Comparison of Automatic De-noising Methods for Phonocardiograms with Extraction of Signal Parameters via the Hilbert Transform

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ABSTRACT

Phonocardiograms (PCGs) have many advantages over traditional auscultation (listening to the heart) because they may be replayed, may be analysed for spectral and frequency content, and frequencies inaudible to the human ear may be recorded. However, various sources of noise may pollute a PCG including lung sounds, environmental noise and noise generated from contact between the recording device and the skin. Because PCG signals are known to be non-linear and it is often not possible to determine their noise content, traditional de-noising methods may not be effectively applied. However, other methods including wavelet de-noising, wavelet packet de-noising and averaging can be employed to de-noise the PCG. This study examines and compares these de-noising methods. This study answers such questions as to which de-noising method gives a better SNR, the magnitude of signal information that is lost as a result of the de-noising process, the appropriate uses of the different methods down to such specifics as to which wavelets and decomposition levels give best results in wavelet and wavelet packet de-noising. In general, the wavelet and wavelet packet de-noising performed roughly equally with optimal de-noising occurring at 3-5 levels of decomposition. Averaging also proved a highly useful de-noising technique; however, in some cases averaging is not appropriate. The Hilbert Transform is used to illustrate the results of the de-noising process and to extract instantaneous features including instantaneous amplitude, frequency, and phase.

Keywords: phonocardiogram, heart sound analysis, heartbeat analysis, wavelets, wavelet packets, averaging, de-noising, Hilbert Transform

1. INTRODUCTION

The stethoscope and human ear have their limitations in diagnosing heart defects and conditions. Modern technology has developed new tools which are capable of revealing information that traditional tools such as the stethoscope alone cannot. For example, digital stethoscopes have been developed which have the capacity to record and to replay the heartbeat sound recordings otherwise known as phonocardiograms (PCGs). The PCG is a particularly useful diagnostic tool because the graphic recordings show timings and relative intensities of heartbeat sounds and may reveal information that the human ear cannot.[1–3] With the aid of computers, the PCG data may be stored, managed, and manipulated for frequency and spectral content.

PCGs are easily obtained by placing the stethoscope against the skin and recording the sounds produced by the heart. The current problem with many PCG systems is that noise, from breath sounds, contact of the stethoscope with the skin, fetal heart sounds if the subject is pregnant, and ambient noise, may corrupt the heartbeat signals.

The PCG would be a much more useful diagnostic tool if unwanted noise was removed clearly revealing the heartbeat sound. Because PCG signals are known to be non-linear and it is often not possible to determine their noise content (because the noise produced in each case will be different), traditional de-noising methods may not be effectively applied. The current study examines methods of removing the noise from the PCG namely using wavelet analysis, wavelet packet analysis and averaging.

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Although it is not exactly known what produces each of the four heart sounds heard through a stethoscope, they are likely to be produced by a number of sources including the opening and closing of valves, vibrations of the cardiac structure, and acceleration and deceleration of blood.[1,4] The first and second heart sounds ("lub-dub") are the two that are generally heard by the human ear, and are usually the most visible on PCG. They may be seen in Figure 1 (a) with the first heart sound, S1, occurring at about 0.1 seconds and the second heart sound, S2, happening at about 0.4 seconds.

An electronic stethoscope is used to record heart sounds. Various digital signal processing tools are employed remove noise from signal. The remainder of this article will introduce the reader to basic wavelet and wavelet packet theory in relation to de-noising heart sounds, averaging in relation to de-noising and the methods and results of de-noising heart sounds which were investigated. The questions that this study attempts to answer are whether wavelet de-noising, wavelet packet de-noising, or averaging best removes noise in a PCG down to such specifics such as which wavelet families and levels of decomposition perform best. This study is an extension of the work described by Maple *et al.* and Messer *et al.*[5,6] It should be noted, however, that the results of the previous study, used an optimized wavelet de-noising tool which was not used in the current study because we wish to fairly compare wavelet and wavelet packet de-noising.

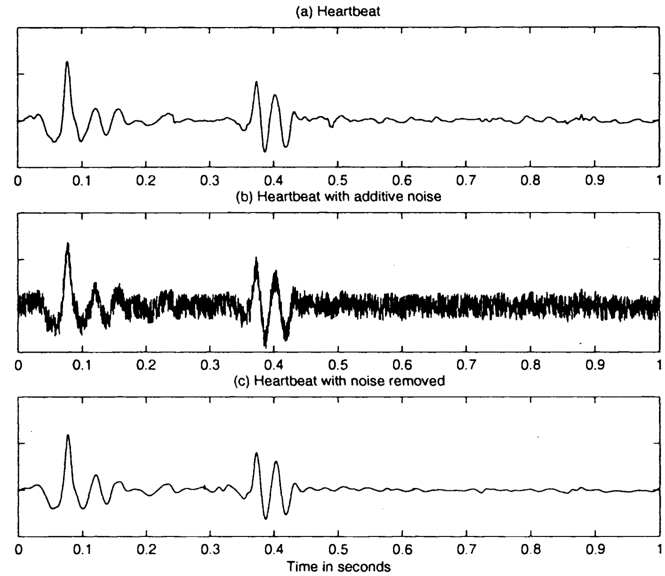

Figure 1. This figure shows the principle of de-noising a heartbeat. (a) is the characteristic heartbeat signal (b) the heartbeat with 1 dB of additive white noise (c) the heartbeat with noise removed using wavelet analysis

2. EQUIPMENT AND DATA

An electronic stethoscope (the Escope from Cardionics), using an electret microphone, outputs the heart sound as an analogue signal. This analogue signal is converted to a digital signal (sampled at 2500 samples/second with 12-bit resolution) and stored on the computer for further use. The electrical activity of the heart is also simultaneously recorded to serve as a reference signal. MATLAB software is then used to analyze the signal and perform the signal processing. For more detail on the equipment and recording procedure, refer to our previous study.[6]

