

## FOCUS ISSUE: Unsolved Problems of Noise and Fluctuations

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### Overview: Unsolved problems of noise and fluctuations

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Noise and fluctuations are at the seat of all physical phenomena. It is well known that, in linear systems, noise plays a destructive role. However, an emerging paradigm for nonlinear systems is that noise can play a constructive role—in some cases information transfer can be optimized at nonzero noise levels. Another use of noise is that its measured characteristics can tell us useful information about the system itself. Problems associated with fluctuations have been studied since 1826 and this Focus Issue brings together a collection of articles that highlight some of the emerging hot unsolved noise problems to point the way for future research. © 2001 American Institute of Physics. [DOI: 10.1063/1.1398543]

**The study of fluctuations crosses many discipline boundaries as “noise” is a ubiquitous phenomenon. Noise is traditionally thought of as an unwanted effect that degrades the performance of a system. However, the emerging paradigm now recognizes that noise can play both a destructive or constructive role, depending on the circumstances. Thus there is now an intense interest in noise in many biological, physical, and other systems. The collection of papers in this Focus Issue examines open problems and debates surrounding the role of noise in both chaotic and nonchaotic systems. In this Overview we introduce the concepts of stochastic resonance, Brownian ratchets,  $1/f$  noise and vacuum fluctuations, and some problems surrounding them.**

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#### I. INTRODUCTION

This issue contains a selection of augmented papers from the Unsolved Problems of Noise and fluctuations (UPoN'99) conference<sup>1,2</sup> held in Adelaide, Australia, 1999. In this 3-yearly conference series, UPoN'96 was held in Szeged, Hungary, and UPoN'02 will be held in the USA.

The roots of noise research, of course, trace back to the Scottish botanist Robert Brown who carried out his famous experiments, observing fluctuating pollen on the surface of a film of water. It is appropriate that UPoN'99 was held in Australia, as this is the country that inspired the very first major unsolved problem of noise. In 1822, Brown traveled to Australia on a voyage with Captain Philip King.<sup>3</sup> Brown was so fascinated by the Australian countryside and unusual plant life that he was inspired to take a deeper look at Australian pollen under a microscope.<sup>3,4</sup> However, on close inspection he found that pollen fluctuated and he became distracted from his main field of expertise to find out why.

Brown was, in fact, not the first person to observe such

motion—in Brown's own writings he acknowledges, for example, that an Italian Jesuit, Lazzaro Spallanzani (1729–1799), and an English–Belgian clergyman, John Turberville Needham (1713–1781), had observed strange random fluctuations before him.<sup>4,5</sup> However, Brown's predecessors incorrectly interpreted the motion, clouded by the ongoing debate on vitalism and spontaneous generation.

Brownian motion is named to give Brown the credit for questioning this position and performing systematic experiments to try to establish the cause of the fluctuations. He ushered in the age of noise research. The first unsolved noise problem was to find out the origin of Brownian motion. Brown died without finding the answer to that question, but he did establish that the motion was not due to bubbles, living organisms, release of matter, etc. The problem of the origin of Brownian motion was perhaps the biggest problem the field has ever seen, as it took over 80 years to fully solve it!

A partial breakthrough was achieved when in 1877 a Belgian priest, Joseph Delsaulx (1828–1891), suggested for the first time the impact of liquid molecules on the fluctuating particles based on a thermodynamic argument.<sup>6</sup> However, there was a long way to go before this could be established with rigor and it was not until around 1905, through the work of both Smoluchowski and Einstein, that the problem was finally settled.

Another milestone was in 1912 when electrons were first considered as Brownian particles,<sup>7</sup> by the Dutch physicist Geertruida de Haas-Lorentz (1885–1973)—the first woman in noise theory, and daughter of H. A. Lorentz. The next crucial unsolved problem was posed by the Dutch physicists Moll and Burger, who in 1925 found they could not indefinitely cascade galvanometers, to increase amplification, because the needle wildly fluctuated.<sup>8</sup> The answer came swiftly in 1926 when the Swedish physicist, Gustav Adolf Ising (1883–1960), correctly explained this<sup>9</sup> in terms of Brownian motion after being inspired by the work of de Haas-Lorentz.

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Then two Swedish immigrants to the US, J. B. Johnson and H. Nyquist, made the next vital step. Johnson began his measurements of the thermal noise in various conductors using a vacuum tube amplifier, in 1925, and published his well-known formula for thermal voltage noise in the 1927–1928 period. When Johnson showed the formula to his friend, Nyquist, it then took only a month to solve the problem of how to theoretically derive it.<sup>10</sup>

The golden age of noise began and many talented workers contributed to the mathematical formalisms we use today such as Wiener, Levy, Ornstein and Uhlenbeck, Ito, and others. Let us now fast forward to the present. What is interesting is that just 20–30 years ago we all thought of noise as a nuisance or a nonideality of a system that we must remove. However, the reader will notice that most of the papers in this issue are about the constructive role of noise: how noise actually induces some kind of ordering in a system or how noise can tell us something useful about a system. This is an exciting new paradigm shift that has been gradually happening over the last two decades and brings with it a new set of unsolved problems for future research.

What are some examples where noise can be useful? It is acknowledged that noise is useful in breaking up the quantization pattern in a video signal,<sup>11</sup> in the random dithering of analog-to-digital converters,<sup>12</sup> in the area of Brownian ratchets,<sup>13</sup> and in the physics of granular flow.<sup>14–16</sup> Also it is known that when training a neural network, adding noise to the training data set can improve *network generalization*, i.e., the neural network's ability to extrapolate outside of the initial training data set.<sup>17</sup> Noise even plays a role in game theory<sup>18</sup> and number theory.<sup>19,20</sup> Also in a range of optimization algorithms, a little noise can be a good thing so that the procedure does not get falsely “stuck” in some local minimum. There are also everyday examples, such as the picking, placement, and fitting of objects together that can sometimes benefit from a little random “jiggling”—perhaps there is a lesson here for the design of control algorithms in industrial robotic arms. Noise in biology is now a fast growing area with implications for DNA expression,<sup>21,22</sup> molecular motors,<sup>23</sup> and ion channel measurement to name just a few. A recent noise study on ion channels has helped to discover crucial details about how malaria parasites feed off red blood cells<sup>24</sup>—perhaps it will soon be possible to claim that “noise research saves lives.”

With the recent surge in interest in using the tools of noise analysis and statistics to analyze stock market fluctuations, DNA data and an increasing number of bioinformatics-related problems, another paradigm shift can be observed. In these areas we are seeing a shift away from the traditional way of doing physics: hypothesis and experiment. We are now seeing a new step added to this loop, namely, collecting huge amounts of data and analyzing, pattern searching and sifting for hypotheses that can be statistically tested on the data. This somewhat new style of doing things explains the recent increase in “strange” journal articles that revolve around analyzing large data sets. This shift in the way we are approaching some important modern problems will no doubt produce a new set of unsolved problems.

I will now briefly introduce two important constructive-

noise paradigms: “stochastic resonance” and “Brownian ratchets,” which are both producing a flurry of research, a rich set of problems, and promise of useful applications.

