

# Winning While Losing: New Strategy Solves 'Two-Envelope' Paradox

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Opening one envelope tells you more than you might think. Random switching based on the value of the envelope can help a player win more money in the long run. Image credit: Wikimedia Commons.

(PhysOrg.com) -- Researchers from Australia have taken a step toward resolving a seemingly simple yet unsolved paradox known as the "two-envelope" problem. They've worked out a new strategy that can enable a player to beat the game in terms of increasing their payoff. The strategy could have applications in optimizing gains in investments and other areas.

Mark McDonnell of the University of South Australia and Derek Abbott of the University of Adelaide have published their results in a recent issue of *Proceedings of the Royal Society A*.

## The Paradox

In the [two-envelope paradox](#), a player must choose between two envelopes, one of which contains twice as much [money](#) as the other. The player can open the envelope they choose, and then they have the option of switching envelopes. The other envelope, of course, has either twice the money or half the money as the first envelope, but the player does not know which.

It may seem that, since a player has a 50-50 chance of choosing either envelope, they have an equal chance of gaining or losing money whether they decide to switch or keep the original envelope. However, probability theory seems to confusingly show that it's always better to switch.

For example, say the first envelope you pick has \$10, so that the other envelope has either \$20 or \$5. Then you can calculate the expected value (i.e. the probability-weighted sum of the possible values) of the second envelope, assuming that each possibility has a 50% chance:  $(0.5 \times \$5) + (0.5 \times \$20) = \$12.50$ . Since \$12.50 is more than \$10, it makes sense to switch. No matter which numbers you use, you always get an expected value for envelope two that is  $5/4$  higher than the value for the original envelope: if  $c$  is the value of the original envelope, the expected value of the second envelope is  $(0.5 \times [0.5c]) + (0.5 \times [2c]) = 5/4c$ . The mathematical difference is determined by the relations between the envelopes' values, but it still doesn't make sense to switch every time, since it could be argued that a player could have started out with the second envelope in the first place - yet still be advised to switch.

Mathematicians have been trying to figure out the problem (or some variation of it) since 1930, though it was not expressed in the two-envelope format until 1988 by Harvard mathematics professor Sandy Zabell. Though several researchers have claimed to have found solutions to the paradox, no consensus has been reached and so the problem is still considered unsolved.

## Randomized Switching

Perhaps, as McDonnell and Abbott suggest, the key to the paradox may occur when the player looks inside the first envelope; knowing this

information breaks the symmetry, since the envelopes are not identical anymore. To demonstrate this idea, the researchers have worked out a formula that can increase a player's chance of picking the envelope with the greater amount of money, if played repeatedly.

The researchers named the new method Cover's strategy, since it originated with a suggestion by Stanford engineering professor Tom Cover during lunch. In the strategy, a player randomly switches envelopes with a probability that depends on the amount of money in the first envelope. The larger the amount, the less likely it is that a player should switch, even without knowing how high or low the values might be (the distribution). Over 20,000 simulations, this strategy increased a player's payoff compared with simple switching. The researchers also found that a deterministic switching strategy - where a player switches whenever the value of the first envelope is smaller than some predetermined threshold - also leads to a gain compared with never switching.

"The apparent paradox arose before because it didn't seem to make sense that opening an envelope and seeing \$10 actually tells you anything, and therefore it seemed strange that your expected value of winning is \$12.50 by switching," Abbott told *PhysOrg.com*. "But we resolve this by explaining it in terms of symmetry breaking. Before the envelopes are opened, the situation is symmetrical, so it doesn't matter if you switch envelopes or not. However, once you open an envelope and use Cover's strategy, you break that symmetry, and then switching envelopes helps you in the long run (with multiple plays of the [game](#))."

The researchers explained that the strategy emerges from recent advances in two-state switching phenomena that are emerging in the fields of physics, engineering, and economics. For example, in stochastic control theory, random switching between two unstable states can result in a stable condition.

"When I had lunch with Tom Cover in 2003 and he suggested that his strategy ought to work, I thought

he was nuts and refused to believe it," Abbott said. "It was that counterintuitive that I thought it was crazy. But I went back to Australia and slowly came around to Cover's viewpoint after careful thought over the years. My expertise in Brownian ratchets was the key to me understanding the physical picture behind it."

As Abbott explained, a Brownian ratchet is a physical device that can organize random particles to flow in a particular direction. "The trick with a Brownian ratchet is that again it uses the idea of breaking symmetry," he said. "It is this idea that is behind the principle of the well-known 'Parrondo's paradox,' which shows that you can mix two losing games and yet win. This solution to the two-envelope problem is a breakthrough in the field of Parrondo's paradox."

### Winning While Losing

Although a player can use the random switching strategy to win money when having prior knowledge of the statistical distribution of the envelopes' values, the significant point is that this knowledge isn't necessary. "What is surprising is that our analysis shows that you can always improve your gain using Cover's method with ignorance of the 'house limit' (the highest value of money allowed) and of the statistical distribution the numbers obey," Abbott said. "That is rather amazing. And the reason it is of importance is that engineers often have to consider what are called 'blind optimization' problems. And so our solution may stimulate new work in this area."

Another type of optimization method that shares similarities with the two-envelope problem is financial investing in the stock market. For instance, in "volatility pumping," switching between poor investments can result in winning an exponentially increasing amount of money.

"Volatility pumping is a 'toy model' that you can't use exactly in its present form on the stock market," Abbott explained. "However, it is a toy model that illustrates underlying mechanisms that are useful. It suggests the power of changing your

portfolio of stocks periodically, buying low and selling high. Both the two-envelope process plus volatility pumping appear closely related to Brownian ratchet phenomena. They both exploit the interaction of asymmetry with randomness.”

This insight also brings with it a number of open questions. For example, when playing a sequence of games, a player could modify the details of the strategy by continually updating the estimated distribution from which the envelopes’ values are chosen. Also, since the strategy relates to two-state switching in other fields, perhaps it may be possible to explain all these phenomena with a common mathematical framework.

More information: Mark D. McDonnell and Derek Abbott. “Randomized switching in the two-envelope problem.” [Proceedings of the Royal Society A](#).  
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