3. THEORY

3.1. De-Noising Theory

As previously mentioned, PCGs are notoriously complex, non-linear signals where the signal genesis is not completely understood and the noise content is different in each case. Thus, traditional de-noising methods, many of which require *a priori* knowledge of the noise content of the signal, may not be applied or may function poorly when used

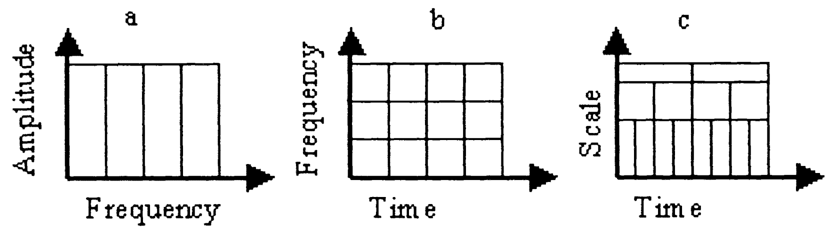

Figure 2. Comparison of a signal represented in different domains with (a) corresponding to the Fourier transform representation, (b) representing the short time Fourier transform, and (c) the wavelet transform

to de-noise PCGs. Three principal methods have been employed to de-noise PCGs: the wavelet transform, wavelet packets, and averaging.

Wavelets may be used to de-noise the PCG as shown in Figure 1. The signal is decomposed by a discrete wavelet transform. Because of the efficient decomposition of heart signals, their wavelet coefficients tend to be much larger than those due to noise. Thus, coefficients below a certain level are regarded as noise and are thresholded out. The signal is then reconstructed without significant loss of information.

Wavelet packet de-noising is very similar to wavelet de-noising, but it offers a wider range of possibilities for signal analysis. For an n-level decomposition, there are $n+1$ possible ways to decompose or encode a signal.

Averaging is a common method used to de-noise signals of a repetitive nature.[4,7] Although heartbeats are considered non-stationary signals, they are periodic in the sense that heartbeats regularly repeat.

3.1.1. Wavelet theory

Wavelet theory dates back to the work of Joseph Fourier, but most of the advances in the field have been made since the 1980's. This section gives a very short introduction to wavelet theory and de-noising. The interested reader may find a further review of wavelet theory in many sources.[8–12]

The Wavelet Transform was developed as a method to obtain simultaneous, high resolution time and frequency information from a signal. The term "wavelet" was first mentioned in 1909 in a thesis by Alfred Haar,[11] although the progress in the field of wavelets has been relatively slow until the 1980's when scientists and engineers from different fields realized they were working on the same concept and began collaborating.[9]

The WT presents an improvement over the Fourier Transform (FT) and the Short Time Fourier Transform (STFT) because it obtains good time and frequency resolution simultaneously by using a variable sized window region (the wavelet) instead of a constant window size. The FT simply shows the frequency content of a signal without any time information as shown in Figure 2(a) because the signal is represented as a sum of sines and cosines integrated over all time. The STFT is an improvement over the FT because it is a time-frequency representation as shown in Figure 2(b). However, the resolution of the STFT is limited by the window size chosen to integrate over. If time resolution is improved, frequency resolution becomes poorer and vice versa. Because the wavelet may be dilated or compressed as is seen in Figure 2(c), different features of the signal are extracted. While a narrow wavelet extracts high frequency components, a stretched wavelet picks up on the lower frequency components of the signal.

A wavelet is a signal of limited duration that has an average value of zero. Examples of wavelets used in this study may be seen in Figure 3.

The mathematical description of the Continuous Wavelet Transform (CWT) is described by,[9]

$$c(a,b) = \int f(t)\psi(at+b)dt \tag{1}$$

where ψ is used to create a family of wavelets $\psi(at+b)$ where a and b are real numbers with a dilating the function ψ and b translating it. The scale of the wavelet may conceptually be considered the inverse of the frequency. As

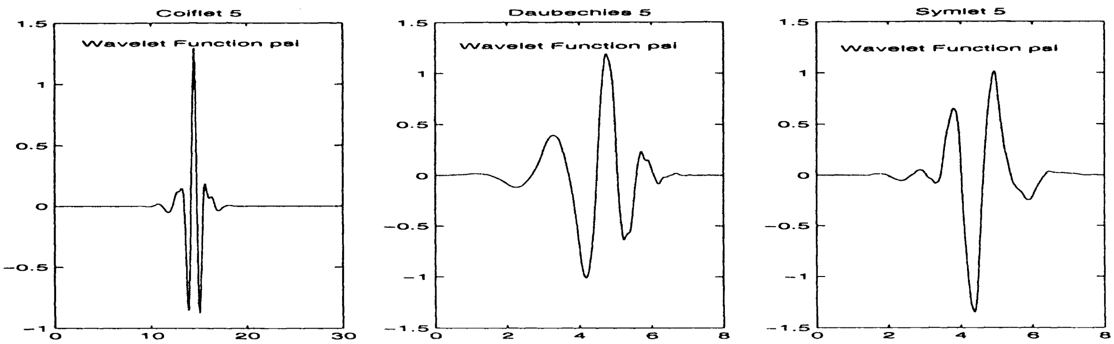

Figure 3. Examples of wavelets used in this study. Time is on the horizontal axis

seen in Figure 2 (c), the wavelet is compressed if the scale is low and dilated if the scale is high. Because the WT is computed in terms of scale instead of frequency, plots of the WT of a signal are displayed as time versus scale.

The process of computing the CWT is very similar to that of the STFT. The wavelet is compared to a section at the beginning of a signal. A number is calculated showing how closely correlated the wavelet and signal section are. The wavelet is shifted right and and the process is repeated until the whole signal is covered. The wavelet is scaled and the previous process is repeated for all scales.