## II. STOCHASTIC RESONANCE

To briefly explain what stochastic resonance (SR) is, consider this amusing analogy: my office. The problem is that when my office gets too messy I can not find anything. Yet when I have the occasional tidy-up and my office is absolutely pristine, I can not find anything either. However, there is an optimum amount of disorder in my office where I can extract information at the maximum rate—that is stochastic resonance! For a very long review of the topic refer to, for example, Gammaitoni *et al.*<sup>25</sup> and for a very short review see Wiesenfeld and Jaramillo.<sup>26</sup>

The term “stochastic resonance,” was coined in 1980 when Benzi first mooted the idea to explain the approximately 100 000 year periodicity of the earth's ice ages.<sup>27</sup> In this example the occurrence of an ice age is the “signal” or information of interest. A weak periodic eccentricity in the Earth's orbit leads to more drastic weather changes than expected due to nonlinear coupling with environmental fluctuations.<sup>28–30</sup>

In electronic systems SR was first demonstrated with a noise driven circuit using a Schmidt trigger<sup>31</sup>—other studies have used tunnel diodes<sup>32</sup> and, more simply, operational amplifiers.<sup>33</sup> SR has also been demonstrated in a wide range of physical and biological systems<sup>25,26</sup> and even social systems.<sup>34</sup>

One of the unsolved problems in SR has been to pin it down to the simplest set of underlying principles. However, in recent years this has been rather like trying to shoot a moving target. At first, SR was thought to be a property of bistable systems, then it was observed in a simple thresholding element,<sup>35</sup> and it is now shown that not even a threshold operation is required.<sup>36</sup> Even more recent viewpoints generalize SR as a nonlinear filtering phenomenon<sup>37</sup> or a natural feature of any nonlinear system with a large, strongly noise-dependent susceptibility.<sup>1</sup> Recent results demonstrate that, surprisingly, these features are not at all necessary for SR to occur.<sup>38,39</sup> A rather cute “rain drop collector” model<sup>39</sup> displays SR for an oscillating aperture—a key feature is that the geometry of the aperture has an asymmetry. The interaction of noise with an asymmetry is certainly a unifying theme that will pop up again in Sec. III (on Brownian ratchets). By reducing SR to its statistical essence, this work shows that noise-facilitated signal transduction is a far more general statistical property of nonlinear systems than previously believed. Perhaps SR can generally be viewed as class of interaction between disorder and asymmetry.

Another unsolved problem of SR has been to find suitable metrics to quantify it. At first, SR was studied with simple periodic signals. However, the extension of SR to general aperiodic signals has led to “aperiodic stochastic resonance,” which is problematic, as simple measures such as signal-to-noise ratio are now not always useful. Also it would be a mistake to exploit familiar metrics found in engineering texts that assume an underlying *linear* system, ex-

cept for sufficiently small signals. Consequently, the concept of information entropy is now becoming widely used as a preferred class of measure.<sup>40–43</sup> A limitation of this approach is that there is no sense of information bandwidth or throughput—to address this shortcoming, Kish *et al.* have recently exploited Shannon channel capacity as the preferred metric and have demonstrated this in a thresholding element.<sup>44</sup> The central result for a single threshold element gives the channel capacity as

$$C = \frac{B_n}{2\sqrt{3}} \exp\left(\frac{-U_t^2}{2B_n S_n}\right) \log_2\left(1 + \frac{(2AU_t)^2}{(B_n S_n)^2}\right), \quad (1)$$

where  $B_n$  is the bandwidth of the input noise,  $U_t$  is the threshold potential,  $S_n$  is the input noise PSD, and  $A$  is the rms amplitude of the input signal. To understand importance of this new SR metric consider, again, the amusing example of my office. Remember I said there is an optimum amount of disorder in my office, where I can extract information at the maximum *rate*. SR based on extracting maximum information would be useless if it took me forever to search my office. Hence for real SR applications, the concept of information rate is the practical metric.

The significance of the vast literature on SR in threshold elements is that the threshold is the major nonlinear operation carried out by a neuron. It turns out that sensory neurons are very noisy, having signal-to-noise ratios (SNRs) of the order of 0 dB.<sup>40,45</sup> One of the holy grails of neural research is to answer the following open questions: (i) why are they so noisy, (ii) do they need to be that noisy, (iii) is the noise helpful (i.e., in what way does SR really play?), and (iv) is that much noise really that useful? These are major unsolved problems. To put 0 dB into context, take the example of a man-made hi-fi system with 80 dB performance—this means that an individual sensory neuron is  $10^8$  times noisier! Why? Answering these questions will not only give us a deeper understanding of neural coding but will almost certainly lead to the design of novel signal processing systems. Obviously the only way that level of noise can be tenable is if many neurons are parallelized—this is an important point for the future of SR research, and I will return to this point shortly.

The next major unsolved problem of SR is in finding an engineering application for it. Many workers have established that SR is used by nature and biological systems, but to date there are no clearly reported engineering applications. To understand this issue, consider a simple threshold element: By adding noise to a subthreshold signal, the output is enhanced. That is SR. However, in a real system the engineer has control over the threshold and can optimally adjust it—it is far better to adjust the threshold than to add noise, if that degree of freedom is available.<sup>33</sup> Classical SR is therefore a nonoptimal method of signal detection—that is, if one is allowed to adjust the threshold to maximize information flow then, after this optimization has been done, SR no longer occurs. This is because maximum information is achieved for suprathreshold signal levels—of course classical SR only occurs for subthreshold signals. Furthermore, if one does use subthreshold signals and SR, then the transmitted information is significantly reduced below what can be achieved for

suprathreshold signals. Because of these considerations, the use of noise to enhance information has to be seen as a compromise—it will enhance subthreshold signals but reduce the information transmitted about suprathreshold signals. There are some applications, such as in sonar, where the degree of freedom to set the threshold is restricted by a trade-off between sensitivity and false-alarm rate—an open question is to establish if classical SR can be utilized in these cases or not.

Fortunately, a new form of SR called suprathreshold stochastic resonance (SSR) has just entered the field<sup>42</sup> and looks to be a most promising solution to some of the aforementioned problems. Inspired by the neuron's ability to cope with large amounts of noise using parallelism, SSR can be demonstrated, for example, in a parallel array of threshold elements.<sup>42</sup> In a recent study on motion detector sensor models, which are inspired by visual sensory neurons, it was found that conventional SR does not enhance single detector performance (in an engineering scenario with access to threshold adjustment) but parallelism does indeed lead to improvement.<sup>46</sup>

Bearing in mind that SSR can only be observed in arrays and not single devices, the advantages are as follows. First, SSR can occur for any size of signal—therefore, in principle, noise can always be used to maximize information—however the actual amount of noise required to achieve this will depend on the signal intensity.<sup>47</sup> Consequently, noise is not interpreted as a compromise—it is of generic benefit. Second, when detecting weak signals that are comparable in size to the internal noise, SSR appears to give a near optimal method of signal encoding<sup>42</sup> and SSR, in the context of parallel thresholding elements, performs better than a conventional analog-to-digital converter—an open question is to formally show if a network configured to maximize the SSR effect is the optimal encoding system when the input SNR is less than 0 dB.<sup>48</sup> Third, SSR can achieve up to 50% of the theoretical capacity calculated in the *absence* of noise—this appears to be much better than what classical SR can do.<sup>49</sup> It is this degree of optimality that may well be relevant to neurophysiological systems<sup>50</sup>—it is known that sensory neurons have SNRs of about 0 dB—an exciting open question is to ask if they make use of SSR to enhance information transmission.

### III. BROWNIAN RATCHETS

The roots of the Brownian ratchet trace back to about 90–95 years ago. A number of physicists of the day debated a simple thought experiment involving a randomly driven ratchet wheel, rather like that shown in Fig. 1.

