Modelling radio refractive index in the atmospheric surface layer

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Knowledge of the refractive index profile at radio frequencies in the surface layer of the atmosphere is required to predict the performance of terrestrial radio systems, and although a constant gradient of refractivity with height is often assumed, both measurements and theory suggest that gradients in the lowest 20 m of the atmosphere may often be greater than those above this level. For the special case of evaporation ducts over water in a neutral atmosphere, a logarithmic refractivity profile is normally assumed, but a general model that includes both this case and the linear profile as special cases is proposed, which may also be used to approximately model stable and unstable surface atmospheres. This new model may be particularly suited to predicting sub-refractive fading.

Introduction: A logarithmic refractivity profile, for a neutral atmosphere or for a stable atmosphere at heights below the Obukhov length, is a consequence of the exchange coefficient $K(z)$ increasing linearly with height [1]. However, in an unstable atmosphere, where the heat flux is directed upwards, the increase in $K(z)$ with height is more rapid, asymptotically proportional to $z^{4/3}$ [1].

Refractivity profiles are often expressed in terms of modified refractivity $M(z)$, given in terms of M units, by

$$M(z) = N(z) + 0.157z \quad (1)$$

where $N(z)$ is the refractive index, as parts per million in excess of unity, at height z metres. The modification term $0.157z$ replaces the physical curvature of the Earth by an added refractive index gradient, to allow the convenience of 'flat Earth' analysis.

Evaporation ducts, resulting from wind over a water surface, are typically modelled [2] as

$$M(z) = M(0) + G_M\left[z - (\delta + z_0)\ln\left(\frac{z+z_0}{z_0}\right)\right] \quad (2)$$

where z_0 is the roughness length, usually assumed to be 0.00015 m in evaporation duct modelling, and δ is the duct height, defined as the height where the modified gradient $M'(z) = 0$. G_M is the standard modified refractivity gradient, with an approximate value of $+0.12$ M units/m. Positive modified gradients greater than this are referred to as sub refractive, and may result in diffraction loss on terrestrial radio paths, because of the strong curvature of ray lines away from the Earth.

Positive modified gradients less than the standard gradient are referred to as super refractive, as line of sight distances are greater than under standard conditions. Linear refractivity profiles are often assumed in the case of sub refraction [3, 4] or mild super refraction, i.e. modified refractivity gradient $M'(z)$ is assumed to be constant.

If the modified gradient becomes negative, the downward curvature of ray lines exceeds the curvature of the Earth. This is referred to as a duct, which may result in strong terrestrial propagation over large distances.

General surface layer model: The logarithmic model of (2), and the linear model with constant $M'(z)$, may both be expressed as special cases of the one model, by considering that $\ln(x)$ is the limit, as p approaches zero, of $(x^p - 1)/p$. We then re state (2) as the special case $p = 0$ of the general model, in the form

$$M(z) = M(0) + G_M\left[z - D_p\ln\left(\frac{z+z_0}{z_0}\right)\right], \quad \text{if } p = 0 \quad (3)$$

and for all other values of p, we have

$$M(z) = M(0) + G_M\left[z - D_p\frac{((z+z_0)/z_0)^p - 1}{p}\right] \quad (4)$$

The modified refractivity gradient $M'(z)$, by differentiating (3) or (4) with respect to z, for all values of p, is

$$M'(z) = G_M\left[1 - D_p\frac{((z+z_0)/z_0)^{p-1}}{z_0}\right] \quad (5)$$

If $D_p > 0$ and $p < 1$, the profile $M(z)$ represents a surface duct, and the duct height δ is determined by setting $M'(z) = 0$, giving

$$\delta = \left(\frac{D_p}{z_0^p}\right)^{(1/(1-p))} - z_0 \quad (6)$$

or alternatively

$$D_p = \left[(\delta + z_0)^{1-p}\right]z_0^p \quad (7)$$

A surface sub refractive layer may be represented by this model, by using a negative value of D_p. This generalises a previous suggestion [5], which described sub refraction as an 'anti duct', using negative D_p in (3).

