October 28, 2013
HUFF POST SCIENCE

Is Mathematics Invented or Discovered?

Posted: 09/10/2013 12:27 pm

Mathematics is the language of science and has enabled mankind to make extraordinary technological advances. There is no question that the logic and order that underpins mathematics, has served us in describing the patterns and structure we find in nature.

The successes that have been achieved, from the mathematics of the cosmos down to electronic devices at the microscale, are significant. Einstein remarked, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?"

Amongst mathematicians and scientists there is no consensus on this fascinating question. The various types of responses to Einstein's conundrum include:

1) Math is innate. The reason mathematics is the natural language of science, is that the universe is underpinned by the same order. The structures of mathematics are intrinsic to nature. Moreover, if the universe disappeared tomorrow, our eternal mathematical truths would still exist. It is up to us to discover mathematics and its workings--this will then assist us in building models that will give us predictive power and understanding of the physical phenomena we seek to control. This rather romantic position is what I loosely call mathematical Platonism.

2) Math is a human construct. The only reason mathematics is admirably suited describing the physical world is that we invented it to do just that. It is a product of the human mind and we make mathematics up as we go along to suit our purposes. If the universe disappeared, there would be no mathematics in the same way that there would be no football, tennis, chess or any other set of rules with relational structures that we contrived. Mathematics is not discovered, it is invented. This is the non-Platonist position.

3) Math is not so successful. Those that marvel at the ubiquity of mathematical applications have perhaps been seduced by an overstatement of their successes. Analytical mathematical equations only approximately describe the real world, and even then only describe a limited subset of all the phenomena around us. We tend to focus on those physical problems for which we find a way to apply mathematics, so overemphasis on these successes is a form of "cherry picking." This is the realist position.

4) Keep calm and carry on. What matters is that mathematics produces results. Save the hot air for philosophers. This is called the "shut up and calculate" position.

The debate over the fundamental nature of mathematics is by no means new, and has raged since the time of the Pythagoreans. Can we use our hindsight now to shed any light on the above four positions?

A recent development within the last century was the discovery of fractals. Beautiful complex patterns, such as the Mandelbrot set, can be generated from simple iterative equations. Mathematical Platonists eagerly point out that elegant fractal patterns are common in nature, and that mathematicians clearly discover rather than invent them. A counterargument is that any set of rules has emergent properties. For example, the rules of chess are clearly a human contrivance, yet they result in a set of elegant and sometimes surprising characteristics. There are infinite numbers of possible iterative equations one can possibly construct, and if we focus on the small subset that result in beautiful fractal patterns we have merely seduced ourselves.

Take the example of infinite monkeys on keyboards. It appears miraculous when an individual monkey types a Shakespeare sonnet. But when we see the whole context, we realize all the monkeys are merely typing gibberish. In a similar way, it is easy to be seduced into thinking that mathematics is miraculously innate if we are overly focused on its successes, without viewing the complete picture.

The non-Platonist view is that, first, all mathematical models are approximations of reality. Second, our models fail, they go through a process of revision, and we invent new mathematics as needed. Analytical mathematical expressions are a product of the human mind, tailored for the mind. Because of our limited brainpower we seek out compact elegant mathematical descriptions to make predictions. Those predictions are not guaranteed to be correct, and experimental verification is always required. What we have witnessed over the past few decades, as transistor sizes have shrunk, is that nice compact mathematical expressions for ultra small transistors are not possible. We could use highly cumbersome equations, but that isn't the point of mathematics. So we resort to computer simulations using empirical models. And this is how much of cutting edge engineering is done these days.

The realist picture is simply an extension of this non-Platonist position, emphasizing that compact analytical mathematical expressions of the physical world around us are not as successful or ubiquitous as we'd like to believe. The picture that consistently emerges is that all mathematical models of the physical world break down at some point. Moreover, the types of problems addressed by elegant mathematical expressions are a rapidly shrinking subset of all the currently emerging scientific questions.

But why does this all matter? The "shut up and calculate" position tells us to not worry about such questions. Our calculations come out the same, no matter what we personally believe; so keep calm and carry on.

I, for one, believe the question is important. My personal story is that I used to be a Platonist. I thought all mathematical forms were *reified* and waiting to be discovered. This meant that I philosophically struggled with taking limits to infinity, for example. I merely got used to it and accepted it under sufferance. During my undergraduate days, I had a moment of enlightenment and converted to non-Platonism. I felt a great burden lift from my shoulders. Whilst this never affected my specific calculations, I believe a non-Platonist position gives us greater freedom of thought. If we accept that mathematics is invented, rather than discovered, we can be more daring, ask deeper questions, and be motivated to create further change.

Remember how irrational numbers petrified the bejesus out of the Pythagoreans? Or the interminable time it took mankind to introduce a zero into arithmetic? Recall the centuries of debate that occurred over whether negative numbers are valid or not? Imagine where science and engineering would be today if this argument was resolved centuries earlier. It is the ravages of Platonist-like thinking that have held back progress. I argue that a non-Platonist position frees us from an intellectual straightjacket and accelerates progress.

More information: Derek Abbott, "The reasonable ineffectiveness of mathematics," *Proceedings of the IEEE,* Vol. 101, No. 10, pp. 2147-2153 , 2013.