Fluctuation and Noise Letters
Vol. 1, No. 2 (2001) L65–L69

WAVELET DE-NOISING OF OPTICAL TERAHERTZ PULSE IMAGING DATA

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Received 23 January 2001
Revised 30 April 2001
Accepted 4 May 2001

Terahertz pulse imaging (TPI) systems are used to obtain sub-millimeter spectroscopic measurements for a wide range of applications. This letter highlights the use of wavelet de-noising to markedly improve the SNR of the obtained data, increasing the SNR by up to 10 dB. A comparison of different wavelet families and properties is presented and the results demonstrated on THz image data of an oak leaf and an Australian $100 note.

Keywords: Terahertz pulse imaging, wavelet de-noising, wavelet order.

1. Introduction

There are a large number of noise sources in terahertz imaging systems. These include the emitter noise, which is a result of random intensity fluctuations in the ultrafast laser [1], Johnson and shot noise in the THz detector [2] as well as thermal background radiation in the THz regime. The coherent nature of T-ray systems allows them to achieve impressive performance despite very high relative noise magnitudes [3]. However, THz imaging systems are still severely noise-limited in terms of their speed and accuracy. This letter shows that wavelet de-noising can be used to drastically reduce the noise and identifies the ideal wavelet family and properties in this context. This work investigates signal processing strategies with the potential to make significant improvements in the speed and clarity of current T-ray systems. Biomedical applications are a particular focus as the high absorption of terahertz frequencies in human tissue results in high relative noise levels.

The data for this study consists of a 100 x 100 pixel image of an oak leaf with an insect sitting on the side of the leaf as shown in Fig. 1(a). The image has a spatial resolution of approximately 1 mm. For each pixel the time response of the terahertz pulse was recorded over 12 ps (10^{-12} s) at an effective sample rate of 25 terasamples/second. The hardware setup used to obtain the data has been described many times in the literature [4–6] and is not repeated here.

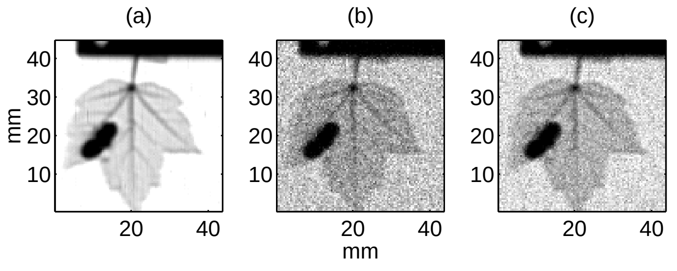

Fig 1. Image of an insect on an oak leaf obtained by terahertz pulse imaging. The image is produced by plotting the peak value of the response for each pixel. (a) Raw image. (b) Raw image with added noise such that the SNR was 3 dB. (c) The noisy image after de-noising using a Coiflet order 4 wavelet, the average SNR is 10 dB greater than in (b).

2. Wavelet De-noising

Discrete wavelet methods are extremely efficient and computationally inexpensive. Wavelet shrinkage de-noising, as defined by Donoho [7] offers a range of attractive properties for efficient representation and de-noising of pulsed functions such as terahertz pulses. The goal of wavelet de-noising is to approximate the noiseless function with as few non-zero wavelet coefficients as possible. The wavelet family ψ should therefore be chosen to produce a maximum number of wavelet coefficients that are close to zero. The relevant properties of the wavelet in this regard are its regularity, the number of vanishing moments and the compactness of its support [8]. To date there is no theoretical method of determining the ideal wavelet for representing a given signal, thus an experimental investigation is the only available recourse. Other authors have suggested using wavelet de-noising for THz imaging data [9, 10] however these are the first results investigating the performance of the technique and analyzing the suitability of different wavelet families.

3. Experimental Determination of the Optimal Wavelet

Experiments were performed to determine which wavelet family performed best in de-noising noisy T-ray data. To do this white, Gaussian noise was added to the measured terahertz responses such that the SNR was 3 dB. The noisy sequence was then de-noised using Daubechies, Meyer, Symlet and Coiflet wavelets of varying order. The order of the wavelet, N, is of particular concern to this discussion as it determines both the number of vanishing moments and the support length for the wavelet.

The quality of the de-noising process was measured based on the resultant signal to noise ratio (SNR). This process was repeated over 100 times to average the results. This procedure was carried out for each of the characteristic T-ray responses, the free air, leaf and insect responses.

4. Results

The wavelet de-noising procedure was very successful in de-noising the terahertz pulses, providing an improvement in SNR ranging from 5 dB up to 10 dB. All of the wavelets tested provided similar results however by averaging over all wavelet orders for each wavelet family the families could be ranked in order of decreasing quality to yield: Coiflet(4), Symlet(7), Daubechies(8) and Meyer(1)[1]. For each wavelet family the variation of de-noised SNR with order was analyzed. It was found that for each family there was an ideal order N^* (shown in brackets above), for which the de-noised SNR was maximized. For orders $N < N^*$ the SNR dropped off rapidly and for higher orders $N > N^*$ the de-noised SNR dropped off gradually in a complex fashion approximating an exponential decay overlaid with an oscillatory function. An example of this response is shown in Fig. 2 for the Daubechies family.

