Identification of static distortion by noise measurement

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Many active devices exhibit gross nonlinearity, which is traditionally mitigated using negative feedback. In those cases where negative feedback is not desired, compensation for the distortion can be carried out at the postprocessing stage by inverting the transfer function of the device. A new technique is demonstrated where the inherent noise in the system is exploited to estimate the required transformation, thus reducing distortion without the need for an offline characterisation step. It is demonstrated that the proposed technique can reduce the total harmonic distortion of a common-emitter amplifier from about 10% to < 1%.

Introduction: All electronic devices exhibit some degree of nonlinearity, which is often combated with negative feedback. In the case of an amplifier, if the desired gain is substantial this places great demands on the specifications and so drives up cost. Conversely, postprocessing after digitisation is relatively inexpensive. For narrowband signals, this can take the form of a harmonic rejection filter; however, wideband signals will contain frequencies whose harmonics fall inside their bandwidth. For these cases, it is desirable to perform compensation in the time-domain, allowing operation down to DC.

Nonlinearity is normally measured with the aid of a reference signal; a ramp [1], sinusoid [1] or noise signal [2] is applied to the input of the device, and its histogram compared to theoretical predictions. This can provide high levels of accuracy, but has two disadvantages: highly precise signal sources are required, and the system must be taken offline to perform the calibration. Alegria *et al.* [3] proposed the use of small test signals with a DC offset in order to overcome the first of these drawbacks; however, we suggest that a similar scheme can also overcome the second – if a small test signal is superimposed on the signal of interest, the response of the system may be characterised while it remains in operation. The novelty that we introduce is to use the internal noise of the system as a test signal, allowing characterisation to occur in real-time with little or no additional hardware.

System identification [4] is a well-established field, but while there are techniques [5] for output-only identification of linear systems, the characterisation of nonlinear systems requires knowledge of the input, whether directly as by Bai [6] or indirectly as by Voss *et al.* [7]. The technique presented in this Letter demonstrates that the internal noise in an electronic circuit can provide sufficient information to characterise its static nonlinearity without knowledge of the input.

Method: We make the assumption that the input noise of the system is dominant, having constant variance σ_i^2. The signal at the input we denote $Z(t) = x(t) + N(t)$, the sum of a deterministic band-limited signal and a small amount σ_i^2 of white noise. The system then produces a distorted output $Y(t) = f(Z(t)) = f(x(t) + N(t))$.

We linearise the transfer function $f(z)$ about $x(t)$, producing the estimate

$$Y(t) \simeq f(x(t)) + N(t)f'(x(t)) \tag{1}$$

which allows us to write

$$f'(x(t)) \simeq \frac{\sqrt{\mathrm{Var}(Y(t))}}{\sigma_i} \tag{2}$$

By our assumption that $x(t)$ is band-limited, one may estimate $f(x(t))$ by lowpass filtering the distorted output $Y(t)$. These two calculations provide an estimate of $f'(x(t))$ for each value of $f(x(t))$, thereby admitting numerical computation of

$$x(t) = \int \frac{\mathrm{d}x(t)}{\mathrm{d}f(x(t))}\,\mathrm{d}f(x(t)) \tag{3}$$

$$\simeq \int \frac{\sigma_i}{\sqrt{\mathrm{Var}(Y(t))}}\,\mathrm{d}f(x(t)) \tag{4}$$

This leaves free σ_i and a constant of integration, which determine the gain and offset, respectively. Although these can be determined using calibration points, a robust linear regression [8] between the distorted and compensated measurements provides sensible choices without modification of the system.

Note that integral (4) is amenable to recursive estimation. However, initial simulations demonstrate an unacceptable level of drift, so we instead choose to wait until a relatively large number of samples are available before computing (4) in its entirety.

A practical implementation can use curve-fitting [9, 10], either on the integrated values or the derivatives directly, to produce a more efficient representation of the transfer function with greater resistance to noise.

Estimation of the derivative: To estimate the gain $f'(z)$, one must determine the local standard deviation at z. However, this poses a dilemma; a small averaging time will produce a relatively noisy estimate, but a large averaging time will have greater bias due to the presence of signal. Highpass filtering can remove much of this unwanted signal, but not all. We empirically find that the best results are achieved in most cases with between 50 and 100 samples. This parameter can be increased as the sample rate rises and so reduces the averaging time. However, if quantisation noise is significant in the signal being measured, the averaging time should be larger. Averaging times significantly shorter than the duration of the quantisation steps will produce impulses in the estimated derivative of the transfer function where the corresponding window contained a step.

Harmonic distortion: We have developed an implementation [11] of the above method using Labview and a National Instruments USB-6341 16-bit 500 kS/s data acquisition (DAQ) unit.