The CWT reveals much detail about a signal, but because all scales are used to compute the WT, the computation time required can be enormous. Therefore, the Discrete Wavelet Transform (DWT) is normally used. The DWT calculates the wavelet coefficients at discrete intervals of time and scale instead of at all scales. The DWT requires much less computation time than the CWT without much loss in detail. With the DWT, a fast algorithm is possible which possesses the same accuracy as other methods. The algorithm makes use of the fact that if scales and positions are chosen based on powers of two (dyadic scales and positions) the analysis is very efficient. Because the algorithm possesses the same accuracy as other methods, this method is often used and is used in the current study. An efficient way to implement this algorithm was developed in 1988 by Mallat which is known as two-channel sub-band coder.[13]

For a single level of decomposition, this algorithm passes the signal through two complementary (highpass and lowpass) filters resulting in approximations which are high-scale, low-frequency components of the signal, and details, which are low-scale, high-frequency components of the signal. This results in twice as many data-points so the data is down-sampled. For further levels of decomposition, successive approximations may be iteratively be broken down into details and approximations as shown in Figure 4. Because of the efficient decomposition of heart signals, their wavelet coefficients tend to be much larger than those due to noise. Thus, coefficients below a certain level are regarded as noise and thresholded out. The signal is then reconstructed without significant loss of information. Then the signal may be reconstructed by up-sampling, passing the approximations and details through the appropriate reconstruction filters and combining the results.

3.1.2. Wavelet packet theory

Wavelet packet de-noising is very similar to wavelet de-noising, but it offers a wider range of possibilities for signal analysis. For n-levels of decomposition the approximations *and* details are broken down into a further level of details and approximations (as shown in Figure 4 resulting in 2^n possible ways to encode the signal).[11] There are more way of decomposing a signal using WP analysis compared to wavelet analysis because wavelet packet atoms are waveforms which are indexed by 3 parameters, position and scale which corresponds to the wavelet decomposition and frequency, instead of 2 as in the wavelet transform. The analysing window size, frequency and position can each be varied separately. For each orthogonal wavelet function, a library of WP bases is generated which can represent the signal in many combinations. With so many ways to represent the signal, a method must be used to select the best decomposition of the signal. An entropy-based search is performed using the adaptive filtering algorithm which is based on work by Coifman and Wickerhauser. If the reader wishes to know more about wavelet packets, there are a number of sources which may be consulted.[14–16]

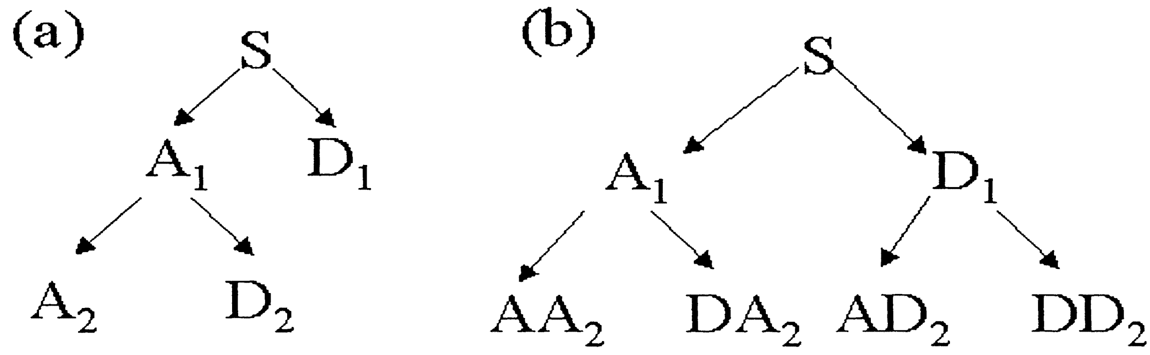

Figure 4. This figure illustrates how (a) the discrete wavelet transform decomposes and signal into details and approximations iteratively decomposing the approximations where in (b) wavelet packets iteratively decompose the approximations and details.

3.1.3. Averaging theory

Averaging is known to reduce white noise because it is randomly distributed throughout the signal and may also be used to produce a "characteristic heartbeat" which is an averaged heartbeat from a series of beats.[4] Over short periods of time, heartbeats have the same statistical properties. Thus, the signal may be considered quasi-stationary over a short period of time.[4]

According to basic probability theory,[7] the intensity of a random signal averaging of n cycles is attenuated by $\sqrt{n}$. Thus, if 20 cycles were averaged, random signals in the heartbeat series would be attenuated by a factor of $\sqrt{20} \approx 4.5$ or if 50 cycles were averaged, the attenuation factor would be about $\sqrt{50} \approx 7$.

An important factor to consider in the use of averaging of heartbeats the type of signal sought. The mechanical activity of the heart can be classified into two categories: "deterministic" and "nondeterministic".[7] In the our case, any process that repeats itself exactly for each beat may be considered deterministic. Thus, if a deterministic series of beats is averaged, any beat should be nearly the same as the averaged beat. Noise or nondeterministic events such as murmurs will be attenuated by averaging. So in some cases, such as removing unwanted white noise from PCGs, averaging is very appropriate, whereas in other cases, for example, if information about the murmurs in a PCG were wanted, averaging would not be the best de-noising method.

The algorithm for averaging the PCG signal uses the ECG as a gating signal because they are both recorded simultaneously. The QRS complex of the ECG signals the beginning of the cycle and is used to separate each heartbeat. A description of the complete algorithm is given by Tinati.[4]

3.2. Hilbert Transform

The Hilbert Transform (HT) may be used to calculate the instantaneous attributes of a signal. The mathematical definition of the Hilbert Transform of is[17]

$$y(t) = \pi^{-1} \int_{\infty}^{-\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{2}$$

The Hilbert Transform (HT) can be considered a convolution between the signal and $\frac{1}{\pi}t$. The HT can be realized by an ideal filter whose amplitude response is unity and whose phase response that is a constant ninety degree lag. The HT is called the quadrature filter because it shifts the phase of the spectral components by $\frac{\pi}{2}t$.

