3-D Low Earth Orbit Vector Estimation of Faraday Rotation and Path Delay

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ABSTRACT An electromagnetic wave propagating through the ionosphere is subject to path delay and the depolarizing effect of Faraday rotation, both of which are dependent on global position and geometry. These effects introduce error and consequently reduce the range resolution of remote sensing polarimetric measurements. Satellite-to-ground communications may be adversely altered by these effects so as to inhibit signal reception. The work presented here introduces a simple vectorized model for a large-field-of-view, low-Earth-orbit, satellite system that yields Faraday rotation and path delay according to global position and geometric parameters. Comparison is made with current models, through the simulation of Faraday rotation and path delay. The presented work may extend the range over which Faraday rotation and path delay estimation are reliable. The work presented forms part of a large-field-of-view, low-Earth-orbit satellite model exploiting multiple-input multiple-output polarimetry in three dimensions.

INDEX TERMS Faraday rotation, ionosphere, path delay, remote sensing, satellite communications.

I. INTRODUCTION
Faraday rotation and path delay are two effects that reduce the effectiveness of remote sensing methods and systems performance of satellite communications. Range resolution of synthetic aperture radar (SAR) systems has been shown to adversely affect data obtained via remote sensing [1], [2]. Data and voice signals may be distorted by these effects so as to render the propagation channel inoperative. Polarimetric systems may make use of compensation techniques to offset these effects. Noise induced biases on Faraday rotation measurements may be estimated and compensated if the system noise and averaging scattering signature of an observed surface is known. Estimates and fittings may be applied as an enveloping technique according to the signal to noise ratio (SNR) and as such may not take account of the subtleties of the components that produce these detrimental effects [3]. In this paper, path delay is used to refer to the less common term of dispersion delay, a first order ionospheric effect due to propagation through a dispersive medium.

The terrestrial magnetic field vector, \( \mathbf{B} \), and plasma density, \( n_e \), are both functions of height as well as latitude and longitude. At a particular (latitude, longitude, height), or \((L, l, h)\) position, they are time-varying. Their values may be determined through simulation and measurement [4], [5]. Indeed, Faraday rotation may be determined through measurement, this being dependent on the satellite operating frequency and orbit [6], [7]. A simple vector-based estimation of Faraday rotation and path delay, taking into account the principle components causing these effects, potentially acts as a useful basis for constructing a model that determines these effects for a unique global position and geometry.

The ionosphere is typically given as a region of atmosphere beginning at a height of 50 km and extending into the magnetosphere. Electromagnetic signals propagating through this region are subject to unwanted interaction with ionised particles [8]. These ionised particles depolarize a propagating signal as well as introduce a propagation or path delay on it. These effects are dependent on signal frequency, atmospheric electron content and terrestrial magnetic field, themselves being dependent on time and global position, and link geometry within the satellite field-of-view (FoV) [9], [10]. In the instance of a low-Earth-orbit (LEO) satellite operating with a large-FoV, link geometry is additionally time-varying due to LEO relative movement. Coupled with low elevational angles, this means that any interaction with a ground receiver may be subject to periods of link failure. A large-FoV, in this instance, is defined relative to the spatial scale on which
the ionospheric phase is approximately linear. This scale structure is subject to temporal and spatial variation [11].

Faraday rotation [12], $\psi_{FR}$, and path delay [13], $\zeta$, affect line-of-sight (LoS) transmission and are more pronounced as transmission frequency decreases. Their effects may be given in many forms [7], [14]–[16]. Propagation through the ionosphere introduces multipath, instrument delays, and noise; the latter being a particular problem for detection of weak signals in radioastronomy [17]. For a more comprehensive understanding of effects that may affect ionospheric signal propagation, the reader is encouraged to review the work in [8], and in [18]. For the model presented here, the interpretations of [19] are used,

$$\psi_{FR} = \frac{e^3 \lambda^2 \sec \alpha}{8\pi^2 \varepsilon_0 m c^2} \int_0^h n_e(h)B_{||}(h) dh$$  \hspace{1cm} (1)$$

$$\zeta = \frac{e^3 \lambda^2 \sec \alpha}{8\pi^2 \varepsilon_0 m c^2} \int_0^h n_e(h) dh$$  \hspace{1cm} (2)$$

where $h$ is the perpendicular height above the Earth’s surface, $B_{||}$ is the terrestrial magnetic field component parallel to the propagation unit vector, $e$ is electronic charge, $\lambda$ is propagating signal wavelength, $\varepsilon_0$ is the permittivity of free space, $m$ is electronic mass, $c$ is the speed of light in vacuo, $n_e$ is the electron density at a specific location in the ionosphere, and $\alpha$ is an off-nadir angle that is used to account for the additional path length of a slanted path. Path delay in Equation (2), being a first order dispersive effect, typically affects the signal more than Faraday rotation, which is a second order dispersive effect.

