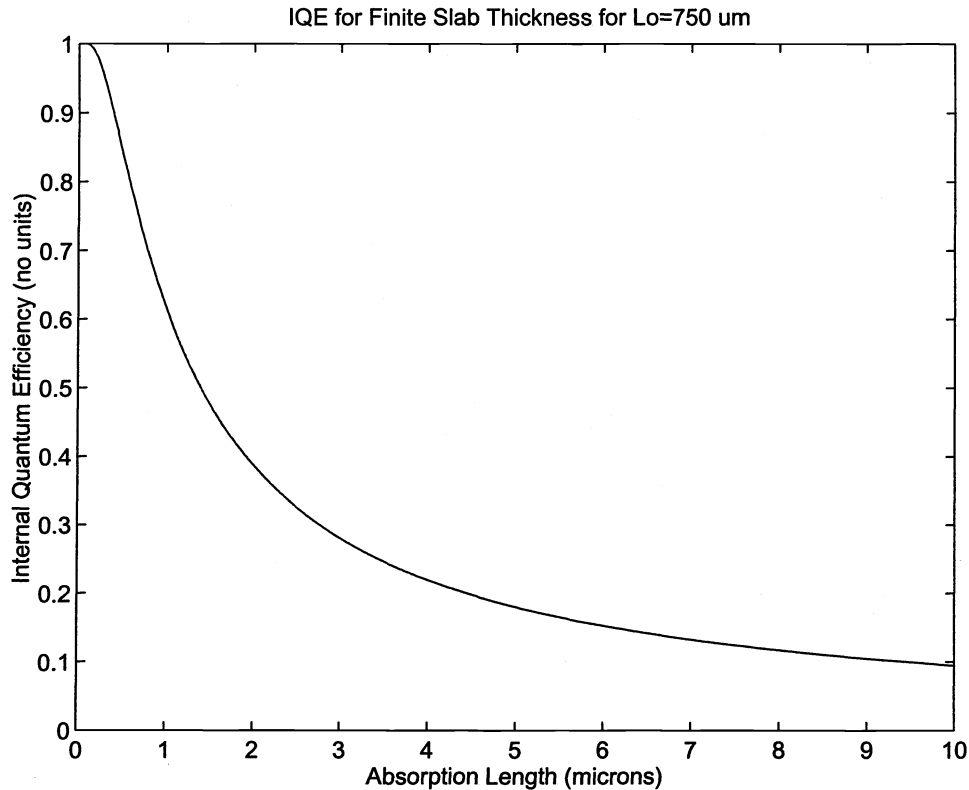


Figure 12. Internal quantum efficiency versus absorption length.

$$J = \Phi(1 - e^{\alpha W}) - DA\alpha - \frac{DB}{L_o} + \frac{DC}{L_o}$$

and by substituting in the expressions for A , B and C , after some manipulation, using $\eta = J/\Phi$, we get,

$$\eta = 1 - \frac{e^{-\alpha W}}{1 + \alpha L_o} - \frac{\alpha L_o e^{-\alpha W}}{\alpha^2 L_o^2 - 1} \frac{e^{-h/L_o} - e^{-h\alpha}}{\sinh(h/L_o)} \quad (12)$$

This equation shows that the quantum efficiency equals that of the semi-infinite case *plus* a (negative) second order term. It is quite remarkable and elegant that these two terms can be separately identified. Both this expression and the realisation that the semi-infinite case *plus* a second order term yields the finite slab case which seems to appear nowhere in the literature. Note that this expression has all the expected features. Each term is dimensionless – ie. each argument is a ratio of two lengths or an absorption coefficient times a length. This is an important feature that is missing in some erroneous attempts found in the literature. Also for $h \rightarrow \infty$ the expression reduces to the semi-infinite case.

The function is also well-behaved in that internal quantum efficiency (IQE) goes down, as expected, for increasing absorption length, Fig. 12, and goes up for increasing diffusion length, Fig. 13. Another important feature is that, by inspection, IQE always stays below unity – this is illustrated for specific examples in the graphs.

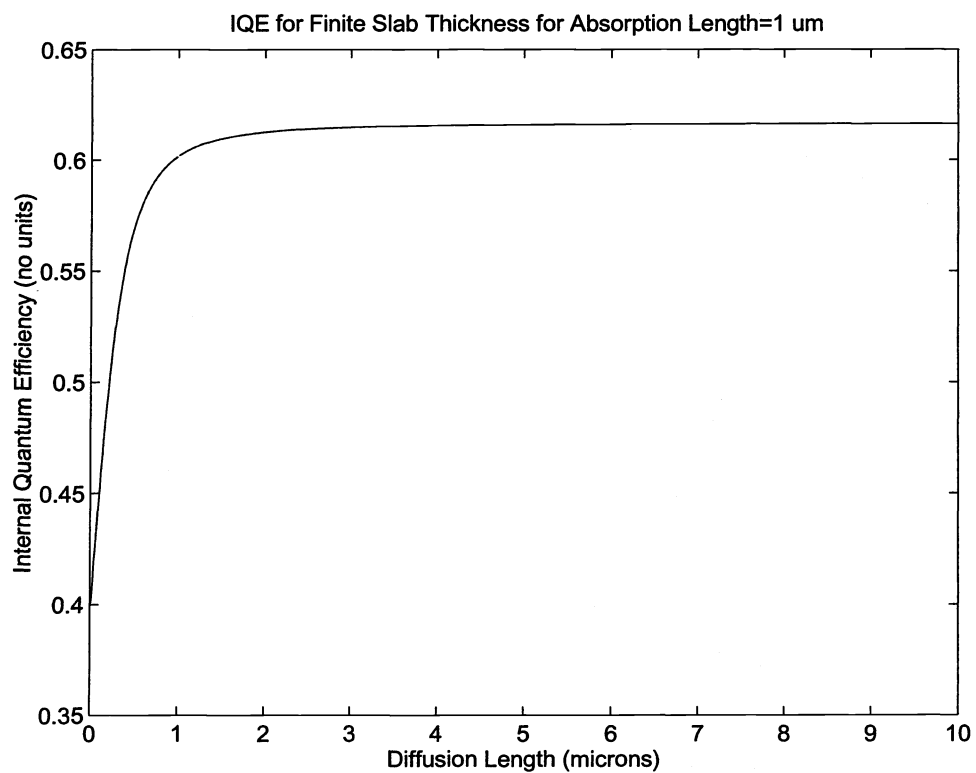


Figure 13. Internal quantum efficiency versus absorption length.