Imagine that all the moving parts in Fig. 1 are so small that the random bombardment of air molecules is significant at that scale. The vane, on the right, will be randomly bombarded and will turn the shaft. However, the ratchet and pawl is designed to only allow motion in one direction. So it seems that it should be possible to lift up a small weight attached to a pulley on the shaft. Thus performing useful work is apparently possible via rectification of random fluc-

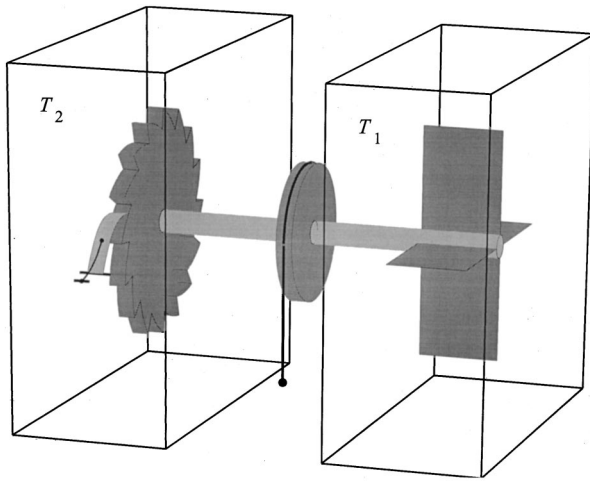


FIG. 1. The ratchet and pawl system. The moving parts are imagined to be so small that random bombardments of air molecules take effect at this scale. The ratchet wheel is connected to a vane, via a shaft. The ratchet rotates preferentially in one direction, as a small latch called the “pawl” blocks the wheel from rotating in the opposite direction. The vane is randomly bombarded and drives the ratchet wheel. A pulley on the shaft winds up a small weight. After Ref. 51.

tuations. Physicists about 100 years ago debated whether this was a violation of thermodynamics or not.

The definitive answer was due to Smoluchowski,<sup>52</sup> who explained that provided  $T_1 > T_2$  the system is perfectly legal and functions as a heat engine—input energy is consumed to maintain the temperature difference and part of this is converted to useful work in lifting the weight. No problem. Now the really interesting question is to ask what actually happens when  $T_1 = T_2$ . In this case there is no input energy into the system and so we are not allowed to lift the weight—however, molecules still bombard the vane and the ratchet wheel still operates, so what is the microscopic description of what happens? Again Smoluchowski brilliantly supplied the correct answer to this apparent paradox. He explained that the spring-loaded pawl will fluctuate and the energy of these fluctuations is of the order of  $kT_1 = kT_2$ . Occasionally, a fluctuation will be large enough that the pawl releases the ratchet wheel to rotate in the wrong direction. So that thermodynamics is preserved, at thermal equilibrium ( $T_1 = T_2$ ), the probability that the wheel rotates counterclockwise (CCW) must equal the probability that the wheel rotates clockwise (CW). This condition is technically called *detailed balance* and ensures that, although the weight jiggles up and down, there is no net work on lifting the weight. Thus there are no violations of physics here.

However, although Smoluchowski’s physical insight was correct, his argument was incomplete, as he did not attempt to calculate the probabilities of CW and CCW rotation to explicitly demonstrate detailed balance. The next milestone occurred in 1963 when Feynman wrote down an explicit probability balance and calculated the efficiency of the system as a thermal engine.<sup>53</sup>

Feynman’s work was the source of inspiration for linear spatial varieties of the Brownian ratchet—this can be visualized as a sawtooth energy potential profile that biases Brownian particles to have preferential motion in one par-

ticular direction.<sup>54–58</sup> Applications from particle separation to DNA sorting are exciting possibilities.<sup>59</sup>

Curiously, Feynman’s work was influential in inspiring these rich set of ideas, despite the fact that his treatment was flawed. In his calculations he incorrectly made the quasistatic assumption, thus arriving at the incorrect expression for efficiency of the ratchet engine.<sup>60</sup> It also turns out that his expressions for detailed balance are problematic<sup>61</sup>—based on Boltzmann statistics, it remains an unsolved problem to carry out Feynman’s analysis from first principles. Only by abandoning his approach and using a level crossing analysis,<sup>61</sup> assuming transition state theory,<sup>62</sup> have we been able to successfully demonstrate detailed balance. To understand why Feynman’s approach was abandoned, consider the following two cases: CW rotation and CCW rotation.

**Case 1: CW rotation.** Let the required energy threshold for the ratchet wheel to rotate one notch passed the pawl be  $\epsilon$ . In general we can say that  $\epsilon = \epsilon_r + \epsilon_p$ , where  $\epsilon_r$  is supplied by the ratchet wheel fluctuation trying to move passed the pawl, and  $\epsilon_p$  is supplied by the pawl fluctuation trying to (partially) disengage. Now the probability of attaining  $\epsilon_r$  is  $e^{-\epsilon_r/kT}$  and attaining  $\epsilon_p$  is  $e^{-\epsilon_p/kT}$ . But note that when the ratchet wheel gets a “kick” of energy equal to  $\epsilon_r$  there is a 50% chance that the kick would be in the CW direction. Similarly, the pawl can fluctuate upwards (to escape the ratchet teeth) or downwards (to dig into the ratchet teeth) and the chance of attaining  $\epsilon_p$  in the upwards direction will be  $\frac{1}{2}e^{-\epsilon_p/kT}$ . Therefore, the probability of CW rotation is

$$P(\text{CW}) = \frac{1}{2}e^{-\epsilon_p/kT} \frac{1}{2}e^{-\epsilon_r/kT} = \frac{1}{4}e^{-(\epsilon_p + \epsilon_r)/kT} = \frac{1}{4}e^{-\epsilon/kT}. \quad (2)$$

**Case 2: CCW rotation.** In this case, we require an energy  $\epsilon$  from the pawl alone to disengage from the ratchet wheel. When the pawl is disengaged, there is a 50% chance that the ratchet wheel will rotate in the CCW direction. Hence,  $P(\text{CCW}) = \frac{1}{2}e^{-\epsilon/kT} \frac{1}{2} = \frac{1}{4}e^{-\epsilon/kT}$ .

Therefore,  $P(\text{CW}) = P(\text{CCW})$  and we have detailed balance. But do we? When calculating  $P(\text{CW})$ , we ignored the case when  $\epsilon_p$  acts in the direction to dig the pawl deeper into the ratchet teeth—in this case the ratchet must attain  $\epsilon_p + \epsilon$  for CW rotation. However, if we alter the probabilities to reflect this, we apparently lose detailed balance.