Satellite channels requiring a large time-varying FoV of the Earth’s surface, such as those encountered in global positioning systems (GPS), remote sensing, satellite broadcast and telephony in remote areas, are often compromised due to the requirement of obtaining adequate orbital height to obtain the required coverage. In the absence of highly directive methods, free space path loss is the overriding factor determining practicability of the system. Highly directive methods require steerable antennas and adaptive processing, both of which are prone to failure. A large-FoV, such as could be expected by omnidirectional antenna deployment, engenders a large variation of components that interact over the FoV to produce inhomogeneous Faraday rotations and path delays [5], [20]–[23]. In addition, these components are time-varying. In the instance of a LEO system with large-FoV, link geometry and consequently determination of these effects is complicated by relative movement.

A one-size-fits-all approach to the estimation of Faraday rotation and path delay may result in over-compensation for certain areas of the FoV and under-estimation for others. This is particularly relevant to a large-FoV, LEO satellite system, exploiting orthogonality of an antenna arrangement, or multiple input multiple output (MIMO) polarimetry, in three dimensions. The aim of such a system is to provide performance comparable to that of current systems, when perfect alignment exists between transmitter and receiver, but in the instance of imperfect alignment. Employment of the ionospheric shell model [24] is limited to higher elevation angles and on-axis signal transfer. A vectorised approach is required for low elevation angles covering off-axis propagation. In order for this to be feasible and to achieve maximum benefit, channel effects on signal transfer in any given direction need to be modelled.

To avoid the effects of Faraday rotation, engineers traditionally employ circular polarization whereby two orthogonally polarized signals are transmitted with a phase shift of 90° separating them. As a result, any depolarising effects are mitigated [3], [25]. To avoid the effects of path delay, two signal frequencies may be employed to measure a time delay on each signal that provides an approximate total electron content (TEC) value to correct for ionospheric aberrations along a signal path [19]. Phase delay may provide information on TEC variations in time and satellite positions [26] but is not used to calculate absolute TEC values, as it is subject to phase unwrapping ambiguities. A path of interest is often taken in isolation and the geometry of the satellite position is not fully taken into account. This approach is problematic for a large-FoV LEO system.

A downside of circular polarization is that up to 3 dB of transmit power is effectively wasted to guarantee reception. In addition, a power transmission maximum is centred at the nadir position where the shortest path length, and thus least amount of ionosphere to traverse, in the FoV exists. At the FoV edge, a power reduction of 3 dB may become critical, particularly as the effects of shadowing are more pronounced at lower elevation angles [27]. The problem of adequate reception is further exacerbated in the instance of a non-geosynchronous orbit such as a LEO where relative movement of ground receiver and satellite are introduced.

Models currently exist for the estimation of TEC, such as [5] which uses solar emission data to predict terrestrial fluctuations, as well as empirical models derived from Global Navigation Satellite System (GNSS) observations. These may be used in conjunction with models which simulate terrestrial magnetic field strength and orientation to determine ionospheric effects [28]. These may be specified by the $(L, I)$ position with data given for a nadir path. The TEC data may be provided in slant path format. The additional propagation length encountered in a slant path is typically accounted for by the inclusion of a secant function. Current models apply this function to a singular value of TEC and the Earth’s magnetic field, at the ionospheric piercing point, for calculation of Faraday rotation along a propagation path. Measurement is often the only means to taking ionospheric variation into account along a propagation path, but is unique to the operating parameters of the measurement system itself. A vectorised approach, taking into consideration the changing value of TEC and the Earth’s magnetic field along the propagation path may provide improved simulated results closer to observable measurement, and not constrained to the measuring system’s operational parameters. A propagation vector varies considerably over a large-FoV and the interaction of signal with ionospheric particles varies accordingly,
in addition to the fluctuations introduced through global position and time. Over a large LEO FoV, this can lead to large variations of Faraday rotation and path delay from one ground position to another. Furthermore, low elevation angles hamper measurement of Faraday rotation due to a severely reduced signal to noise ratio (SNR) at the receiver.

In the case of a satellite-to-ground system, typically in the absence of fast fading where multipath effects may be included and estimated using a Ricean distribution for multiple fading channels all arriving at once, individual LoS analysis is required. A vector-based approach is seen as a viable means of estimating Faraday rotation and path delay over a large-FoV.