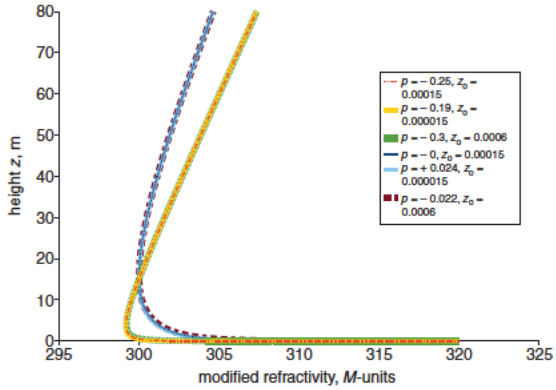

Fig. 1 *Modelling two different ducts, each with three different values of z_0*

Refractivity profiles for $M(0) = 320$, $M(15) = 300$ and $G_M = 0.12$ M-units/m, the two duct profiles being generated firstly with the usual z_0 value for evaporation ducts of 0.00015 m, and then super-imposed profiles with smaller and larger z_0 values, produced by adjusting the value of p in (4)

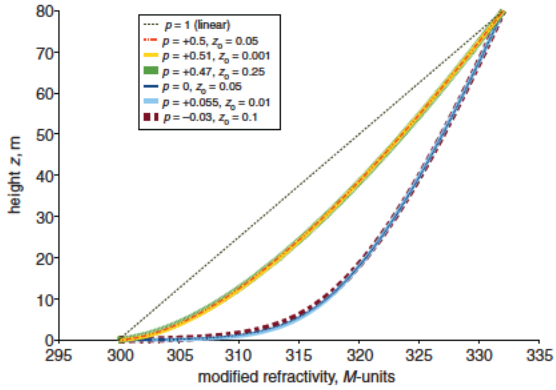

Fig. 2 *Modelling three sub-refractive profiles, using various values of z_0*

Refractivity profiles for $M(0) = 300$, $M(80) = 332$ and $G_M = 0.12$ M-units/m, for the usual linear assumption $p = 1$, and two nonlinear profiles, each generated firstly with $z_0 = 0.05$ m, and then super-imposed profiles with smaller and larger z_0 values, produced by adjusting the value of p in (4)

Fitting the model to observations: If refractivity at the surface $M(0)$, and $M(z_1)$ at height z_1 above the surface, are both known, then D_p is given by

$$D_p = \frac{z_1 - [M(z_1) - M(0)]/G_M}{\ln[(z_1+z_0)/z_0]}, \quad \text{for } p = 0 \quad (8)$$

or otherwise

$$D_p = p\frac{z_1 - [M(z_1) - M(0)]/G_M}{[(z_1+z_0)/z_0]^p - 1} \quad (9)$$

Substituting D_p in (3) or (4) then provides a family of possible refractivity curves for the same values of $M(0)$, z_1, $M(z_1)$ and z_0, depending on the value of p.

For the $p = 0$ case of (3), z_0 has a specific physical meaning, as the roughness length of the surface, but this physical meaning may not necessarily hold for (4), and in fact z_0 becomes irrelevant in (4) if $p = 1$.

In practice, model variations that result from varying the value of z_0 can be fairly accurately compensated for, by adjusting the value of p.

This is demonstrated in Fig. 1 for two different ducting refractivity pro files, each produced in three ways, each having significantly different values of z_0. Despite widely different z_0 values, closely replicated pro files are achieved, by varying the value of p.

Fig. 2 is a similar demonstration for two nonlinear sub refractive profiles, as well as the linear $p = 1$ case, which is independent of z_0.

Thus we may, for practical purposes, choose to define z_0 to be the physical parameter roughness length, for all values of p and then let p be the parameter which controls the shape of the refractivity profile.

Practical significance of different p values super refraction: Considering ducting propagation, different propagation characteristics would be expected because of differences in duct height. In the example of Fig. 1, with $z_0 = 0.00015$ m, and the same mean gradient in the lowest 15 m, duct height is 15.8 m with $p = 0$ and 3.8 m with $p =$ 0.25.

Practical significance of different p *values sub refraction:* Modelling of sub refraction conventionally assumes a linear refractivity profile [3, 4] or $p = 1$ in terms of (4), but nonlinear profiles, particularly in the region of $p = 0.5$, may result in greater diffraction loss than the linear case. This may be an important for predicting sub refractive fading, as this type of fading predominantly occurs in a stable atmos phere prior to sunrise [4]. In a stable atmosphere, the refractivity profile is expected [1] to be logarithmic at heights below the Obukhov length L; and essentially linear at heights exceeding L. Although some thing of the order of 20 m may be considered typical, L is proportional to the cube of wind speed and inversely proportional to heat flux [1], hence the value of L would vary considerably. The model presented here, with $p = 0.5$, may be a useful compromise for predicting sub refractive fading, as it seems to be close to a worst case profile, and mid way between the type of profiles expected above and below the varying Obukhov length.