The Hilbert Transform may be used to calculate the instantaneous frequency, phase and amplitude of the signal. The instantaneous frequency is calculated through the analytic method using the Hilbert Transform and is mathematically defined in Equation 3 where s is the signal and $H(s)$ is the Hilbert Transform of the signal,[18]

$$\Theta = \frac{1}{2\pi} \frac{d}{dt} [\arctan(\frac{H[s(t)]}{s(t)})]. \tag{3}$$

The instantaneous frequency of a signal may be used to demonstrate how effective de-noising is.[19] The instantaneous phase is simply the derivative of the instantaneous frequency. The instantaneous amplitude is the magnitude of the complex analytical signal found by using the Hilbert Transform.

4. METHODS FOR MEASURING DE-NOISING RESULTS

Signal-to-noise-ratio (SNR) is a traditional method of measuring the amount of noise present in a signal. SNR is defined as

$$SNR = 10 * \log_{10}(\text{Power}_{\text{signal}}/\text{Power}_{\text{noise}}) \tag{4}$$

measured in decibels. Two tests are performed using the SNR to measure the performance of wavelet and wavelet packet de-noising.[5]

Because there is currently no known mathematical method to calculate which wavelet, wavelet packets, and levels of decomposition best de-noise a signal, simulations must be performed to evaluate the de-noising capabilities of wavelets, wavelet packets and decomposition level combinations. A known amount of noise was added to a "clean" heart sound recording. ("Clean" refers to the fact that although attempts were made to eliminate all environmental noise during the recording, there is still some noise present in small amounts.) Varying various parameters, the wavelet and wavelet packet de-noising techniques were applied to the heart sound recording which has noise added. Then the SNR will be calculated for the de-noised signal and the original signal. The higher the SNR, the less noise there is present. The second test measures how much of the original signal is recovered after the de-noising process or how much information in the original signal is lost by the de-noising process. The wavelet or wavelet packet de-noising techniques are applied to a clean recording and the SNR of the resultant signal and the original signal is computed. In other words, the more of the original clean signal that remains after applying the de-noising, the better, because we want to retain the signal but discard the noise.

The concept of adding a known amount of noise to clean heartbeats, then de-noising the signal, and seeing how much noise remains is also employed to measure how well averaging performs as a de-noising technique.

5. EXPERIMENTAL RESULTS AND DISCUSSION

It was expected that wavelet packet de-noising would perform better than wavelet de-noising because wavelet packet analysis adaptively chooses the best basis based upon an entropy search and a study comparing wavelet and wavelet packet de-noising for knee-joint vibrations, which are complex, non-stationary signals, concluded that wavelet packet de-noising performed better for knee-joint vibrations.[15] However, in the current study, wavelet de-noising and wavelet packet de-noising proved to perform similarly. Averaging proved to be a useful de-noising technique.

The two previously mentioned tests were performed for wavelet and wavelet packet de-noising using all possible combinations of orthogonal wavelets, because they allow for perfect signal reconstruction, and levels of decomposition from 1 to 10. The orthogonal wavelets used here were the Daubechies wavelets orders 1-45, Coiflets orders 1-5, and Symlets orders 1-15.

The wavelet transform process of decomposition and recomposition was applied to several clean heartbeats. The SNR of the original signal and the signal after the WT is applied are calculated. This number represents how much information was lost in the wavelet analysis process. The higher the SNR, the more of the original signal content that remains. As shown in Figure 5, the wavelet packet de-noising seems to lose more of the original signal content than the wavelet de-noising process. It is also interesting to note that most wavelets appear to lose about the same amount of the original signal content. The notable exceptions to this statement would be the lower order wavelets such as the Daubechies orders 1-3. This fact may be explained by the influence of support length, regularity, and the number of vanishing moments. For Daubechies wavelet of order N, the support length of ψ and ϕ is $N-1$ and the vanishing moment of ψ is N.[11] The order of regularity of a wavelet is the number of continuous derivatives which it possesses.[9] Poor regularity may introduce artifacts.[9] Regularity may be increased by increasing the length of support[9] which increases with N. Vanishing moments influence what signal content is picked up by the wavelet transform.[9] With 1 vanishing moment, linear functions are not seen, and with 2 vanishing moments, quadratics are not picked up. Thus, by increasing the number of vanishing moments, the lower order components of the signal may be seen.

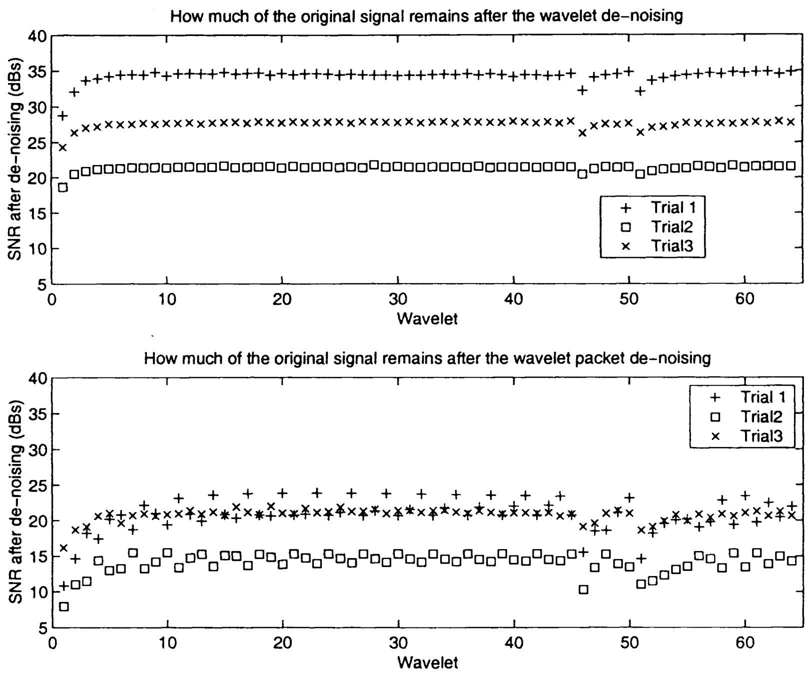

Figure 5. This figure shows how much of the original signal content remains (expressed as an SNR in dBs) after wavelet and wavelet packet de-noising (with 4 levels of decomposition) are applied to 3 "clean" PCGs. The x-axis represents the different wavelets respectively: Daubechies Orders 1-45, Coiflet Orders 1-5, Symlets Orders 1-15 with 4 levels of decomposition used in each case.