The model presented here forms part of a (L, l) specific, large-FoV, LEO satellite channel model incorporating MIMO polarimetry in three dimensions. Such a system offers the perspective of improved capacity performance over the FoV in its entirety, as compared to currently employed techniques. Faraday rotation and path delay are two effects requiring effective estimation if such a model is to demonstrate this benefit.

II. METHODS

A. LINK GEOMETRY

Link geometry is given according to [13] and [29] and is illustrated in Figure 1. Relevant nomenclature is now introduced. The transmitter at the satellite shall be referred to as T with the receiver as R.

An Earth centred Earth fixed (ECEF) coordinate system is invoked for this work. The x-axis is nominally positioned at (0°, 0°). The FoV may subsequently be generated about this axis. At the FoV centre, an easterly direction [0 1 0]T, where T refers to transpose, coincides with an azimuthal angle θT of 0°. Within the FoV, anti-clockwise rotation is deemed as positive. Figure 2 demonstrates the concept. Link geometry is determined according to the following equations,

\[
\gamma = \arcsin \left( \frac{u s \sin \gamma}{s} \right) - \gamma
\]

where \(r_e\) is the 6378 km radius of the Earth and \(u = r_e + d\), \(d\) being orbit height.

Elevation at T is given by \(\alpha\) with 0° in the nadir direction, otherwise positive. Elevation at R is given by \(\kappa\) with 0° in the zenith direction, otherwise positive. The azimuthal angle \(\theta_R\) at R of the vector representing signal propagation \(k\) differs to that of the corresponding angle \(\theta_T\) at T by 180°.

The receiver R is assumed to be at a distance \(s\) from T which changes according to FoV location. The FoV may subsequently be positioned, according to its centre, at any user specified (\(L_u\), \(l_u\)) global position.

\[
\kappa = \arccos \left( \frac{r_e}{u} \right)
\]

\[
s_{\text{max}} = \sqrt{u^2 - r_e^2}
\]

\[
\alpha_{\text{max}} = \arcsin \left( \frac{r_e}{u} \right)
\]
**B. Latitude, Longitude and Height**

We begin by calculating latitude and longitude positions for every position within the FoV. The number of positions within the FoV are user defined through increments of the azimuthal and off-nadir angles, $\theta_T$ and $\alpha$ respectively. Global positioning through $(L, l)$ doublets for each of these positions may be given by the following formulas [30]. These formulas are based on the Haversine formula giving great-circle distances between two points on a sphere from their latitudes and longitudes [31]. The FoV centre is assumed to be at $(0^\circ, 0^\circ)$, $L = \arcsin \{ \sin(L_{0,0}) \cos(r_{plot}/r_e) + \cos(L_{0,0}) \sin(r_{plot}/r_e) \cos(\theta_h) \}$ (10)

$$l = l_{0,0} + \arctan \{ \sin \theta_h \sin(r_{plot}/r_e) \cos(L_{0,0}), \cos(r_{plot}/r_e) \sin(L_{0,0}) \sin(L) \}$$ (11)

where $\theta_h$ is a bearing angle from the FoV centre and is related to $\theta_T$ by,

$$\theta_h = \left| 2.5\pi - \theta_T, 2\pi \right|.$$ (12)

The function $\arctan_2(x, z)$ is given as,

$$\arctan_2(x, z) = \begin{cases} \arctan(z/x) & (z > 0) \\ \arctan(z/x) + \pi & (x \geq 0, z < 0) \\ \arctan(-z/x) - \pi & (x < 0, z < 0) \\ +\pi/2 & (x > 0, z = 0) \\ -\pi/2 & (x < 0, z = 0) \\ \text{undefined} & (x = 0, z = 0) \end{cases}$$ (13)

where $x$, $y$, and $z$ are lengths scaled by $r_e$ along respective ECEF axes. At the FoV centre in Figure 2, $x = 1$, $y = 0$, and $z = 0$.