An even more tricky problem is as follows. Now,

$$P(E_p > \epsilon_p) = e^{-\epsilon_p/kT}, \quad P(E_r > \epsilon_r) = e^{-\epsilon_r/kT},$$

therefore the pdfs are

$$p(E_p) = \frac{1}{kT} e^{-E_p/kT}, \quad p(E_r) = \frac{1}{kT} e^{-E_r/kT}.$$

So if  $E = E_p + E_r$ , then  $p(E) = p(E_p) \otimes p(E_r)$  or

$$\begin{aligned} p(E) &= \int_0^E \frac{1}{kT} e^{-E_p/kT} \frac{1}{kT} e^{-(E-E_p)/kT} dE_p \\ &= \frac{1}{(kT)^2} E e^{-E/kT} \end{aligned}$$

and so

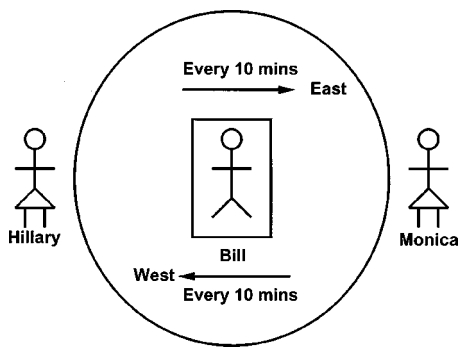


FIG. 2. The “two-girlfriend” paradox. This is an example of a Brownian ratchet in the time domain. The circle represents a city and the box is a station at the center of the city. A train leaves every 10 min to the East and a train leaves every 10 min to the West. Bill arrives at the station at a random time every morning and takes whichever train is there first. However on average he ends up seeing one girl 9 times more than the other.

$$P(E > \epsilon) = \int_{\epsilon}^{\infty} \frac{1}{(kT)^2} E e^{-E/kT} dE = (1 + \epsilon/kT) e^{-\epsilon/kT}. \quad (3)$$

For CW rotation, the requirement is for  $\epsilon_r + \epsilon_p > \epsilon$ . Hence we have that  $P(\text{CW})$  is always unequal to  $P(\text{CCW})$ , which is clearly not allowed, in order to prevent a *perpetuum mobile*. The unsolved problem is to find the flaw in this and find the correct approach. One possibility is to say the above-mentioned analysis assumes  $E_r$  and  $E_p$  are independent—however for a coupling between  $E_r$  and  $E_p$  we would expect the prefactor in Eq. (3) to become even more complicated. It is not obvious how to alter the assumptions to make the equations balance. A possible solution may be to look more deeply into Feynman’s hidden assumptions in linking pawl and ratchet states with energy-based probabilities.

The Brownian ratchet is an interesting example of a system where ordering can be induced by noise. The essential feature is a system with an asymmetry, where energy is inputted into the system, which is exchanged for order, via random jiggling. There are some simple everyday examples of this such as longshore drift on a beach. There is also the famous “Brazil nut problem”—on shaking a bag of mixed nuts, the Brazil nuts (the biggest ones) rise to the top<sup>14</sup> against your expectation. In a sense that is a kind of ordering effect due to fluctuations. What is interesting is to ask where the asymmetry in this system lies. In the classical Brownian ratchet the asymmetry is in a spatial dimension, such as in a sawtooth energy potential profile, whereas with the nut problem one of the sources of asymmetry is in the field itself—because gravitation has a direction.

Another interesting form of Brownian ratchet is an emerging idea in game theory that you can combine two losing strategies and get a winning strategy. This is the concept of a discrete-time Brownian ratchet<sup>63</sup> and will be discussed in detail in this Focus Issue.

To illustrate another type of Brownian ratchet, I will introduce an excellent “old chestnut” brainteaser that has been around a few decades<sup>64</sup>—I shall modernize it and explain this useful problem in detail, as it has been largely forgotten. The puzzle goes like this. Bill has two girlfriends—one lives

TABLE I. Solution to the “two-girlfriend” paradox. This is an example segment of a train time table. Looking at the first two columns, if Bill arrives in the 1 min slot between 12:00 and 12:01 the Eastbound train will appear first. In the next 9 min slot the Westbound train will appear first. His bias will therefore be to the West. This bias can be reversed to the East by considering the different timetable in the second two columns.

| Westbound | Eastbound | Westbound | Eastbound |
|-----------|-----------|-----------|-----------|
| 12:00     | 12:01     | 12:00     | 11:59     |
| 12:10     | 12:11     | 12:10     | 12:09     |
| 12:20     | 12:21     | 12:20     | 11:19     |
| 12:30     | 12:31     | 12:30     | 12:29     |
| 12:40     | 12:41     | 12:40     | 12:39     |

in the East and another lives in the West of a city, as shown in Fig. 2. He arrives at the train station, at the center of the city, once every morning at a random time. A train leaves for the East every 10 min and a train leaves for the West every 10 min—he chooses whichever train arrives first. The paradox is that he ends up with one girl nine times more than the other! Why?

The answer is that it is a form of Brownian ratchet, but the interesting point here is that the asymmetry is in the time variable, not in a spatial variable. The asymmetry occurs because there can be a phase difference between the trains, such that if Bill arrives in a certain 9 min window he will always get the same train, but in the other 1 min window he gets the other. Although initially counterintuitive, it becomes trivial to understand this problem if a train schedule is examined, such as shown in Table I.

So far I have illustrated Brownian ratchets that involve spatial and time variables. What about the money variable? Recall that all a Brownian ratchet really does is to exploit an asymmetry to rectify random fluctuations to get a flow of information or particles in a preferred direction. So in the money domain, an everyday example of a ratchet would be if Bill complains when a restaurant check is overcharged, but keeps quiet when it is undercharged. The asymmetry in this behavior is rectifying random price fluctuations in Bill’s favor. Another asymmetry is the paradigm of “buy-low, sell-high” on the stock market—this also attempts to capture favorable fluctuations—however the reason why this is not as sure-fire as the restaurant payment example is the problem of market volatility. Ratchets can also occur in more social situations: take the example of job promotion. The asymmetry is that it is harder to be promoted within a company than to enter that level from outside. Couple this with the random movements of employees quitting and being recruited, and we have a “people ratchet” that creates a flow of old employees out of a company. The trade-off between retaining stale people and the cost of new recruitment would make an interesting analysis where the science of Brownian ratchets may be of assistance. Other unexplored areas that remain good open questions are the application of Brownian ratchets to biological evolution and even biogenesis.<sup>65</sup> A systematic approach to stock market risk management based on Brownian ratchet models is also open for consideration—Brownian ratchets with volatility is an unexplored area that could be of

interest to studies of biogenesis, as well as stock market modeling.

#### IV. $1/f$ NOISE

No forum on unsolved problems of noise would be complete without some mention of  $1/f$  noise and UPoN'99 was no exception.  $1/f$  noise refers to the fact that in many real systems, at low frequencies, the noise spectrum takes on an approximately  $1/f$  shape or similar. It has become the holy grail of noise research to get a deeper understanding of  $1/f$  noise; many reviews have been written<sup>66–68</sup> and so we will only briefly touch on it here. Two main unsolved problems in this area are the question of (i) what are the underlying mechanisms and (ii) why is  $1/f$  noise so ubiquitous and what are the unifying features.

Some researchers have delved deeply into study of  $1/f$  noise in various electronic devices in order to gain some understanding. Others have shunned this approach, as different electronic devices come and go, change with processing conditions, and make interpretation very complex. Consequently many have concentrated their research on  $1/f$  noise in bulk materials, others have looked at natural phenomena (e.g., weather-related fluctuations), and others have preferred still further reductionism by studying computer simulated toy models.

The frustrating thing about  $1/f$  noise is that it has eluded consensus of thought and virtually every noise researcher has a “pet” theory. However, there are some poles of thought emerging that I will briefly touch on. One type of viewpoint is based on recognizing that high energy events tend to occur less frequently than low energy events. Another type of viewpoint seeks to find underlying statistical properties. An underlying property that is now in vogue is the idea of scale invariance.

To illustrate how scale invariance leads to a  $1/x$  like property, I will now use the amusing analogy of Benford's law. Benford's law<sup>69</sup> states that if you write down a set of numbers (e.g., the lengths of all the rivers in the world) you will find that the numbers begin with the digit “1” more often than any other number. What is even more spooky is that this holds true no matter whether your measurement units are in miles, inches, or whatever scale you chose! This is scale invariance.