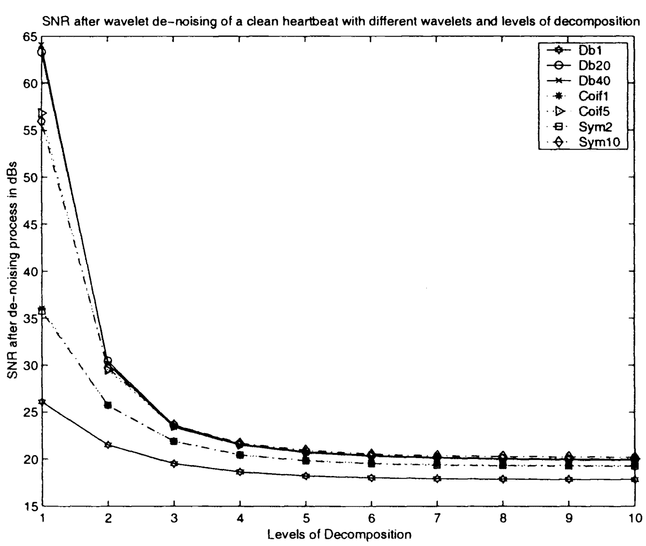

Figure 6. This figure shows the SNR after wavelet de-noising of a "clean" PCG of a 79 year old male with a heart murmur and high blood pressure for various wavelets at different decomposition levels. The higher the SNR after the de-noising process the more of the original signal content that remains.

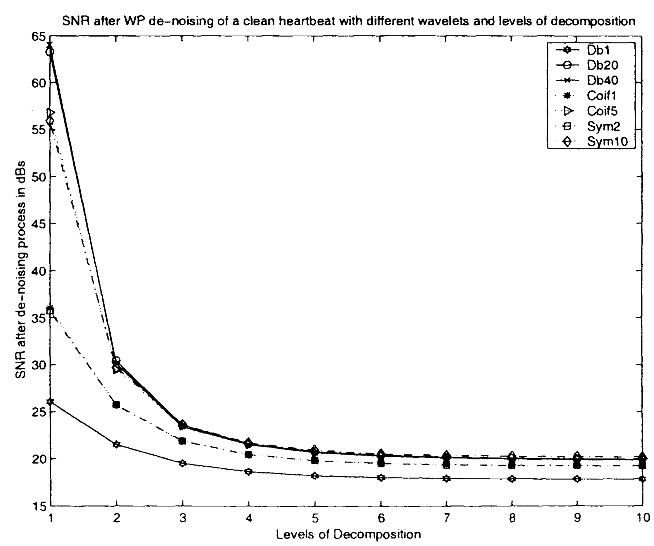

Figure 7. This figure shows the SNR after wavelet packet de-noising of a "clean" PCG of a 79 year old male with a heart murmur and high blood pressure (same as in Figure 6) for various wavelets at different decomposition levels. The higher the SNR after the de-noising process the more of the original signal content that remains.

In Figures 6 and 7, we can see how much information is lost in the wavelet and wavelet de-noising process at various decomposition levels for some different wavelets. It is obvious and logical that as we increase the number of decomposition levels more information from the original signal is lost. From 1-2 levels of decomposition, there is a lot of information discarded with the amount of information lost decreasing less steeply to 4-5 levels of decomposition where the amount of information lost remains relatively constant.

In Table 1, the best results of wavelet and wavelet packet de-noising for all the combinations may be seen. The best results are very similar for the two methods, and most were achieved using 3-5 levels of decomposition with higher order wavelets. Higher order wavelets all perform rather equally better over their lower order counterparts as seen in Figure 8 for reasons previously explained. From Figure 9, it is demonstrated that decomposition levels of 3-5 for both wavelet and wavelet packet de-noising produce the best de-noising results.

Amount of White Noise Added	Trial 1		Trial 2		Trial 3	
	WP	W	WP	W	WP	W
1 dB	13.55	12.89	11.79	11.14	11.90	11.91
5 dBs	17.23	17.24	13.15	13.54	14.65	14.96
10 dBs	20.07	20.07	16.90	16.78	18.37	18.64
20 dBs	28.17	28.22	24.97	24.80	26.09	25.87

Table 1. This table lists the best results (using SNR measured in dBs) of all the combinations tried for wavelet and WP de-noising with varying amounts of white noise added. Trial 1 is a 24 year old, healthy, female. Trial 2 is a 43 year old female with hypertension, Trial 3 is an 84 year old female with atrial fibrillation. Wavelet and wavelet packet de-noising seem to work equally well.