Height $h$ along the propagation path is determined as,

$$h = (r_e + d) - \left\{ r_e \{1 - \cos \gamma_{\text{FR}} \} + d \right\} \times \left( \frac{k_{\text{FR}} - 1}{(l_{\text{FR}} - 0.999)} \right)$$ (14)

In Equation (14), $l_{\text{FR}}$ is the $i^{\text{th}}$ incremental position in terms of $\alpha$ from the nadir and $k_{\text{FR}}$ is a variable running from the nadir position along a constant azimuthal path, $\theta_T$, to the $i^{\text{th}}$ incremental position of $\alpha$. The curvature of the Earth, important in the instance of a large-FoV, is included in the model. Figure 3 illustrates the concept. Current models, such as those in [7] and [19], opt for a flat Earth approximation where the curvature of the Earth is not considered. Propagation to an FoV $(L, l)$ doublet position is along a path with a constant azimuthal angle, $\theta_T$. A propagation unit vector $\hat{k}$ from $T$ to $R$ is determined for all positions in the FoV by,

$$\hat{k} = \begin{bmatrix} -\cos \alpha \\ \cos \theta_T \sin \alpha \\ \sin \theta_T \sin \alpha \end{bmatrix}.$$ (15)

The simulation computes $\hat{k}$ and all investigated effects starting at the FoV centre or nadir position at $\theta_T = 0$. The simulation refers to each FoV position in turn through increments of $\theta_T$ which, after completion of one circle, moves radially outward by an increment of the off-nadir angle, $\alpha$.

Consider transmission to a receiver position, $R$. The introduction of $h$ creates $(L, l, h)$ triplets where $h$ is calculated at each $(L, l)$ doublet position along the unique path, given by $\hat{k}_{R1}$, to the receiver position, $(L_{R1}, L_{R1})$. Once the accompanying effects of Faraday rotation and path delay have been computed for propagation from $T$ to $R$, the simulation moves to position $R_2$. New values of $h$ must be assigned to each $(L, l)$ position along a propagation path each time a new receiver position is considered. Receiver positions on the same concentric circle utilise the same values of $h$ along the path.

The values of $h$ are used in the determination of the Earth’s magnetic field vector and this is considered shortly.

With a set of values of $\hat{k}$ generated, one for each $(L, l)$ doublet position in the FoV centred at $(0^\circ, 0^\circ)$, we may then position this FoV set at a user specified $(L_u, l_u)$ global position through simple matrix rotation,

$$\hat{k}_{l_u,l_u} = R_{l_u} R_{u} \hat{k}$$ (16)

where,

$$R_{l_u} = \begin{bmatrix} \cos(L_u) & 0 & \sin(L_u) \\ 0 & 1 & 0 \\ -\sin(L_u) & 0 & \cos(L_u) \end{bmatrix}$$ (17)

and

$$R_{u} = \begin{bmatrix} \cos(-l_u) & -\sin(-l_u) & 0 \\ \sin(-l_u) & \cos(-l_u) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ (18)

Along a propagation path from $T$ to $R$, globally referenced positions are passed through at progressively reducing heights with each set of encountered $(L, l, h)$ triplets subsequently introducing an accompanying set of magnetic field vector and TEC data. This data is then used to generate the estimates of Faraday rotation and path delay.

**C. Magnetic Field Vector and Total Electron Content**

Models of terrestrial magnetic fields include the International Geomagnetic Reference Field (IGRF) model, the World Magnetic Model (WMM), the British Geological Survey Global Magnetic Model (BGSGMM), and the Model of the Earth’s Magnetic Environment (MEME), and the International Reference Ionosphere model (IRI). The two most referenced of these are the IGRF model and the WMM. The WMM is a predictive-only model and is valid for a quinquennial epoch (currently 2010–2015). The IGRF is retrospectively updated and the latest update, IGRF-11 is valid for the years 1900–2015. The WMM is used extensively for navigation and in attitude and heading referencing systems by the UK Ministry of Defence, the US Department of Defense,
the North Atlantic Treaty Organization and the International Hydrographic Organization. It is available in the Matlab programming environment and so may be readily interfaced with proprietary coding. Determination of the Earth’s magnetic field vector at a triplet position in this research is according to the WMM [4]. Each magnetic field unit vector may be orientated into the ECEF coordinate system beginning with an ECEF northerly pointing unit vector, \( \hat{z} \) or \([0 \ 0 \ 1]^T \), at (0°, 0°). This is rotated by the declination, \( \delta \), and inclination, \( \iota \), of the magnetic field vector for a given height,

\[
\mathbf{R}_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & -\sin \delta \\ 0 & \sin \delta & \cos \delta \end{bmatrix} (19)
\]

\[
\mathbf{R}_\iota = \begin{bmatrix} \cos \iota & 0 & \sin \iota \\ 0 & 1 & 0 \\ -\sin \iota & 0 & \cos \iota \end{bmatrix} (20)
\]

Convention gives \( \delta \) as positive in an easterly direction and \( \iota \) as positive in a downward direction. The magnetic field unit vector may then be rotated to the appropriate \((L, l)\) position in the FoV,

\[
\mathbf{b}_{L,l,h} = \mathbf{R}_{\delta} \mathbf{R}_{\iota} \mathbf{R}_s \mathbf{R}_t \mathbf{z}_{0.0}. (21)
\]