It is easy to show that only numbers that follow a  $1/x$  distribution can lead to this property. One might believe that the lengths of rivers follow an approximate  $1/x$  distribution because you would expect shorter rivers to occur more frequently than longer rivers. So to find the probability of a first digit beginning with “1” you have to integrate the  $1/x$  probability density function (pdf) and look at the interval from 1 to 2. The integral of  $1/x$  is a logarithm. On a logarithmic scale, the interval from 1 to 2 is the largest. This explains Benford's law.

Notice also that a logarithm is a unitless function, and thus you would get the same frequencies of first digits no matter what units you measure the rivers in. The fact that the rivers can be measured in any scale units you like, and Benford's law still holds, is what we call *scale invariance*. No-

tice that scale invariance can only result from a  $1/x$ -type distribution. Any other distribution would lead to a nonlogarithmic function and units would then matter. However, note that it is possible for other distributions to have no characteristic scale—but this is *different* from scale invariance—and the correct term in these cases is *scale free*. Beware that many papers incorrectly talk about “scale invariant” when they really mean “scale free.” Fractals are self-similar over different scales and this explains the current interest in linking  $1/f$  noise to fractal-based models.

#### V. VACUUM FLUCTUATIONS

A problem with thermal noise formula  $\langle v_n^2 \rangle = 4kTR\Delta f$  is that it classically predicts infinite noise power for  $f \rightarrow \infty$ . This is an analogous situation to the blackbody radiation problem where the Rayleigh–Jean's law suffers from the so-called ultraviolet catastrophe—the divergent blackbody curve having infinite area over all frequencies. Anticipating this, Nyquist<sup>70</sup> in 1928 suggested replacing  $kT$  with the one-dimensional form of Planck's law

$$\frac{hf}{e^{hf/kT} - 1}, \quad (4)$$

which reduces to  $kT$  as  $f \rightarrow 0$  and rolls off for  $hf > kT$ .

So far so good. Nyquist's quantum term successfully removes the unwanted infinity, but introduces a new set of problems. First, this quantum term alone is obviously inadequate as it predicts that we can communicate with noiseless channels if  $hf > kT$  (i.e., in the terahertz band). A second problem is that the quantum term, in Eq. (4), predicts zero energy at  $T=0$ , which is a violation of the Uncertainty Principle. As we shall see the solution to this creates a further conundrum.

During 1911–12, Planck's “second theory” produced the following modification to the quantum term:<sup>71</sup>

$$\frac{hf}{e^{hf/kT} - 1} + \frac{hf}{2} = hf \coth\left(\frac{hf}{2kT}\right). \quad (5)$$

The extra  $hf/2$  term is called the *zero-point energy* (ZPE) and in this case, at  $T=0$ , the Uncertainty Principle is not violated. This creates a further conundrum in that  $hf/2$  is infinite when integrated over all frequencies, which is an apparent return to the type of “catastrophe” problem we saw in the classical case. One can only assume that Nyquist accordingly did not suggest this form and probably would have been aware of Planck's own misgivings concerning the experimental objectivity of  $hf/2$ . The inclusion of  $hf/2$  in standard noise texts only became popular after 1951 following the classic work of Callen and Welton<sup>72</sup> that produced the  $hf/2$  ZPE term as a natural consequence of their generalized treatment of noise in irreversible systems using perturbation theory.

The solution to the “quantum catastrophe” problem is that  $hf/2$ , in fact, turns out to be the *ground state* of a quantum mechanical oscillator. If  $n$  is the quantum number, which is a positive integer, then the allowed energy states for a quantum oscillator are  $(n + \frac{1}{2})hf$  and thus the ground state is

given when  $n = 0$ . As there is no lower energy state than the ground state, there is no energy level transition available to release the ZPE. Therefore it can be argued that  $\hbar f/2$  should be dropped before integration of the quantum expression. This procedure is an example of *renormalization*, which basically redefines the zero of energy. Renormalization is a significant area of quantum field theory and is usually presented in a more formal manner. The problem of renormalization is an open question in the theory of gravitation where there is the apparent catastrophe of *total* energy becoming infinite. For most laboratory measurements there is no catastrophe as we are only interested in energy *differences*. It is rather vexing that many basic texts herald quantum theory as removing the classical catastrophe, without admitting to the new set of catastrophe-type problems it introduces such as in gravitation—a modern fully covariant theory of renormalization<sup>73</sup> resolves some problems, but the case is not yet fully closed.

The fact that the ground state energy, which we call ZPE, cannot be released means that texts that quote the Callen and Welton  $\hbar f/2$  term as an observable noise component are not strictly correct. However, by coincidence it turns out that the mean square of the zero point fluctuation (ZPF) also has the form  $\hbar f/2$ .<sup>74</sup> The mean square does not vanish with renormalization, of course, and this ensures the Uncertainty Principle survives renormalization. The mean square fluctuation is a detectable quantity and represents the magnitude of the ZPF. This noise starts becoming significant just when the thermal noise begins to roll-off, in the THz band, thus preventing the possibility of noiseless communication.

Each mode contributes  $\hbar f/2$  toward the mean square fluctuation and, for an infinite number of frequencies, the magnitude is infinite. It is considered that this infinity is not fundamental, since the measurement conditions have not been specified. It can be shown<sup>74</sup> that for any finite observation bandwidth and volume of space the magnitude of the fluctuations of a quantum field is finite—if either the bandwidth is infinite or the measurement is evaluated at a *point* in space then the fluctuations become infinite.

In 1982, Grau and Kleen expressed the view that  $\hbar f/2$  is both unextractable and unobservable, adding their memorable rejoinder in the *Solid-State Electronics* journal that  $\hbar f/2$  is not “available for grilling steaks.”<sup>75</sup> Uncannily, about the same time Koch, Van Harlingen and Clarke (KVC) published noise measurements in superconductors reporting to have observed ZPF.<sup>76</sup> Over the next 3–4 years a number of independent superconductor papers followed, all nonchalantly quoting the KVC interpretation of ZPF as standard. In reply, Kleen (1987) essentially restated his case pointing out an unanswered question in the superconductor measurements.<sup>77</sup> As far as we are aware there has been no published KVC reply. This debate epitomizes the tension in schools of thought between  $\hbar f/2$  merely producing a measurement artifact (school of Kleen) and  $\hbar f/2$  being a real noise power (school of KVC). (It is curious to note that KVC consistently always refer to the term “ZPF” in their papers, whereas Kleen always uses the term “ZPE”—hence there is the added confusion of semantics entangled with valid points of disagreement.)

The orthodox position is that the effects of ZPF are observable such as in the Casimir effect.<sup>78</sup> ZPF also has an orthodox status in explaining the observations of Mullikan,<sup>79</sup> Lamb,<sup>80</sup> and the nature of liquid helium.<sup>81</sup> The view that ZPF cannot give rise to a detectable noise power itself, but can indirectly modulate or induce a detectable noise power has been expounded by Senitzky.<sup>82</sup>

The quantum zero field should be regarded as a conservative field as far as the extraction of energy is concerned. We can illustrate this using the thought experiment of a pair of parallel plates being pulled together by the Casimir effect—we can imagine one of the moving plates attached to a cord over a pulley with a minuscule mass on the end. As the mass is raised, the plate therefore does work and hence a small amount of energy is extracted from ZPF. However, external energy must be put into the system, to separate the plates to restart the process. Hence we have a conservative field. It could be argued that the ZPF is merely releasing externally introduced energy, stored by the system, and this may be a mechanical analogy of Senitzky's view.<sup>82</sup>

On the other hand, Jaynes has pointed out<sup>83</sup> that the energy density of the Lamb shift, in a hydrogen atom, caused by ZPF, would give rise to a Poynting vector about three times the power output of the sun. This had led to a view that ZPF has no reality.<sup>84</sup> Hence the level of reality of ZPF, in this example, is in tension with the previous example. This also reflects the tension between KVC and Kleen.

