

# Paradox in Game Theory: Losing Strategy That Wins

By SANDRA BLAKESLEE

A Spanish physicist has discovered what appears to be a new law of nature that may help explain, among other things, how life arose out of a primordial soup and why investing in losing stocks can sometimes lead to greater capital gains.

Called Parrando's paradox, the law states that two games guaranteed to make a player lose all his money can generate a winning streak if played alternately.

Named after its discoverer, Dr. Juan Parrando, who teaches physics at the Complutense University in Madrid, the newly discovered paradox is inspired by the mechanical properties of ratchets — the familiar saw-tooth tools used in automobile jacks and in self-winding wristwatches.

By translating the properties of a ratchet into game theory, the scientific discipline that seeks to extract rules of nature from the gains and losses observed in games, Dr. Parrando discovered that two losing games could combine to produce winnings.

"The importance of the paradox in real life remains to be seen," said Dr. Charles Doering, a mathematician at the University of Michigan, who is familiar with the research.

"It gives us a new and unexpected view of a variety of phenomena," he said, "and who knows? Sometimes finding that one piece of the puzzle makes the whole picture suddenly

clear," Dr. Doering said.

Dr. Derek Abbott, director of the Center for Biomedical Engineering at the University of Adelaide in Australia, said that many scientists were intrigued by the paradox and had begun applying it to engineering, population dynamics, financial risk and other disciplines.

Dr. Abbott and a colleague at his center, Dr. Gregory Harmer, recently carried out experiments to verify and explain how Parrando's paradox works.

Their research is described in a recent issue of the journal *Nature*.

The paradox is illustrated by two games played with coins weighted on one side so that they will not fall by chance to heads or tails.

In game A, a player tosses a single loaded coin and bets on each throw. The probability of winning is less than half.

In game B, there are two coins and the rules are more complicated. The player tosses either Coin 1, loaded to lose almost all the time or Coin 2 loaded to win more than half the time. He plays Coin 1 if his money is a multiple of a particular whole number, like three.

If his money cannot be divided evenly by that number, he plays Coin 2. In this setup, the second coin will be played more often than the first.

"Sure enough," Dr. Abbott said, when a person plays either game A or game B 100 times, all money taken to the gambling table is lost. But when the games are alternated — playing A twice and B twice for 100 times — money is not lost. It accumulates into big winnings.

Even more surprising, he said, when game A and B are played randomly, with no order in the alternating sequence, winnings also go up and up.

Because these results seem so surprising, the outcome is paradoxical — Parrando's paradox. Switching between the two games seemed to create a ratchet-like effect. With its saw-tooth shape, a ratchet allows

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Unfortunately, these tactics won't work at the casino.

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movement in one direction and blocks it in the other.

"You see ratchets everywhere in life," Dr. Abbott said. "Any child knows that when you shake a bag of mixed nuts, the Brazil nuts rise to the top. This is because smaller nuts block downward movement of larger nuts." This trapping of heavier objects — moving them upward when one would expect them to fall down — is the essence of a ratchet.

The same is true for particles that tend to move randomly within cells but can be captured, or ratcheted, into performing useful work. This is how many proteins and enzymes are designed, Dr. Abbott said.

Sharing an interest in microscopic ratchets, Dr. Abbott and Dr. Parrando met in a coffee shop in Madrid

in 1997 to discuss the phenomenon. They started to wonder what might happen with a so-called flashing ratchet.

First, they imagined two tilted slopes that could be laid on top of each other or held apart. One is smooth and straight, the other saw-toothed.

Particles placed at the top of either slope would fall down to the bottom under the pull of gravity. Particles placed at the bottom of either slope would go nowhere. But if the two slopes were superimposed and alternated or "flashed" back and forth, particles resting at the bottom could be made to move uphill.

Dr. Parrando then translated a flashing ratchet into the language of game theory. Then, he devised the two coin games. Dr. Abbott confirmed their outcomes in recent experiments. Game A is like the smooth slope. The single loaded coin produces steady losses, just like particles sliding straight downhill. Game B is like the saw-tooth slope that can catch objects. Each tooth on a ratchet has two sides, one that goes up and one that goes down. The two coins, one good and one bad, are like two sides of a single saw-tooth. In a computer, the games are played 100 times, mimicking a ratchet with many teeth.

When the games are run in a computer, Dr. Abbott said, capital starts accumulating. Switching the game traps any winnings before new rounds of the game can cause the money to be lost.

Unfortunately, Parrando's para-

dox will not work for the kinds of games played in casinos, Dr. Abbott said. Games A and B must be set up to copy a ratchet, which means they must have some direct interaction. In the experiments carried out by Dr. Abbott, game B depends on the amount of capital being played and game A affects those amounts. They are subtly connected, he said.

Parrando's paradox may help scientists find new ways to separate molecules, design tiny motors and understand games of survival being played at the level of individual genes. Life itself may have been bootstrapped by ratchets, Dr. Abbott said. When simple amino acids were formed by chance, environmental forces would tend to destroy incipient order. Factors that acted like ratchets could prevent that destruction, helping move life along its evolutionary pathways toward greater complexity.

Economists are studying Parrando's paradox to help find the best strategies for managing investments. Dr. Sergei Maslov, a physicist at Brookhaven National Laboratory in Upton, N.Y., recently showed that if an investor simultaneously shared capital between two losing stock portfolios, capital would increase rather than decrease. "It's mind-boggling," Dr. Maslov said. "You can turn two minuses into a plus." But so far, he said, it is too early to apply his model to the real stock market because of its complexity.