OPTICAL COMMUNICATIONS
SUMMARY OF SOME USEFUL FORMULAE

General

$$\Delta(\tau/L)_{\mathrm{dis}} = -(M + M_g)\Delta\lambda$$

$$f_{3dB}(\mathrm{opt.}) \times L = \frac{0.5}{\Delta(\tau/L)}$$

$$f_{3dB}(\mathrm{elec.}) \times L = \frac{0.35}{\Delta(\tau/L)}$$

$$R_{RZ} \times L = \frac{0.35}{\Delta(\tau/L)}$$

$$R_{NRZ} \times L = \frac{0.7}{\Delta(\tau/L)}$$

Resonant Cavities

$$\Delta f_c = \frac{c}{2Ln}$$

Optics

$$R(\text{normal incidence}) = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Optical Fibres

Step Index:

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\Delta(\tau/L)_{\mathrm{mod}} = \frac{n_1\Delta}{c}$$

$$\text{Single mode:} \quad a/\lambda_0 < \frac{2.405}{2\pi.\mathrm{NA}}$$

Graded Index (Parabolic Profile):

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$n(r) = n_1\sqrt{1 - 2(r/a)^2\Delta} \text{ for } r \leq a$$

$$\mathrm{NA} = n_1\sqrt{2\Delta}\sqrt{1 - (r/a)^2}$$

$$\Delta(\tau/L)_{\mathrm{mod}} = \frac{n_1\Delta^2}{2c}$$

$$n_{\mathrm{eff}} = n_1 - (p + q + 1)\frac{\sqrt{2\Delta}}{k_0 a}$$

Source Bandwidth

$$\begin{aligned} P_{SP} &= \frac{a_1 I_{SP}}{\sqrt{1+\omega^2\tau^2}} \\ f_{3dB} &= \frac{1}{2\pi\tau} \\ t_r &= \frac{0.35}{f_{3dB}} \end{aligned}$$

Source Coupling

For a source with pattern $\cos^m\theta$:

$$\begin{aligned} \eta_{\mathrm{SI}} &= 1-\left[1-(\mathrm{NA})^2\right]^{(m+1)/2} \text{ to SI Fibre} \\ \eta_{\mathrm{GRIN}} &= \eta_{\mathrm{SI}}\left\{1-0.5\left(\frac{a_e}{a_f}\right)^2\right\} \text{ to GRIN Fibre where } a_e \leq a_f \end{aligned}$$

Optical Detectors

$$\begin{aligned} \rho &= i/P = \frac{M\eta e\lambda}{hc} \\ f_{3dB} &= \frac{0.35}{t_r} \\ t_r &= 2.19 R_L C_d \end{aligned}$$

Analogue Modulation

AM/IM:

$$\begin{aligned} i &= I_0 + I_s(1+m\cos\omega_m t)\cos\omega_{sc}t \\ P &= P_0 + P_s(1+m\cos\omega_m t)\cos\omega_{sc}t \end{aligned}$$

FM/IM:

$$\begin{aligned} i &= I_0 + I_s\cos(\omega_{sc}t+\beta\sin\omega_m t) \\ P &= P_0 + P_s\cos(\omega_{sc}t+\beta\sin\omega_m t) \\ B &= \frac{\omega_m}{\pi}(1+\beta) \end{aligned}$$

LED Operating Characteristics

$$\begin{aligned} \lambda_0 &= \frac{1.24}{W_g(\mathrm{eV})} \\ P &= \eta i W_g(\mathrm{eV}) \\ P_s &= \frac{a_1 I_s}{\sqrt{1+\omega^2\tau^2}} \end{aligned}$$

Harmonic Distortion

$$
\begin{aligned}
P &= P_{dc} + a_1 i_s + a_2 i_s^2 + \ldots \\
\text{THD} &= 0.25 \left(\frac{a_2 I}{a_1} \right)^2 \quad \text{for } i_s = I \sin \omega t
\end{aligned}
$$

Receiver Design

General Signal to Noise Ratio:

$$
S/N = \frac{(m^2/2)(M\rho P)^2 R_L}{M^n 2 e R_L \Delta f (I_D + \rho P) + 4kT\Delta f F}
$$

Thermal noise limited probability of error:

$$
\begin{aligned}
P_e &= \frac{1}{2} - \frac{1}{2} \text{erf} \left(0.354 \sqrt{S/N} \right) \\
\text{erf}(x) &\approx 1 - \frac{e^{-x^2}}{\sqrt{\pi} x} \quad \text{for } x \geq 3
\end{aligned}
$$

Peak S/N (with threshold set at $0.5 i_s$) for specified bit error rates.

P_s	S/N
10^{-4}	17.4dB
10^{-6}	19.5dB
10^{-9}	21.5dB

Quantum noise limited probability of error:

$$
\begin{aligned}
P_e &= e^{-n_s} \\
n_s &= \frac{\eta P \tau}{hf}
\end{aligned}
$$

Heterodyne Receivers

$$
\begin{aligned}
i_{\text{dc}} &= \frac{\eta e}{hf}(P_L + P_S) \\
i_{\text{IF}} &= \frac{2\eta e}{hf} \sqrt{P_L P_S} \cos \left[\omega_{\text{IF}} t - \theta(t) \right] \\
S/N &= \frac{2 \left(\eta e / hf \right)^2 R_L P_S P_L}{2 e R_L \Delta f \left[I_D + (\eta e P_L / hf)(1 + P_S/P_L) \right] + 4kT\Delta f}
\end{aligned}
$$

Some Physical Constants

Speed of Light in Free Space	$c = 3 \times 10^8$ m/s
Boltzmann's Constant	$k = 1.38 \times 10^{-23}$ J/K
Planck's Constant	$h = 6.626 \times 10^{-34}$ Js
Electron Charge	$e = 1.6 \times 10^{-19}$ C