Magnitudes given by [4] are attributed to each triplet position. Determination of the TEC value at a \((L, l)\) position is given by the Coupled Thermosphere Ionosphere Plasmasphere Electrodynamics (CTIPe) model [5]. The data is coarse with resolution of \((2^\circ, 18^\circ)\) and is renewed every 10 minutes. The CTIPe model is a physics-based model that aims to compete with empirical models during quiescent periods while providing more reliable global TEC value forecasts during periods of geomagnetic disturbance. In addition, it is a truly global model, and is not limited to certain geographical zones, as are many of its contemporaries. Each simulation run is generally ahead of real time by 10–20 minutes, as it uses Advanced Composition Explorer (ACE) data for inputs, and so offers advantages over real-time empirical data offered by GPS. In order to produce a smooth interpretation of TEC for each \((L, l)\) doublet in the FoV, bilinear interpolation is invoked. This is pertinent as most \((L, l)\) positions do not align with the global positions used in the data collection performed in [5].

The total electron content for a \((L, l)\) nadir path is determined by the multiplication of a TEC value given by [5] with a TECU or TEC unit defined globally as \(10^{16} \) electrons per m² [19]. The resultant path value, VTEC, refers to the number of free electrons in a vertical column of cross section 1 m² along the nadir path at a specified \((L, l)\) position and may be given as,

\[
\text{VTEC} = \int_{n_e} \text{d} s_{(\text{nadir})} (22)
\]

where \(n_e\) refers to the electron density along the nadir path [32].

Variability of interaction between ionised particles and an electromagnetic signal propagating along a path between transmitter and receiver is accounted for by the introduction of an interaction length over which the magnetic field vector and TEC, respectively \((L, l, h)\) and \((L, l)\) specific, interact with the signal. In Figure 3, \(s\) is assumed to be 1500 km. The FoV position is at the end of a constant azimuthal or \(\theta_T\) trajectory path, at a position specified by \(\alpha\). For simplicity, it shall be known as \(\alpha_3\).

![Figure 3](image-url)

**FIGURE 3.** Heights \(h_1, h_2, \) and \(h_3\) are calculated at respective FoV positions along the propagation path. Each of these FoV positions has an accompanying \((L, l)\) reference so forming a unique \((L, l, h)\) along a propagation path of length, \(s\), and of direction given by the unit propagation vector, \(\mathbf{k}\). Associated magnetic field and TEC data are respectively attached to each triplet along the path. Interactions between the ionosphere and the propagation vector \(\mathbf{k}\) along the path at each encountered FoV position are then integrated to provide an estimate of Faraday rotation and path delay, both specific to a \((L, l)\) FoV position. This is repeated for all \((L, l)\) FoV positions.

Three interaction lengths, shown as purple, blue and green zones, are delineated in Figure 3. Each length is 500 km and attributed to each of the three \((L, l)\) doublet positions along the path. Over an interaction length, the TEC is given according to the accompanying \((L, l)\) position. Magnetic field vector data is obtained using a \((L, l, h)\) triplet position. A scaling factor is introduced to account for ionospheric electron density, as a function of height, along the propagation length. The factor is based on the electronic layer distribution and reflects how height, in addition to \((L, l)\) position, affects the amount of Faraday rotation on a propagating signal at that point [8]. It is noteworthy to observe that, as a result, TEC becomes \((L, l, h)\) specific for path effect calculations.

Over each interaction length along the propagation path, values of magnetic field and TEC associated with each respective \((L, l, h)\) triplet and \((L, l)\) doublet are then used in Equations (1) and (2). The resulting values from all interaction lengths are then integrated to provide an estimation of Faraday rotation and path delay for each \((L, l)\) FoV position. Movement radially outward to the next concentric circle through increment of \(\alpha\) redefines the parameters. In the case of Figure 3, the path is now split into four interaction lengths.
It is evident that the user is, in effect, able to control the resolution of the simulated Faraday rotation and path delay. Improved resolution would require improved TEC data resolution. Estimated errors of TEC values associated with the CTIYe model range between 1 and 12 TEC units, as compared with other estimation methods [5], [33].

Finally, the system may be seen from the perspective of the receiver R or propagation from R to T. In this instance, the azimuthal angle $\theta_R$ is invoked with a propagation path determined by unit vector $\mathbf{\hat{k}}$. A doubling of Faraday rotation occurs for T to R followed by R to T propagation, as experienced in SAR systems [1], [2], [34].