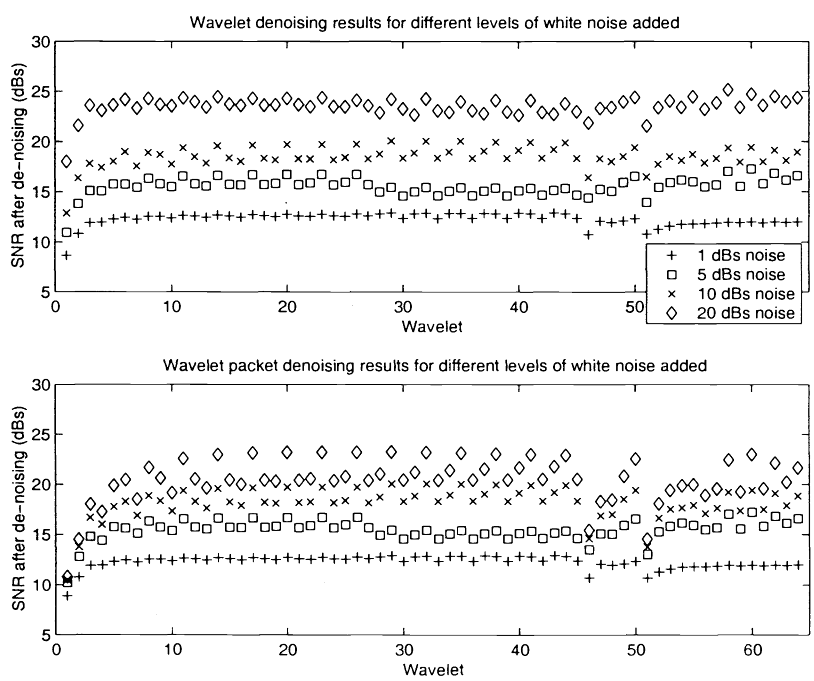

Figure 8. This figure shows wavelet and wavelet packet de-noising results (as an SNR in dBs) for different levels of white noise added to a heartbeat sample. The x-axis represents the different wavelets respectively: Daubechies Orders 1-45, Coiflet Orders 1-5, Symlets Orders 1-15 with 4 levels of decomposition used in each case. Wavelet and WP de-noising appear to perform about equally.

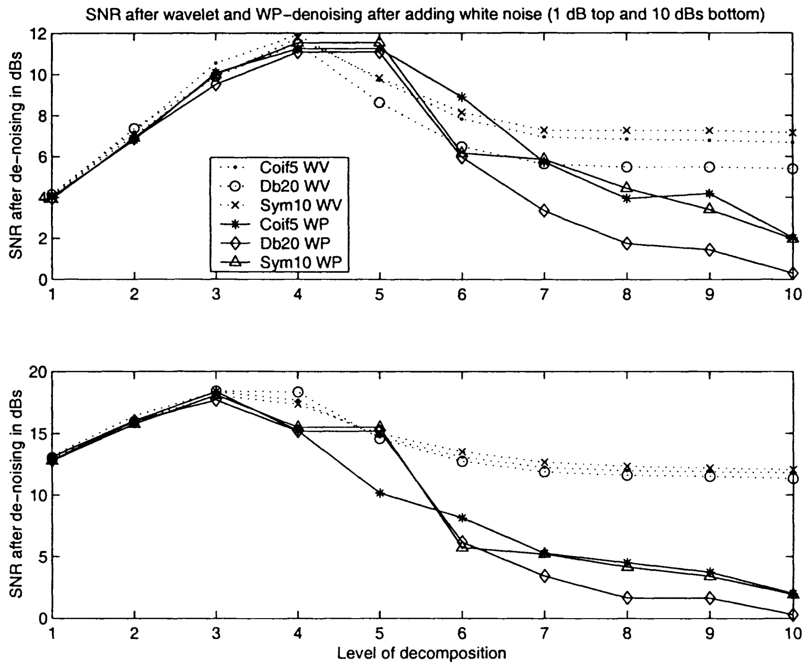

Figure 9. This figure illustrates the effect of varying the level of decomposition for wavelet and wavelet packet de-noising for various wavelets with additive white noise at levels of 1 dBs and 10 dBs. It appears that a decomposition level of about 4 is the best. After 4-5 decomposition levels, more of the original signal is lost due to the de-noising process. Below these levels of decomposition, much of the noise remains in the signal.

Averaging seemed to produce significant improvements especially if there is a large amount of noise present in the signal. Figure 10 shows that averaging a series of 50-75 beats seems to give the best result in terms of recording and computation time tradeoff. It is difficult to obtain a long, clean recording and also increases the computation time required. There seems to be marginal improvement in SNR when little noise is present in the signal and the signal is not averaged a fair number of times. For example, with an SNR of additive noise at 1 dB, after averaging the signal 10 times, we see that after de-noising the noise levels decrease as the SNR approaches 11 dBs, but with an initial SNR of additive noise at 20 dBs and averaging 10 heartbeats, the SNR is still about 20 dBs after de-noising meaning the noise remains.

We can also examine the PCG by plotting the instantaneous frequency. Figure 13 shows a characteristic heartbeat, then the beat with noise added, and finally the noisy beat with noise removed both by the wavelet and wavelet packet de-noising processes. The corresponding instantaneous frequencies are also shown. We can clearly see from the instantaneous frequency plots when there are large amounts of noise present. It is also interesting to note that around S1 and S2, the instantaneous frequencies remain relatively constant at low-frequencies supporting the well known fact that S1 and S2 are composed of several low-frequency sinusoidal components.

The instantaneous amplitude is an alternative method of looking at the PCG data. Figure 11 demonstrates that recording the PCG of a patient is reproducible because plots of the instantaneous amplitude of a PCG recorded on 4 different occasions are very similar. Figure 12 shows the instantaneous amplitude of PCGs for patients with various pathological conditions and patients with normal hearts. We were limited by the number of PCG recordings available, but by examining this plot we may see that the healthy patients appear to have a well defined and compact S1 and S2 whereas some of the patients with pathological conditions do not.

Figure 14 borrows the concept of a complex trace from seismic data analysis.[20] The signal and its Hilbert Transform are projected on their prospective axes with the complex trace being a vector sum of the two. This view

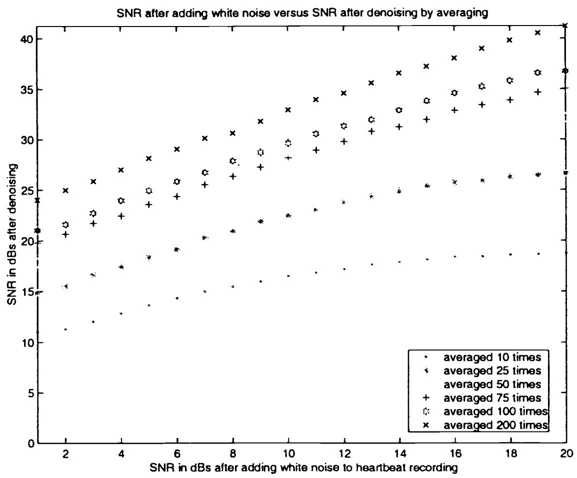

Figure 10. This chart shows the SNR after adding white noise to a series of heartbeats and the SNR after averaging these series of heartbeats to obtain a characteristic heartbeat and reduce noise.