Another consequence of a literal view of ZPE is that via the  $E = mc^2$  relation and general relativity, this energy can also act as the source of a gravitational field—call this energy density in space  $W$ . Then the Kepler ratio for a planet with mean distance  $R$  from the sun and period  $T$  is proportional to  $m_{\text{sun}} + (V/c^2)W$ , where  $V$  is the volume of the sphere of radius  $R$ . To agree with observed ratios for the planets the upper frequency cutoff for  $W$  can be no higher than optical frequencies.<sup>85</sup> But any attempt to account for the Lamb shift with ZPF requires a cutoff thousands of times higher, at the Compton wavelength.<sup>85</sup> This gravitational energy would not only disturb the above-mentioned ratios, but it would radically disrupt the solar system. This *ad hoc* selection of frequencies for the operation of ZPF for the convenience of explanation is problematic.

Although in the literature terminology is not standard, I suggest to prevent confusion that the unextractable and unobservable ground state be called ZPE, whereas the vacuum fluctuations themselves be called ZPF. We noted that the mean square fluctuation of ZPF has the form  $\hbar f/2$  and ZPE also has the form  $\hbar f/2$ . This has caused some consternation in the literature and we highlighted that these quantities are different. ZPE can be removed by renormalization, whereas the effects of ZPF can be seen in a number of physical phenomena. It is clear that noise measurements are affected by an  $\hbar f/2$  law, as seen experimentally, otherwise communication channels would be noiseless above a certain frequency. However unresolved debate surrounds whether this represents a real noise power or is some quantum disturbance of a measurement (with no power to grill steaks). Also, Senitzky proposed a third option that ZPF cannot do work, but can modulate power from an outside source.

## VI. PERSPECTIVE ON THIS FOCUS ISSUE

One of the particularly exciting defining characteristics of the UPoN series is a focus on “open questions.” As the old saying goes, “90% of the solution is in asking the right question.” So I believe it is significant that we have a conference that is focused on highlighting the open problems, to define our goals and lead the noise research community forward.

Another characteristic of the UPoN series is “multidisciplinary.” It is exciting that we have papers on everything from cosmology to neurons and from biology to electronic devices. If we look back 100 years ago there used to be just physics, chemistry, and biology. If we now look at the scientific disciplines we have today we have things like: biophysics, biochemistry, biomechanics, biophotonics, econoengineering, econophysics, and the list goes on. It is interesting to note how the prefix “bio” occurs more often than not in the cross-disciplinary fields. So it should be no surprise that the third UPoN (2002, USA) will have a biophysics and biomedical engineering flavor.

If we look at some of the great discoveries over the last 20–30 years, we see an increase in multidisciplinary teamwork. Multidisciplinarianism is the research paradigm to lead us in this new millennium. It is therefore appropriate that UPoN is a conference about fluctuations—because fluctuations appear in every physical science (and even in many nonphysical disciplines).

Another point that characterizes this series is that most of the papers are about the constructive role of noise: how noise actually induces some kind of ordering in a system or how noise can tell us something useful about a system. This is an exciting new paradigm shift that has been gradually happening over the last few decades.

A further interesting characteristic of UPoN’99 is that refereeing was carried out to a high standard using a strict double-blind refereeing process. Even Invited and Keynote manuscripts were refereed. Accepted manuscripts then had to be corrected according to the referee and editorial comments. Revised manuscripts were screened and a further round of corrections were solicited. Then the papers were placed in a “draft proceedings” made available at the conference. All committee members, speakers, and attendees were then invited to comment on and recommend corrections to anyone’s papers after their oral presentation. Hence UPoN’99 also had a process of “community peer review” built into it. After these three stages of screening and corrections, after the conference, the editors reserved the right to still reject papers from the final proceedings. Invited papers had four and Regular papers had three referees and in some cases as many as five or six referees had to be called in. Each referee was asked to grade each paper on the basis of (i) substance, (ii) significance, (iii) technical quality, and (iv) clarity of open questions. Being a double-blind process, authors’ names, addresses, and acknowledgments were deleted for the reviewing process. Referees were asked to guess the name of any author or coauthor they were sure of. As a test of the efficacy of the double-blind process, it was found that only 30% of the guesses were correct—and even then each referee could

never be totally certain. In nearly all cases of a correct guess, it turned out that those authors had zealously self-referenced themselves (at their own risk). For those papers with moderate or no self-referencing, it was remarkable how totally inept the referees actually were at guessing correct names!

After this lengthy process, selected papers were then chosen for this Focus Issue. Authors were asked to augment their papers and then they were sent to four to five reviewers. This time the reviewing was the traditional single-blind and not a double-blind process. Therefore from beginning to end each paper has undergone a number of thorough revisions and has been reviewed by seven to ten independent anonymous reviewers. Being a series on “unsolved problems” authors were all asked to end each paper with a section called “Conclusions and open questions.”

## VII. PAPERS IN THIS ISSUE

It is appropriate that our first paper by Paul Davies is on the subject of vacuum fluctuations, also known as zero point fluctuations (ZPF). The study of these fluctuations impacts physics both at the atomic level all the way through to the cosmic level, with issues such as black holes and how the universe began. Vacuum fluctuations pervade everything from the very small to the very big. The significance of Hawking’s milestone papers on black holes is that his theory brings quantum mechanics, gravitation, and thermodynamics together—thus providing a focal point for attempts at a unification of these ideas. What is stimulating about Davies’ paper is that open questions center around the relationships between entropy, information, vacuum fluctuations, and gravitation.

The investigations reported in the paper by Dan Gillespie *et al.* began as an attempt to see if classical stochastic process theory could shed any light on the intrinsically stochastic behavior of measurement outcomes in quantum mechanics. The authors focus on a very simple system, the so-called two-state quantum oscillator, for which it is known that any viable two-state modeling process cannot be of the relatively simple Markov type. This complication propels the authors on an odyssey through the seldom traveled hinterlands of non-Markovian stochastic process theory. They ultimately succeed in finding not one but three classical processes that faithfully model the measurement statistics of the two-state quantum oscillator. But in the end, they conclude that their journey revealed much more about classical stochastic process theory than it did about quantum mechanics.

Gabor Balazsi and colleagues demonstrate spatiotemporal stochastic resonance (STSR) on neural models, exploiting internal noise to enhance signal transmission. Previous studies indicate the enhancement of signal transmission as a function of the internal noise of individual neurons. Nevertheless, the existence of stochastic resonance (SR) and STSR in neural systems using their internal noise is an open question and yet to be proven experimentally. The present paper discusses two possible ways of the realization of STSR in neural systems. This may help experimentalists to design appropriate experiments to study these phenomena. The dis-

cussed models may be suitable to copy the behavior of a neuronal system, and moreover that of an astrocyte tissue propagating a calcium wave. Can an understanding of these mechanisms result in helping patients with diseases related to neuronal transmission (such as diseases of the myelin sheath)?