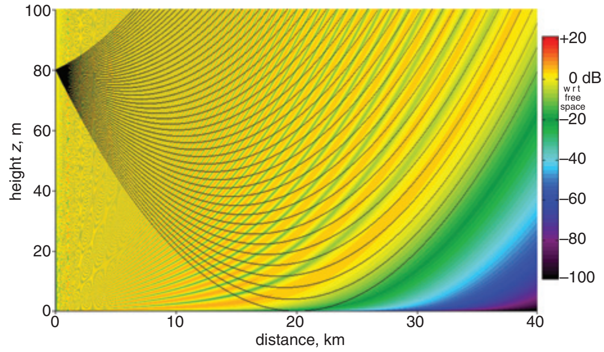

Fig. 3 *Predicted field relative to free-space, and ray tracing, for p 1*
Sub-refraction +400 M-units/km: $M(0)$ 300, $M(80)$ 332
Transmitter: 10 GHz at 80 m, with traced rays 0.2 milliradians apart

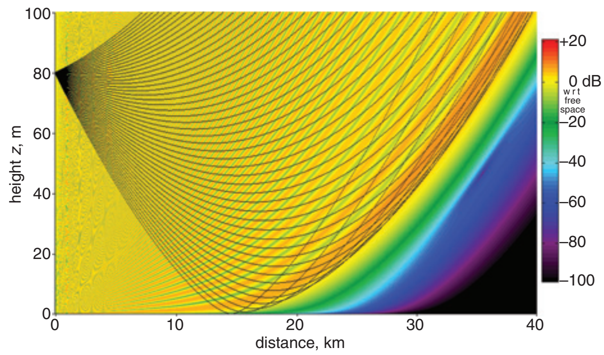

Fig. 4 *Predicted field relative to free-space, and ray tracing, for p 0.5*
Sub-refraction +400 M-units/km: $M(0)$ 300, $M(80)$ 332
Transmitter: 10 GHz at 80 m. G_M 0.12 M-units/m and z_0 0.05 m

The impact on radio propagation of different refractivity profiles, for the same mean gradient, may be studied using the parabolic equation method [6, 7]. Field strength predictions for the three refractivity profiles of Fig. 2, at 10 GHz, with a transmitter height of 80 m, are shown in Figs. 3 5, for p values of 1, 0.5 and 0, respectively. Considering diffraction loss to receivers at low heights, more than 30 km from the transmitter, the linear case $p = 1$ of Fig. 3 suffers less loss than the logarithmic case $p = 0$ of Fig. 5, but the greatest loss is encountered by the $p = 0.5$ case of Fig. 4.

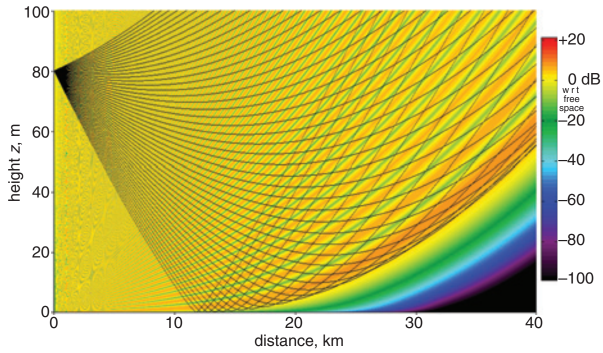

Fig. 5 *Predicted field relative to free-space, and ray tracing, for p 0*
Sub-refraction +400 M-units/km: $M(0)$ 300, $M(80)$ 332
Transmitter: 10 GHz at 80 m. G_M 0.12 M-units/m and z_0 0.05 m

Conclusion: We have shown that the conventional logarithmic evapor ation duct refractivity profile model, and the simple linear refractivity profile model, are both special cases of a general 'log power' model, readily differentiable, with parameters p and D_p, which may both be varied to produce a range of surface refractivity profiles, while maintain ing the same values of the parameters of roughness length z_0 and standard modified refractivity gradient G_M.

We have demonstrated that for a given mean refractivity gradient in the surface layer of the atmosphere, varying the power parameter p may result in considerable differences in the predicted radio field strength. Information about a shape parameter such as this may be required for accurate field strength prediction, in addition to the usual parameters of surface refractivity gradient or duct height.

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One or more of the Figures in this Letter are available in colour online.

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