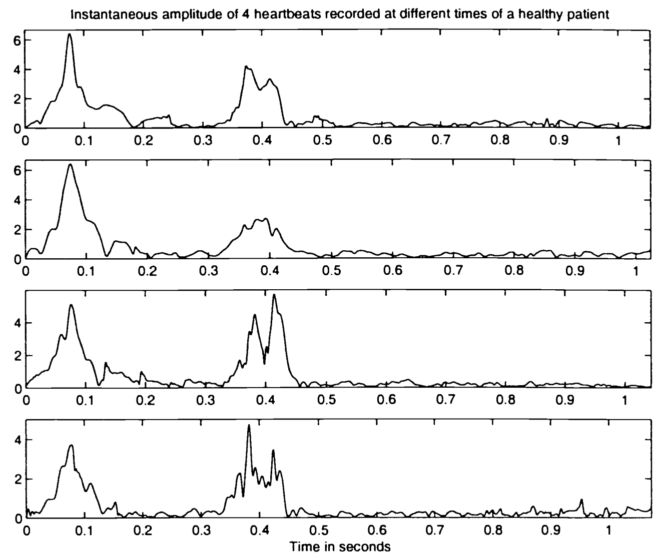

Figure 11. This figure shows the instantaneous amplitude of 4 heartbeats recorded at different times from the same healthy patient. They are all fairly similar demonstrating that this technique is reproducible.

reveals many features of the signal. The length of the complex trace vector is the instantaneous amplitude. The orientation angle (usually measured relative to the positive axis of the plane where the real signal is projected) is the instantaneous phase. The time rate of change of the phase angle is the instantaneous frequency.

6. CONCLUSIONS AND FUTURE DIRECTIONS

Wavelet and wavelet packet de-noising perform roughly equally in de-noising of PCGs. Wavelet or wavelet packet de-noising in combination with averaging would be very useful. However, there may be certain clinical cases, for example if a pathological condition was only present in some beats and not others, where wavelet and wavelet packet de-noising alone should be employed as averaging attenuates non-deterministic events. Decomposition levels of 3-5 were found to perform the best in wavelet and wavelet packet de-noising. Future topics to research in this area of phonocardiògram de-noising include the application of the matching pursuit method.[21]

The use of the Hilbert Transform was explored in relation to the analysis of heartbeats. Much information is displayed in the complex PCG trace. Further studies could be performed to investigate the clinical use of instantaneous PCG parameters as indicators of cardiac health.

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REFERENCES

1. L.-G. Durand and P. Pibarot, "Digital signal processing of the phonocardiogram: Review of the most recent advancements," *Critical Reviews in Biomedical Engineering* **23**(3/4), pp. 163–219, 1995.

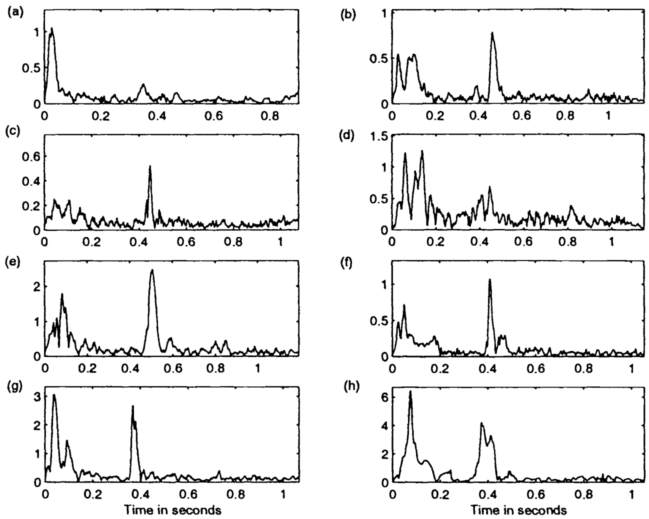

Figure 12. (a) Instantaneous amplitude of a heartbeat of a patient with mitral valve prosthesis (b) Instantaneous amplitude of a heartbeat of a patient with a heart murmur and hypertension (c) Instantaneous amplitude of a heartbeat of a patient with a aortic stenosis and hypertension (d) Instantaneous amplitude of a heartbeat of a patient with a systolic murmur and angioplasty (e) Instantaneous amplitude of a heartbeat of a patient with atrial fibrillation (f) Instantaneous amplitude of a heartbeat of a patient with hypertension (g) Instantaneous amplitude of a heartbeat of a healthy patient (h) Instantaneous amplitude of a heartbeat of a healthy patient

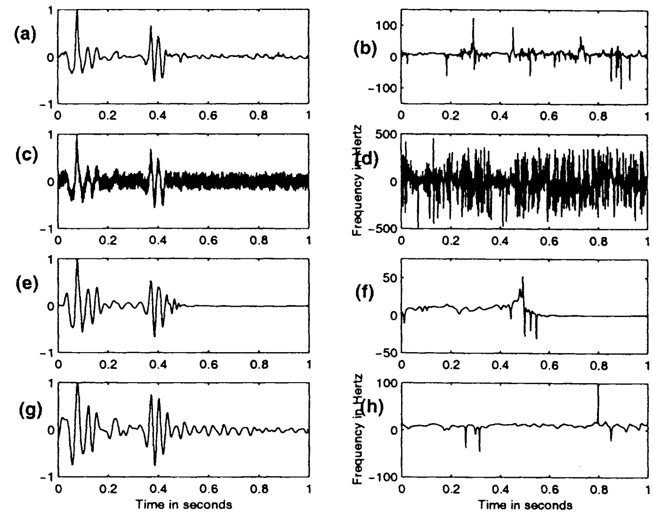

Figure 13. (a) Characteristic heartbeat, (b) instantaneous frequency of (a), (c) 1 dBs of white noise added to (a), (d) instantaneous frequency of (c), (e) Noisy heartbeat de-noised by wavelet technique (f) instantaneous frequency for (e), (g) Noisy heartbeat de-noised by wavelet packet technique, and (h) instantaneous frequency of (h)

