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Scientists crack 'two-envelope' problem

Anna Salleh
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Australian mathematicians say they have solved a long-standing problem about how a person chooses between two envelopes containing different amounts of money.

Dr Mark McDonnell of the University of South Australia and Dr Derek Abbott of the University of Adelaide report their new solution to the two-envelope problem today in the journal *Royal Society Proceedings A* .

In the problem a person is presented with two envelopes and told that one contains twice the amount of money as the other.

They are invited to open the envelope but then must decide if they'd like to switch it with the other.

The dilemma is they could double their money - but they could also halve it.

Current wisdom says that people have an equal chance of gaining or losing no matter whether they decide to switch or stay put with the original envelope, says McDonnell.

But he and Abbott have now worked out a formula they say can be used to increase the chance of winning the envelope game, if it's played repeatedly with different amounts of money each time.

The new formula tells you how often you should switch envelopes to make sure you win in the end *(Source: iStockphoto)*

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Switching probability

The formula relates a particular amount of money (y) found in the first envelope to the probability (P) that you should switch envelopes to gain in the long term.

For example, if you open the first envelope and see $10, the formula might tell you that the probability you should switch envelopes is 0.1 - that is once in every 10 games played.

The formula calculates a smaller probability of switching the larger the original amount (y) is.

Using the previous example, if y is $100, the probably might drop to 0.01, or 1 in every 100 games.

Random strategy

What's key to this strategy is that the decision when exactly to switch - in which game - must be random, says McDonnell, who studies random processes in telecommunications and the brain.

McDonnell says their solution is different to those that have gone before because of this random element.

"Our solution is to switch randomly," he says.

"Each time you are offered two envelopes, you observe the amount in one envelope, and then the larger the amount observed, the less likely it is that you should switch, but the choice is still random."

"The key result is that this kind of random switching leads to a long-run financial gain in comparison with either (i) never switching or (ii) switching randomly in a manner that ignores the observed amount in the opened envelope."

Optimising stocks

A related formula developed by McDonnell and Abbott calculates how much a player actually gains as a result of applying the random switching strategy.

In real life, the actual gain will obviously depend on how much money is actually put in the envelopes, says McDonnell.

Abbott says the growth in money seen in the two envelope problem appears to have some similarities to a theory known as "volatility pumping".

"Volatility pumping is a way of switching between poor investments and yet winning an exponentially increasing amount of money," he says.

"It suggests the power of changing your portfolio of stocks periodically, buying low and selling high."

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