### III. RESULTS

Estimates have been generated of path delay over the FoV, using the most recent TEC data acquired on 28th May 2014. The estimates generated are contrasted with those according to [19], and are shown in Figure 4. The date corresponds to a period of relative high solar activity.

Estimates are generated of Faraday rotation over the FoV, using magnetic field and TEC data acquired on 15th January 2010. This date was chosen, in the absence of any specific reference date, as the positions of the North Pole, South Pole, and Magnetic Equator were readily available to the author. The date corresponds to a period of relative solar inactivity. Northern latitudes and eastern longitudes are deemed positive, conforming to an ECEF reference system. On this date, positions of magnetic north and south pole, and magnetic equator are identified as $(85^\circ, -133^\circ)$, $(-64^\circ, 137^\circ)$, and $(-10^\circ, -71^\circ)$ respectively [4]. The estimates generated are contrasted with those according to [7], based on published work in [19], and [35], and are shown in Figure 5. The estimate of Faraday rotation in [7] is given as,

\[
\psi_{FR} = 6950 \cdot \text{VTEC} \cdot \mathbf{\hat{b}}_{400\text{km}} \cdot \mathbf{\hat{k}} \sec \alpha. \tag{23}
\]

As the vectorised model presented is contrasted with references based on trigonometrics, and using the same data in all model simulations, the choice of these dates is not seen as affecting the results.

Both path delay and Faraday rotation estimates of [19], and the Faraday rotation estimate of [7], use magnetic field and TEC data obtained at a single $(L, l)$ position corresponding to the mid-point position of the propagation path. This type of model used is described as a thin-shell ionospheric model. The shell is defined as infinitesimally thin, and is often given as being at the altitude of maximum electron density of 300–400 km. The ionospheric electron density is concentrated into this shell. An ionospheric piercing point is defined as the point at which the LoS signal propagating between satellite and ground receiver pierces this shell. This point defines the $(L, l)$ position at which TEC information is obtained, so defining first and second order effects such as path delay and Faraday rotation. A secant function may then be applied to derive an approximate value of TEC along a slanted propagation path. This model is valid for small FoVs, where neither the surrounding TEC values, magnetic field nor propagation path length vary considerably. However, the model may not operate correctly for large-FoVs, where towards the edges of the FoV these parameters may change considerably from those at the FoV centre. The angle of signal propagation, shallower to the outer edges of the FoV, affects the interaction with the magnetic field, and hence the amount of Faraday rotation to be expected. A vectorised integration of these parameters, from the point of transmission to the
point of reception, offers a credible approach to resolving this issue.

The magnetic field data is confined to a height of 400 km as a reference in the case of [7]. No height is given in [19], although a single mean value lying at 400 km is suggested in the relevant reference, [1]. This height corresponds to the mid-height of the propagation path and is approaching the dense F2 electron layer. Both examples of Faraday rotation reference formulas in these models are hence based on a thin-shell representation of the ionosphere. The presented model deviates from this representation, as it takes the three-dimensional structure of the ionosphere and off-axis propagation into consideration, and does not limit ionospheric parameters to a singular point at which the signal is deemed to pierce the ionosphere at 400 km. The estimates of the presented model deviate from the thin-shell model as they are calculated via integration of the varying strength of Faraday rotation and path delay to which a signal is subjected along the entire path.

The ionospheric scaling factor is calculated by assuming the ionosphere to range from 50 km to 1000 km in altitude above the Earth. It differs from the thick-shell model, which assumes a simplified Chapman profile of ionospheric electron distribution as a function of altitude [36]. This in turn contrasts with the computationally easier thin-shell model, as used in the comparison models of [7] and [19]. Both thick- and thin-models consider on-axis, or LoS, propagation only. In the presented model, a non-simplified layered profile of electronic distribution is assumed, based on a daily solar maximum profile [8]. The profile is divided into 10 km altitude segments and an electron density is attributed to each segment. An average electronic density is calculated using these values. Certain altitudes have an electron density higher than this average, while others have lower densities. A weighting may then be applied to each segment. The weighting is calculated by dividing the electron density in a segment by the average electron density. By treating the ionosphere in this manner, and by using multiple \((L, I)\) dependent values of TEC and magnetic field strength, the model provides on-axis and off-axis estimates of path delay and Faraday rotation using a non-simplified approach to
ionospheric distribution. Each estimate of Faraday rotation and delay is hence \((L, l, h)\) specific, and so takes three dimensions into account.

An operating frequency of 1.616 GHz and orbit height of 780 km, typical of remote satellite telephony systems [37], is employed. All figures presented in this paper make use of bilinear interpolated TEC data. The edge of the FoV is the point at which elevation at R is 0° and is orbit specific.