Alexei Zaikin and Jürgen Kurths discuss spatially extended systems, which consist of coupled overdamped oscillators, where additive noise can induce first- and second-order phase transitions, and in particular cases manifest themselves in the appearance of spatially ordered patterns. Another interesting behavior occurs if a system works as a signal processor. Then additive noise is able to optimize the response of a system to an external periodic signal, if this system possesses a property of multiplicative noise induced bistability (doubly stochastic resonance, DSR). DSR is SR in a bistable potential, which is also induced by noise; but DSR differs from SR significantly, because both multiplicative and additive noise is needed. Hence DSR can be controlled by multiplicative noise, which is not the case in a conventional SR. The authors have already designed a simple electronic circuit to demonstrate DSR, and the open question is where it can be observed in biological systems. Also is noise-induced propagation in a monostable medium, which possesses DSR, possible? Until now noise-induced propagation is reported only in bistable or excitable media.

Peter Ruzsyczynski *et al.* survey a new stochastic resonance phenomenon, originally introduced by Kish and Bezrukov, which is called “spectral stochastic resonance” and is relevant for the propagation of interacting neural spikes as well as for traffic of cars. The existence of  $1/f$  noise in neural systems and car traffic seems to be useful, due to the fact that the best spike propagation and traffic is detected for the case of  $1/f$  noise. However, the underlying reasons for the  $1/f$ -like spectra remain an open question.

Mark Dykman and colleagues deal with the problem of control of activated processes, such as escape from a metastable state or nucleation, by time-dependent fields. The problem is of fundamental physical interest, as systems driven by ac fields form one of the most important types of nonequilibrium systems, and there are no known general principles that would describe fluctuations in such systems. At the same time, it follows from the results of the paper that ac fields can be used for novel types of selective and highly efficient control of activated processes. This has a bearing on many applications, from crystal growth to separation science. One of the unexpected results is that the effect of ac driving on probabilities of activated processes can be described in a general and simple form. The authors show theoretically, by analog and digital simulations, and through experiments with optically trapped particles, that even for high-frequency driving, the activation energy is often linear in the field amplitude. Can we learn about the dynamics of a system, away from stable or metastable states, using large fluctuations?

Slava Soskin *et al.* explore activation over a barrier due to thermal fluctuations—this is a problem that is central to many branches of physics, chemistry, and biology. It was tackled and solved, for different parameter ranges, by Arrhenius, Kramers, and Mel’nikov. All of these authors,

however, restricted their treatment to the problem of escape from a metastable potential in the quasistationary regime, i.e., for times much larger than the typical relaxation time inside the potential. The authors, here, extend the analysis to encompass the case of much shorter time scales, comparable with, or shorter than, the time scale over which equilibrium in the metastable potential is established. It also covers the case where the metastable potential has an internal structure, for instance more than one minimum. The authors find that, quite counterintuitively, the escape flux may depend on time and friction in a very complicated way, with nonmonotonic behaviors, steps, oscillations, and cusps. Quite apart from its fundamental interest, what is reported may have important applications, e.g., by exploiting the sensitivity to friction and inertia in the control of femtosecond chemical reactions, or in optimizing the separation of different atomic or molecular species, or in improving the efficiency of optical tweezers.

S.A. Guz and M. Sviridov pose problems associated with “green” noise—defined as noise with zero spectral density at zero frequency. They find that stochastic systems driven by green noise become more stable than in the case of white noise. An open question is the following: Can this study be of benefit to problems involving blackbody radiation, broadband random radiation passing through a finite aperture, noise current passing through a capacitance, phonon gas in crystals and clusters, zero-point fluctuations of a quantum oscillator, random stationary modulation of the phase of an input signal in a phase-locked loop, random vibration bias in a ring laser, the phase modulation of a radar signal at random oscillations of an object along a probe beam, the effect of noise on a mechanical system through a viscous friction, and so forth?

Level-crossing statistics have long been one of the more intractable topics in the theory of stochastic processes. Twenty-four years ago Hurst and co-workers in New Zealand, followed by Corotis and co-workers in the USA, drew attention to the self-scaling (power law) distribution of atmospheric wind speed level-crossing intervals. Paul Edwards and Robert Hurst now revisit this issue by reinterpreting atmospheric wind speed level-crossing statistics as a fractal stochastic point process. They have also synthesized an artificial single point wind field based on a two-dimensional Ornstein–Uhlenbeck process with power spectral density, Rayleigh speed distribution, and self-scaling level-crossing statistics which closely mimic those of the real wind. While leaving unanswered a number of fundamental questions in the theory of stochastic point processes (such as the conditions necessary for the generation of fractal point statistics), this work nevertheless suggests several new lines of attack. In immediate practical terms the work demonstrates the success of a simple Markov model in reproducing most of the characteristics of atmospheric turbulence relevant to the stochastic control of aerogenerators.

Zoltan Gingl and co-workers pose questions about noise whose power spectral density follows a power law, as commonly found in physical and biological systems, however the general occurrence and properties are not yet understood. They have found a special invariant property against a nonlinear transform (truncation) of  $1/f$  noise. Such nonlinear

transforms are found frequently in real systems including biological systems and in data communications as well. The open question is how can we use this curious discovery of invariance under truncation to further understand the underlying principles and ubiquity of  $1/f$  noise?

Juraj Kumicak uses a new approach to the study of fluctuations in thermostated nonequilibrium systems. Two models are analyzed using a Galton board model and the generalized baker map. The author finds (i) ergodic strange attractors, (ii) the character of noise appearing in the models can be brought into relation with fractal properties of the attractors, namely with their information dimension, (iii)  $1/f$  noise is found only in thermostated systems. The approach links specific properties of noises to specific properties of phase-space structures—the open question is whether this can be of conceptual significance for future theoretical investigations.

Balaram Das examines the interplay of influence dominance between the cognitive insights of an intelligent agent and the algorithmically derived results, of the computational tool being employed by the agent, as he or she ponders the next step in the iterative solution of some problem of interest. The thesis of the work is that the highest level or ergonomic efficiency is achieved when the computational tool is designed such that the time series of dominance fluctuations displays the ubiquitous self-similarity of  $1/f$  noise. The computational tool may be a “black box” as far as the agent is concerned, but if it is properly designed it will engender within the cognitive agent the appropriate level of “trust” that is necessary for it to become an equal (but not dominant) partner in this interplay. The establishment of general principles for the design of such computational aids is of utmost importance for the information age and the author attempts to set forth such a set of general principles. An intriguing open question would be to ask if the author’s prescription for the design of ergonomically efficient software may have already been employed, namely, by nature in the evolution of human intelligence.

Physiological signals generated by complex regulatory systems, which process inputs with a broad range of characteristics, are extremely inhomogeneous and nonstationary. This is a challenge to conventional methods of analysis and modeling. Plamen Ivanov and co-workers present an exciting review of recent work on the application of concepts and methods from modern statistical physics and nonlinear dynamics to physiological signals. The authors demonstrate that in spite of their “noisy” and “erratic” appearance, physiological fluctuations exhibit unexpected hidden scale-invariant structures. In particular, the authors show that healthy heartbeat fluctuations exhibit complex temporal organization characterized by long-range power-law correlations. Power-law correlations indicate the absence of a characteristic scale (i.e., scale free) and suggest that the underlying mechanisms regulating the heartbeat dynamics have statistical properties which are similar on different time scales. Such statistical self-similarity is an important characteristic of fractal objects.

Arun Holden and Vadim Biktashev apply Karhunen–Loève decomposition to image sequences of modeled and

optical recordings of the electrical activity on the inside and outside surfaces of the wall of the heart during ventricular fibrillation. This activity is an example of spatiotemporal irregularity in an extended system with local interactions. In the cardiological context, a practical problem is whether or not it is generated by two- or three-dimensional processes: This is approached by evaluating the complexities of the signals from the two surfaces separately and conjointly. Many irregular time series are in fact the activity, at a point, of spatiotemporal irregularity in a three-dimensional spatially extended system, and quantitative measures of the complexity of the full spatiotemporal process can be helpful in characterizing the process. Measures of apparent complexity provide descriptive statistics, while measures of behavioral complexity identify the number of independent mechanisms that could generate the process. The methods illustrated in the paper may be applied to contexts where three-dimensional tomographic imaging of spatiotemporal irregularity is not available, but the irregular surface can be monitored.