The DAQ generates a 10 Hz sinusoidal voltage, which is applied to a common-emitter amplifier without feedback – a single BC547 transistor with a 100 Ω pull-up resistor to $V_{cc} = 5$ V. The amplified signal is then digitised and processed in real-time, estimating the transfer function in Fig. 1 using a combination of noise from the transistor and electronic and quantisation noise from the DAQ.

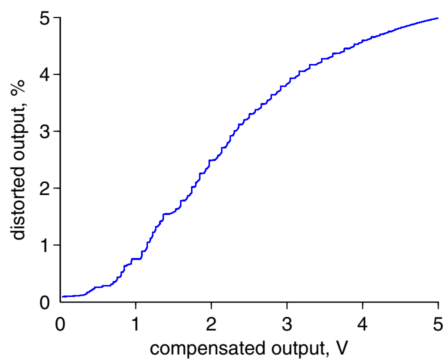

Fig. 1 *Experimentally measured voltage transfer function f(z) of tested amplifier, as estimated using its noise variance*

Saturation causes the gain to fall substantially near supply rails at 0 and 5 V

The effect of compensation on harmonic distortion is shown in Fig. 2. The total harmonic distortion (THD) remains low even when the amplifier is driven well into saturation, extending the useful dynamic range of the amplifier by an order of magnitude.

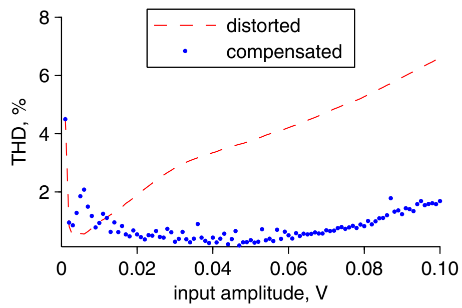

Fig. 2 *THD against signal amplitude for tested amplifier*

10 Hz sinusoid of each amplitude is applied to transistor base and THD of digitised output measured using Labview. Large increase near zero amplitude is due to quantisation of test signal. Each point shown is median of three runs

Static error: We claimed earlier that time-domain compensation is necessary for systems that operate near DC, where filtering cannot be used to suppress harmonics. In these situations, static error [1] provides a more useful measure of performance than THD. We apply a voltage ramp to the amplifier and then compensate the measured response. The deviation from an ideal ramp is shown in Figs. 3 and 4.

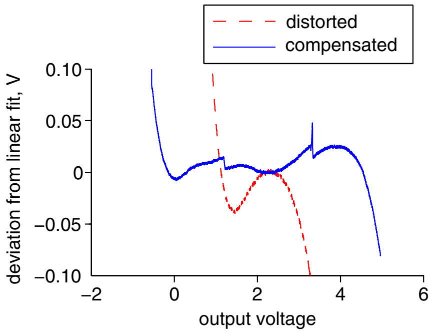

Fig. 3 *Experimentally measured static error of tested amplifier*

Ramp between 0.7 and 0.8 V is applied over 1 s and linear fit to central region subtracted to estimate nonlinearity. Compensation made system linear over almost entire output range despite heavy distortion of signal. Discontinuities are caused by impulsive noise, which is removed in Fig. 4 along with quantisation noise from DAQ

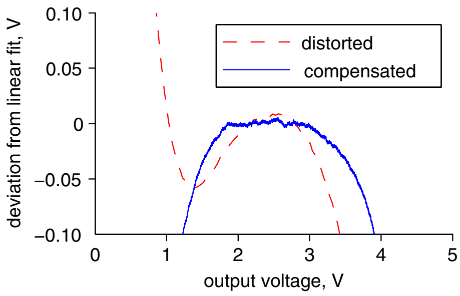

Fig. 4 *Experimentally measured static error of tested amplifier with quantisation noise excluded*

Ramp between 0.7 and 0.8 V is applied over 1 s and linear fit to central region subtracted to estimate nonlinearity. Input data are identical to that used in Fig. 3, but a second implementation is used that ignores regions containing quantisation steps. This process also removes the quantisation noise visible in Fig. 3

Compensation allows recovery of the ramp with much improved linearity. Although quantisation noise assists the reconstruction, its removal does not prevent the algorithm from functioning, as shown in Fig. 4, demonstrating that electronic noise provides for a significant enhancement of linearity. We stress again that no preliminary calibration is required, and that this enhancement is achieved entirely in postprocessing.

Conclusion: We have demonstrated a technique for output-only nonlinear system identification that can significantly enhance linearity. This enhancement is possible without a synthetic test signal, enhancing the utility of extremely low-cost feedback-less amplifiers in static and wideband applications. We have developed and made available proof-of-concept implementations for both Labview and MATLAB. The technique is suitable for implementation on a microcontroller, and so is a promising basis for a future adaptive post-distortion device that operates down to DC.

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One or more of the Figures in this Letter are available in colour online.

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