The vector nature of Faraday rotation is evident in Figures 5, as both positive and negative rotations are observed over a large extent of the FoV. This highlights the need to avoid an enveloping offset approach to this effect.

Near the nadir, all models provide similar estimates of Faraday rotation and path delay. This is as both TEC and the Earth’s magnetic dipole do not change considerably for small values of \(\alpha\), the off-nadir angle. The additional path distance encountered as propagation moves away from the nadir is well approximated by a secant function.

However, moving out towards the FoV edge, the presented model estimates of both Faraday rotation and path delay diverge from those of [7] and [19]. The full use of geometry in the presented model, including consideration of the Earth’s curvature and replacement of trigonometrical approximation by three dimensional vector analysis, extends the FoV. The assignment of \((L, l, h)\) triplets along the path leads to estimates of both Faraday rotation and path delay that take into account the variable nature of the ionosphere, allowing for a more realistic approximation of these effects to be obtained in any propagation direction.

As the FoV edge is approached, more ionosphere is traversed than at the nadir. The effect of both Faraday rotation and path delay is dependent on the variability of TEC and the Earth’s magnetic field along the path. Dense regions of electrons may be traversed but may not adversely affect the signal if the propagation vector does not align with the Earth’s magnetic field at the \((L, l, h)\) triplet position being considered. However, at an incremental distance further along the path, this may no longer be the case. As such, it is suggested that, for the outer edges of the FoV, a vectorised treatment of the ionosphere, through integration along the path, should be considered.

The presented model demonstrates favourable magnitudinal and angular trend correlation of both Faraday rotation and path delay with those of [7] and [19]. However, there is divergence in the overall FoV patterns, as a result of a vectorised approach and a geometric calculation of propagation path.

It is noteworthy to recall that LEO systems may use lower frequencies, around 160 MHz, than those employed in this work, which would typically increase both path delay and Faraday rotation seen in Figures 4 and 5 by a factor of one hundred. At these lower frequencies, ionospheric disturbances are more prevalent and third order effects such as scintillations, decoherence, variable refraction and phase instability become significant. Around 10 MHz, these effects become comparable to second order Faraday rotation effects.

The point is made in [7] that Faraday rotation measurements obtained for values of \(\kappa\) greater than 60° are unreliable due to low SNRs. This angle corresponds to a radial distance, from the FoV centre, of 1000 km. The vectorised approach, through integration along the path, may offer the possibility to model the FoV radially outward beyond \(\kappa\) greater than 60°. This is useful for simulating and improving measurement through the method of tri-orthogonal polarimetric MIMO signalling. Such a method may extend the range of measurement as SNR, towards the outer edge of a large-FoV, is typically higher for such a method than for a conventional dual-polarized system.

In the instance of a constant TEC and B field magnitude and direction along the propagation path, and no measurement errors in any of the required parameters, the estimates in Faraday rotation and path delay can differ between the thin-shell models of [7] and [19] and the presented vectorised model. This results from the interpretation of the ionospheric layers and was found to be of the order of 6%. As a result, at the FoV centre, the vectorised model estimates are 6% higher and 5.3% higher respectively while at the FoV edge, the vectorised model estimates are 13.7% higher and 12.1% higher respectively. This can be factored out at the nadir to mitigate this effect. The ionospheric interpretation provides layered distribution as a function of height which is not a function of \((L, l)\) and so is assumed to extend in all directions.

If we introduce a 10% decrease in a TEC value at a particular \((L, l)\) position that coincides with the 400 km altitude point of the thin-shell model of [7] and of [19], the difference in Faraday rotation and path delay estimates at the outer edge increases commensurately to the order of 22%. The singular value dependency of the thin-shell model has a large effect on the estimate. The vectorised model takes the direction of magnetic field into account at multiple positions along the propagation path, whereas the thin-shell model uses a singular value. This would further alter the estimate difference between models, as the alignment of propagation and magnetic field determines the amount of coupling that exists between them.