Raul Toral and colleagues study chaos synchronization, which is an essential ingredient in the recovery of a message that is masked by a chaotic carrier. Although the usual way to synchronize two chaotic systems is by injecting part of the emitted signal into the receiver, the possibility of synchronization using a common noisy forcing has also been suggested. This method has the additional advantage that the synchronizing signal carries no information. The paper reviews the existing, and sometimes contradictory, literature and gives explicit examples (both analytical and numerical) of chaotic systems that can be synchronized in this way. Finally, the authors analyze the expected degree of synchronization that can be achieved with nonidentical chaotic devices, such as those that are commercially available to construct electronic circuits.

Kish and co-workers have introduced a working model, inspired by experimental carbon nanotube formation, for the tubular shape formation of an ensemble of ultrafine particles; these particles are captured by microscopic eddies in a fluid or gaseous medium. The authors have shown that the particles are trapped around a particular stationary orbit and the noise (temperature) acts as a perturbation, repelling the particles out of this orbit. The strength of the noise controls the thickness of tube wall and a sufficiently strong noise prohibits tubular formation. Though the proposed toy model is obviously not applicable directly to nanotube formation, a number of experimentally observed features have striking resemblance to the model behavior.

Chaotically moving particles move more or less independently from each other—they can be accelerated in a local field and are restricted only by the (potential) barriers they encounter in their free flight. The only interaction existing between the particles is that generated by the field they source individually. Their motion can therefore be described statistically. By following the flight histories of the individual electrons, Canute Moglestue has simulated the electrical characteristics of microelectronic devices. The open question is whether this approach can be applied generally to transistors, diodes, particle detectors, lasers etc. and whether

it can predict certain types of noise behavior.

Nonlinear trans-resonant wave phenomena have been little studied, although the trans-resonant oscillations are a classical problem. Shulim Galiev *et al.* consider these phenomena in different dispersive–dissipative systems. They found that in the trans-resonant band, harmonic waves can amplify and transform into mushroom-like waves and then into vortex clusters. The authors speculate that similar processes could model galaxy generation and galaxy clusters. Also, the authors suggest that atom and electron structures are formed by nonlinear resonant waves—they can change the form and velocity, compress and stop. These effects depend on properties of the resonant structure and whether there is forced or parametric excitation. An open question is to extend these models to physical cases and see if there is promise in areas such as quantum optics and quantum computation.

Gregory Harmer and colleagues analyze “Parrondo’s paradox,” which involves combining losing games of chance to win. The significance of these games is that they are physically motivated from the concept of a flashing Brownian ratchet. The games can be thought of as a discrete-time Brownian ratchet. Many phenomena in nature exhibit ratchet-like behavior, and an open question is to ask if the principles from this toy model can be extended to model various biological phenomena where losing strategies appear to win out in the end.

Andrew Allison *et al.* take the toy model principles of Parrondo’s games and map them onto a control theory context—where it is shown that noisy switching between unstable systems can counterintuitively lead to stability. A key unifying feature is that Parrondo’s games rely on curvature in the lose–win boundary of the game parameter space, and in the control example there is a curvature in the stable–unstable boundary. This condition allows for the possibility of convex linear combinations, which geometrically explains these counterintuitive effects. An open question is to now extend these ideas to other systems.

Brownian ratchets and noise induced directed motion have revived interest in old problems surrounding the foundations of statistical mechanics—such as the probabilistic nature of the second law of thermodynamics and the relationship between entropy and information. These problems are well illustrated by gedanken experiments like the *Maxwell demon* or the *Szillard engine*, and ratchet-like systems have been dubbed as “automatic” Maxwell demons. Juan Parrondo gives an original interpretation of the Szillard engine by using recent results of the energetics of ratchets and more general stochastic systems. The author shows that one of the key ingredients in the Szillard engine is a spontaneous symmetry breaking phase transition. To support this idea, a Szillard engine is devised using an Ising model. The original Szillard engine and the proposed “Ising engine” are very different from the physical point of view. However, since both exhibit spontaneous symmetry breaking transitions, the energetics of both systems is very similar. The Ising engine works, as any Maxwell demon, by measuring some quantity of the system. The novelty here is that this quantity is the global magnetization of the system, i.e., it is a macroscopic

magnitude. Therefore, the Ising engine could be considered as a macroscopic Maxwell demon. We see that the paper presents a new approach to the problem of the relationship between entropy and information. In particular, it points out that, in a spontaneous symmetry breaking transition, the state of the system no longer obeys the usual Fokker–Planck equation and prompts the open problem to find the evolution of the state of the system. Further research on this problem and on the energetics of stochastic systems, undergoing symmetry breaking transitions, will probably help to clarify important issues on the thermodynamics of computation and on the foundations of statistical mechanics.

Shunya Ishioka and Nobuko Fuchikami thoroughly reconsider the thermodynamics of computation. The point is that thermodynamic (Clausius) entropy is an objective quantity but not the measure of lack of information. The distinction of Clausius entropy from information leads to the exact correspondence between logical and physical irreversibilities. The authors point out that Clausius entropy decreases in a symmetry breaking process, which implies that the residual entropy is not a thermodynamic one.

Bruce Davis *et al.* pose a cute conundrum involving a capacitor driven by a noisy resistor. The twist is that the plates of the capacitor are allowed to move. If a demon were to restore the plates everytime the voltage across the plates is small, you could get work out of the system. As this is impossible, by the second law of thermodynamics, it means the demon must be doing an equivalent amount of work. But this is not immediately obvious and the open question is to find the correct microscopic description. The authors do not attempt to directly answer this problem but try to simplify it by replacing the imaginary demon with a restoring spring. Unfortunately this “simpler” system brings with it a new set of open problems. The open question is to find the simplest model that removes any violations. Could it be that contact of the plates should be modeled by a time-varying resistance? Or should the effects of inductance be included in the system? For such an apparently simple system, the definitive solution appears both elusive and strangely complex.

## VIII. QUO VADIS?

To conclude, the constructive role of noise is a very fascinating area. There are interesting questions we can ask, such as what constructive role does noise play in neural networks and how does it affect cognition and consciousness. We have highlighted that a possible unifying concept between stochastic resonance and Brownian ratchets is the idea of disorder interacting with an asymmetry. Traditionally “doing physics” has focused on looking for symmetries in nature. Perhaps the new paradigm is to start deliberately searching for asymmetries and see how they interplay with disorder or a fluctuating environment. By searching for asymmetry in nature, can we identify more instances where noise plays a good role? Does chirality in nature have a role in the context of noise? What constructive role do vacuum fluctuations play in the universe? Can vacuum fluctuations drive a Brownian ratchet?

The quotation “stop telling God what to do,” attributed

to Niels Bohr as a rejoinder to Einstein's oft quoted "God does not play dice with the universe," was chosen as the signature theme of UPoN'99. This is a conference series on unsolved random phenomena—asking how the "dice" are played in various aspects of our physical world. The first step in finding the answers is to remove all our preconceptions, i.e., stop telling God what to do! If we do this we then open ourselves to the deeper answers that invariably shock and surprise us, humbling us to realize what little we really do know.

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