2. S. Lukkarinen, A. Nopanen, K. Sikio, and A. Angerla, "A new phonocardiographic recording system," *Computers in Cardiology* **27**, pp. 117–120, 1997.
3. S. Lukkarinen, K. Sikio, A.-L. Nopanen, A. Angerla, and R. Sepponen, "Novel software for real-time processing of phonocardiographic signal," in *Proceedings of the 19th International Conference of the IEEE Engineering in Medicine and Biology Society*, pp. 1455–1457, IEEE Engineering in Medicine and Biology Society, October/November 1997.
4. M. A. Tinati, *Time-Frequency and Time-Scale Analysis of Phonocardiograms with Coronary Artery Disease Before and After Angioplasty.* PhD thesis, The University of Adelaide, October 1998.
5. J. Maple, L. Hall, J. Agzarian, and D. Abbott, "Sensor system for heart sound biomonitor," *Proc. SPIE Electronics and Structures for MEMS* **3891**, pp. 99–109, October 1999.
6. S. Messer, D. Abbott, and J. Agzarian, "Optimal wavelet denoising for smart biomonitor systems," *Proceedings of SPIE: Smart Electronics and MEMS* **4236**, 2000.
7. R. Beyar, S. Levkovitz, S. Braun, and Y. Palti, "Heart-sound processing by average and variance calculation–physiologic basic and clinical implications," *IEEE Transactions on Biomedical Engineering* **31**, pp. 591–596, September 1984.
8. C. S. Burrus, R. A. Gopinath, and H. Guo, *Introduction to Wavelets and Wavelet Transforms: A Primer*, Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1998.
9. B. B. Hubbard, *The World According to Wavelets*, A K Peters Ltd., 289 Linden Street, Wellesley, MA 02181, 1996.
10. Y. Meyer and R. D. Ryan, *Wavelets Algorithms and Applications*, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1993.

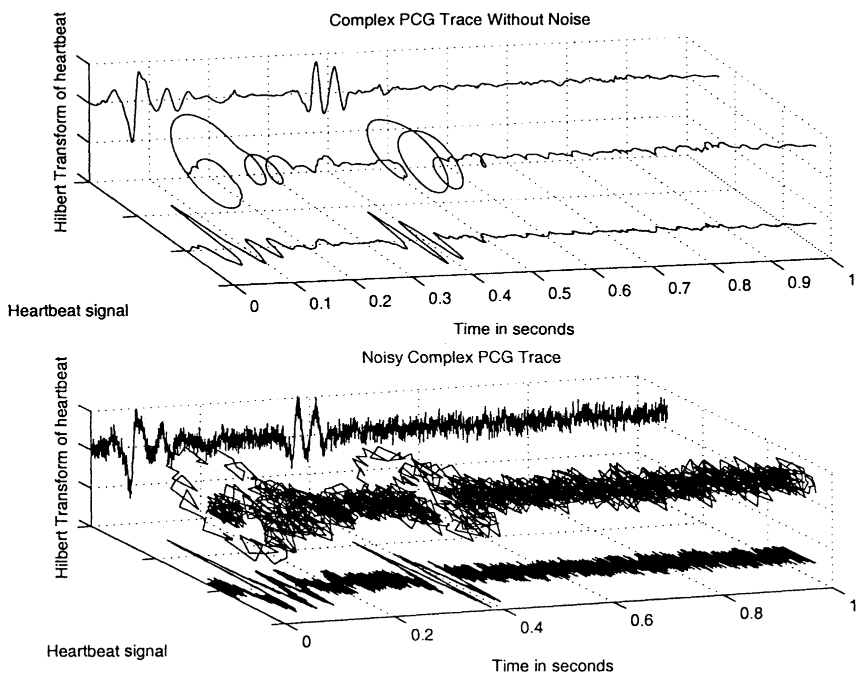

Figure 14. This figure shows a complex PCG trace first with additive white noise and secondly without noise.

11. M. Misiti, Y. Misiti, G. Oppenheim, and J.-M. Poggi, *Wavelet Toolbox: For Use With Matlab.* The Math Works Inc., 1996.
12. G. Strang and T. Nguyen, *Wavelets and Filter Banks*, Wesley-Cambridge Press, 1996.
13. S. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.* **11**(7), pp. 674–693, 1989.
14. A. Jiménez, M. R. Ortiz, M. A. P. na, S. Charleston, A. T. Aljama, and R. González, "The use of wavelet packlets to improve the detection of cardiac sounds from the fetal phonocardiogram," *Computers in Cardiology* **26**, pp. 463–466, 1999.
15. S. Krishnan and R. M. Rangayyan, "Automatic de-noising of knee-joint vibration signals using adaptive time-frequency representations," *Medical and Biological Engineering and Computing* **38**, pp. 2–8, 2000.
16. M. V. Wickerhauser, *Adapted wavelet analysis from theory to software*, A. K. Peters, Ltd., 1994.
17. O. Ersoy, *Fourier-Related Transforms, Fast Algorithms and Applications*, Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1997.
18. J. Gao, X. Dong, W.-B. Want, Y. Li, and C. Pan, "Instantaneous parameters extraction via wavelet transform," *IEEE Transactions on Geoscience and Remote Sensing* **37**, pp. 867–870, March 1999.
19. P. Carré, H. Leman, C. Fernandez, and C. Marque, "Denoising of the uterine ehg by an undecimated wavelet transform," *IEEE Transactions on Biomedical Engineering* **45**, pp. 1104–1113, September 1998.
20. M. T. Taner, F. Koehler, and R. E. Sheriff, "Complex seismic trace analysis," *Geophysics* **44**, pp. 1041–1063, June 1979.
21. S. Mallat and Z. Zhang, "Matching pursuits with time-frequency dictionaries," *IEEE Transactions on Signal Processing* **41**, pp. 3397–3415, December 1993.