At the outer edge of a large-FoV, the Earth’s curvature needs to be taken into account, which the vectorised model does. We assume that any measurement error in TEC and magnetic field is constant over the FoV. This is reasonable as the measuring system is likely to be calibrated to the same degree over the entire FoV. This alters both estimates of thin-shell model and vectorised model by the same amount, or percentage change in TEC and magnetic field measurement, due to the measurement error. For the error in Faraday rotation due to a constant measurement error across the FoV in TEC and magnetic field component aligned with the direction of propagation, we have,

\[
\Delta_{\text{rms}, \psi_{\text{FR(thin-shell)}}} = \left\{ \left( B_{||(400\text{km})} \times \delta\text{TEC}_{(400\text{km})} \right)^2 + \left( \text{TEC}_{(400\text{km})} \times \delta B_{||(400\text{km})} \right)^2 \right\}^{\frac{1}{2}} \tag{24}
\]
The ability to ensure that a correctly polarized signal, typically through circular polarization, arrives at the receiver is obviously paramount since, without this, the possibility of no reception arises. However, a circularly polarized system will concentrate transmit power along the nadir path that is often subjected to the least amount of ionospheric interaction. A loss of 3 dB, typical of a circularly polarized waveform when compared to linear techniques, can adversely reduce capacity performance at the FoV edge, rendering signal transfer severely degraded. The time-varying nature of both the ionosphere and link LEO geometry further complicates efforts to provide reception at any position in the FoV. The use of MIMO polarimetry, in three dimensions, provides a solution whereby the benefits of both linear and circular polarization techniques may be possible. This provides improved capacity performance to areas in the FoV where this would not have previously been the case.

The tri-orthogonal arrangement may allow improved measurement of Faraday rotation at greater off-nadir angles, or towards the FoV edge, since the arrangement promises higher SNR values in this region.

Whereas the model is presented according to demonstrable geometry, and compares favourably with both [7] and [19], further revisions are expected. These may include the following points:

- Data for total electron content remains coarse and, despite the use of bilinear interpolation, may require resourcing in any future revision to the model.
- The ionospheric scaling factor is currently based on a daytime maximum profile of the ionosphere. As the ionosphere is time varying, this requires consideration in any future iteration. Differentiation between daytime and nighttime profiles, and local and seasonal variances, would be required; thus greatly expanding the model beyond the scope of this paper.
- Magnetic field vector data has been obtained at any height along the path, including very low levels. Although this has been demonstrated in [4], further investigation into the validity of these low level techniques is required.
- Measured data is required to fully validate the model at these outer lying regions of the FoV.

REFERENCES


\[
\Delta_{\text{rms, } \psi_{\text{FR(vectorised)}}} = \frac{1}{ifr} \int_{0}^{ifr} \left\{ \left( B_{||}(kfr) \times \delta \text{TEC}(kfr) \right)^2 + \left( \text{TEC}(kfr) \times \delta B_{||}(kfr) \right)^2 \right\}^{\frac{1}{2}} \, \text{d}fr
\]

An identical analysis supports the RMS error in path delay due to a constant measurement error in TEC across the FoV.

IV. CONCLUSION

This research takes the well documented effects of Faraday rotation and path delay and presents them over a large-FoV, using vectorisation of time-varying parameter values and link geometry.

Estimates of Faraday rotation and path delay for a large-FoV, LEO system have been presented using parameter vectorisation in three dimensions. Results have been demonstrated for a suggested system operation of remote satellite telephony. A comparison has been made with existing theoretical modelling, and by consequence measurement [7], of Faraday rotation over FoVs centred at the magnetic north and south poles, and at the magnetic equator. This comparison demonstrates favourable magnitudinal and angular trend correlation of both Faraday rotation and path delay. However, there is divergence in the overall FoV patterns, as a result of a vectorised approach and a geometric calculation of propagation path.

Whereas previous models employ a single value of the Earth’s magnetic field dipole and TEC, the presented model extends the estimates of Faraday rotation and path delay into three dimensions. These effects may then be calculated for receiver positions toward the edge of the FoV, according to the integration of variables along a propagation path. The non-uniform nature of the ionosphere is taken into account, according to height as well as global (L, I) position.

The research here forms part of work on a large-FoV, LEO satellite channel model employing MIMO polarimetry in three dimensions. This is to model capacity over the entire FoV, in an effort to develop performance that is independent of signal propagation direction. This is particularly relevant at the FoV edge, where received signal power is typically much lower than that received by a receiver at the FoV centre. To avoid severe depolarization, and so ensure signal reception, the traditional use of circular polarization may lead to a halving of signal power at the receiver, when compared to linear techniques. The use of MIMO polarimetric techniques, in three dimensions, provides the power transfer characteristics of such linear techniques, while providing diversity performance and reception characteristics typical of those provided through the use of circular polarization.

An all encompassing approach to the mitigation of Faraday rotation and path delay, often in the form of circular polarization may not always be the best solution. This is pertinent at the FoV edge where the propagation path is longest, and a priori subject to the most variability along its length. Any propagating signal along this path is most likely to suffer the greatest depolarizing effect.


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