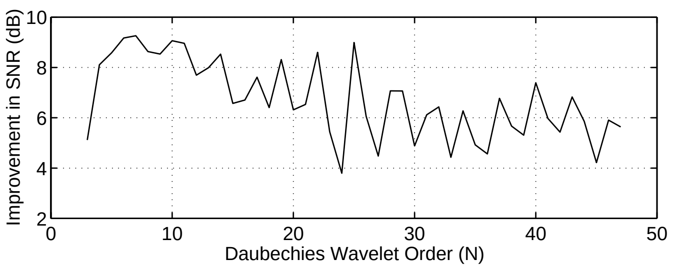

Fig 2. The improvement in SNR provided by de-noising noisy signals with Daubechies wavelets of varying order N. The average improvement in SNR of the de-noised terahertz pulses is plotted against the wavelet order.

The ideal order arises because the de-noising efficiency is decreased with increasing support width, and increased with the number of vanishing moments. Both of these properties are proportional to the wavelet order N, thus for each wavelet family there is an order at which this compromise is optimal.

The improvement in image quality when the noisy responses are de-noised using the optimal wavelet (Coiflet order 4) is demonstrated in Fig. 1(b) and (c).

4.1. *Increasing the speed of TPI systems*

One of the factors limiting the speed of TPI is the Lock In Amplifier (LIA). This component provides signal averaging at the expense of acquisition speed. The LIA time constant determines the amount of time spent averaging each sample. Wavelet de-noising using a Coiflet wavelet of order 4 was used to de-noise a THz signal measured with a small LIA time constant of 1ms after transmission through an

[1]only one Meyer wavelet was considered in this study

Australian \$100 note. The results of de-noising this signal are shown in Fig. 3. It can be seen that the noise is significantly reduced, the SNR was improved from 7 dB to 8.5 dB. This technique could potentially allow effective systems to be constructed without the LIA, this would reduce the hardware complexity and, if the slow stepper motor that typically provides the optical delay is replaced by a scanning optical delay line, would dramatically increase the acquisition speed of the system.

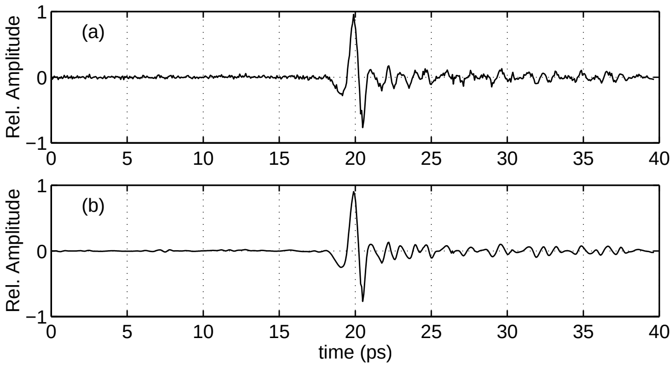

Fig 3. Illustration of wavelet de-noising of genuine noisy data. (a) The terahertz response of an Australian \$100 note measured with a short LIA time constant of 1ms. (b) The same signal after wavelet de-noising using a Coiflet wavelet of order 4.

Wavelet de-noising also promises to increase the power of the terahertz imaging technique in measuring samples that have high attenuation and correspondingly high relative noise levels. This is of particular benefit in biomedical applications where the absorption in human tissue can be prohibitive.

5. Conclusion

Wavelet de-noising was found to be well suited to de-noising terahertz pulses after transmission through various samples, achieving up to 10 dB improvement in SNR when applied to waveforms with an initial SNR of 3 dB. This is significantly better than the other de-noising techniques investigated. Four wavelet families were compared, their performance was ranked in order of success and the ideal order for each family was identified. The Coiflet order 4 wavelet was found to be optimal for this application. This wavelet was found to perform well for all the sample responses investigated and therefore promises broad applicability across the scope of T-ray spectroscopic and imaging applications.

This is a dynamic field of research and much work remains. In particular signal processing strategies must be adapted to deal with $1/f$ noise originating from the ultrafast laser. Many other signal processing techniques hold potential, notably adaptive signal processing, and techniques adapted from speech recognition applications.

Acknowledgments

The authors would like to acknowledge the support of X.-C. Zhang's group at Rensselaer Polytechnic and the support of Professor Doug Gray from the CRC for Sensor, Signal and Information Processing (CSSIP). Funding from the Australian Research Council (ARC) is gratefully acknowledged.

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