Example Exam Question:

Consider a heterodyne receiver for a digital optical fibre communications system.

(a) Briefly explain how a heterodyne receiver detects phase, given that photodetectors can only detect amplitude or optical power.

(2 marks)

(b) What type of modulation scheme can a heterodyne system permit that is not otherwise achievable with direct detection using a standard receiver?

(1 mark)

(c) State the key advantage of phase detection over amplitude detection.

(1 mark)

(d) Heterodyne receivers offer increased sensitivity. Briefly state why.

(1 mark)

(e) Using a heterodyne receiver, compute the local-oscillator (LO) power required to make the SNR 1 dB *less* that the quantum limit. You may assume the IF bandwidth is 500 MHz and the received optic power is constant at 5 nW when a binary "1" is received. The dark current of the photodetector is $I_D = 2$ nA, and its responsivity is $\rho = 0.5$ A/W. Assume the temperature is 27ºC and a load resistance of 100 Ω.

(11 marks)

(f) If this were *not* a heterodyne system, then the receiver's bandwidth could be as small as 250 MHz. For this case determine the signal power required to achieve a SNR equal to that in part (e). (4 marks)

Model solution:

Consider a heterodyne receiver for a digital optical fibre communications system.

(a) Briefly explain how a heterodyne receiver detects phase, given that photodetectors can only detect amplitude or optical power.

A heterodyne system detects phase with an amplitude detector via mixing the signal with that of a local oscillator (LO) to produce beats. Thus phase changes in the optical carrier are converted to changes in optical intensity, which can then be measured by a photodetector. (2 marks)

(b) What type of modulation scheme can a heterodyne system permit that is not otherwise achievable with direct detection using a standard receiver?

Heterodyning makes optical FM detection possible. (1 mark)

(c) State the key advantage of phase detection over amplitude detection.

Phase detection is less prone to errors due to non-linearities. (1 mark)

(d) Heterodyne receivers offer increased sensitivity. Briefly state why.

The IF signal is proportional to local oscillator power. Thus, in effect, the LO acts as a signal amplifier, increasing sensitivity of the receiver. (1 mark)

(e) Using a heterodyne receiver, compute the local-oscillator (LO) power required to make the SNR 1 dB *less* than the quantum limit. You may assume the IF bandwidth is 500 MHz and the received optic power is constant at 5 nW when a binary "1" is received. The dark current of the photodetector is I_D = 2 nA, and its responsivity is ρ = 0.5 A/W. Assume the temperature is 27ºC and a load resistance of 100 Ω.

Since $\rho = \eta e/hf$ we can write the quantum limited SNR of the heterodyne receiver as,

$$\text{SNR} = \frac{2\rho^2 P_S P_L}{2e\Delta f[I_D + \rho(P_L + P_S)]}$$ (2 marks)

which for large local oscillator power $(P_L >> P_S)$ and negligible dark current yields a quantum limited SNR of

$$\text{SNR} = \frac{\rho P_S}{e\Delta f} = \frac{0.5 \times 5 \times 10^{-9}}{1.6 \times 10^{-19} \times 500 \times 10^6} = \frac{50}{1.6} = 31.25 = 15\text{ dB}$$ (2 marks)

The question wants an SNR 1 dB *less* than this limiting SNR, thus we require an SNR of 15 dB - 1 dB = 14 dB. In linear units, this gives an $\text{SNR} = 10^{14/10} = 25.$ (1 mark)

To calculate the LO power needed to give us an SNR of 25, we can write down a simplified expression for total SNR (containing thermal plus shot noise) with a simplification assuming $P_L >> P_S$ and that the dark current, I_D, is negligible. This gives:

$$\text{SNR} = \frac{2\rho^2 P_S P_L R_L}{2eR_L \Delta f \rho P_L + 4kT\Delta f}. \qquad \text{(2 marks)}$$

Rearranging to obtain P_L, gives:

$$P_L = \frac{4kT\Delta f \times \text{SNR}}{2\rho^2 P_S R_L - 2eR_L \Delta f \rho \times \text{SNR}}. \qquad \text{(1 mark)}$$

Putting as the temperature is 27ºC, we put $T = 300$ K, giving:

$$P_L = \frac{4 \times 1.38 \times 10^{-23} \times 300 \times 500 \times 10^6 \times 25}{2(0.5)^2 \times 5 \times 10^{-9} \times 100 - 2 \times 1.6 \times 10^{-19} \times 100 \times 500 \times 10^6 \times 0.5 \times 25}$$

$$\therefore P_L = 4.1 \text{ mW} \qquad \text{(1 mark)}$$

Our assumption that $P_L >> P_S$ is confirmed, as 4.1 mW >> 5 nW. (1 mark)

Also our assumption that dark current can be neglected is justified because 5 nA << ρP_L = 2 mW. (1 mark)

(f) If this were *not* a heterodyne system, then the receiver's bandwidth could be as small as 250 MHz. For this case determine the signal power required to achieve a SNR equal to that in part (e).

For a direct detection the SNR is:

$$\text{SNR} = \frac{(\rho P_S)^2 R_L}{2eR_L \Delta f (I_D + \rho P_S) + 4kT\Delta f}$$

Neglecting dark current and rearranging to get signal power gives a quadratic in P_S:

$$\rho^2 R_L P_S^2 - \text{SNR} \times 2eR_L \Delta f \rho P_S - \text{SNR} \times 4kT\Delta f = 0 \qquad \text{(1 mark)}$$

The three quadratic coefficients come to:

$$a = \rho^2 R_L = 0.5^2 \times 100 = 25$$

$$b = \text{SNR} \times 2eR_L \Delta f \rho = 25 \times 2 \times 1.6 \times 10^{-19} \times 100 \times 250 \times 10^6 \times 0.5 = 10^{-8}$$

$$c = \text{SNR} \times 4kT\Delta f = 25 \times 4 \times 1.38 \times 10^{-23} \times 300 \times 250 \times 10^6 = 10^{-10}\,\text{W}$$

The second coefficient is negligible and taking just the positive solution of the quadratic (as negative power has no meaning in this context), gives a source power of,

$$P_S = \frac{\sqrt{-4ac}}{2a} = \frac{\sqrt{4 \times 25 \times 10^{-10}}}{2 \times 25} = \frac{10^{-4}}{50} = \boxed{2\,\mu\text{W}}$$ (2 marks)

Our assumption that dark current can be neglected is justified because 5 nA << $\rho P_S = 1$ μW. (1 marks)

Note that $b \times P_S = 10^{-8} \times 2 \times 10^{-6} = 2 \times 10^{-14}\,\text{W}$ is the shot noise power and this is considerably lower than the thermal noise which is of the order of $c = 10^{-10}$ W. Thus the system is thermal noise limited, whereas the heterodyne system in part (e) was shot (quantum) noise limited. This makes sense because the LO power in part (e) is about 2000 times larger, thereby having many more photons to create shot noise. (3 bonus marks)