Practical method for determining inductance and capacitance of metamaterial resonators

J.N. Li, W. Withayachumnankul, S.J. Chang and D. Abbott

A practical approach to determining the distributed capacitance and inductance of individual metamaterial resonators is demonstrated. By loading a set of lumped capacitors onto the resonator, the resonance frequency can be measured as a function of the load capacitance. The capacitance and inductance values of the resonator are then determined by relating the equivalent circuit model to the measurements. Although the measurement is limited to the microwave region owing to the availability of suitable lumped capacitors, the concept is useful in simulations over a wider frequency range. The obtainable parameters are useful for further metamaterial analyses or syntheses.

Introduction: Metamaterials are artificial composite materials engineered to possess unusual electromagnetic properties, such as negative permeability or negative refractive index. These properties are usually derived from a collection of sub-wavelength periodic metallic resonators [1]. Various designs of metamaterial resonators have been proposed to date, e.g. split-ring resonators (SRR) [2] or electric *LC* (ELC) resonators [3, 4]. In the quasistatic limit, a metamaterial single unit cell can be considered as an *LC* resonance circuit with its effective inductance and capacitance mainly governed by the resonator geometries and materials. To estimate the capacitance and inductance values, several analytical models based on the transmission line theory have been proposed but with a limited accuracy [5, 6].

In this Letter, an empirical means is introduced to determine the *LC* parameters of metamaterials. In brief, a series of capacitors with different capacitances are loaded onto the resonator to alter the resonance frequency [7, 8]. Then, based on an equivalent circuit model of the resonator, the distributed inductance and capacitance can be determined from the observed relationship between the resonance frequency and the loaded capacitance. This reliable parameter estimation method is helpful in understanding and designing new functional metamaterials.

Design and method: Three common metamaterial resonators, as shown in Fig. 1*a*, are chosen to demonstrate the method. The first two designs, M1 and M2, are ELC resonators, and the third design, M3, is an SRR. In all of the designs, the dielectric gaps form the capacitors, while the conducting loops form the inductors. In normal operation, when the electric field polarisation of the incident wave is perpendicular to the gap, a surface current is induced in the loops. Similar to an *LC* resonance circuit, the structure becomes resonant when the energies stored in the capacitive gap and the inductive loop are equal. The resonance frequency follows the condition of $\omega_0 = 1/\sqrt{LC}$.

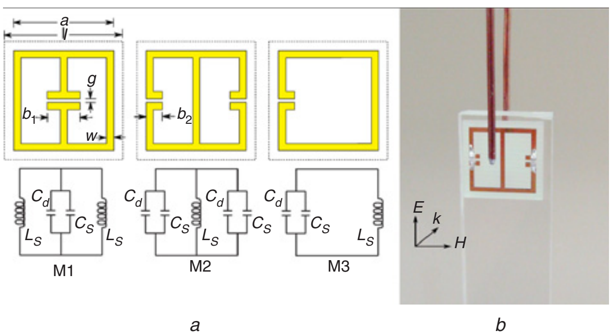

Fig. 1 *Schematics of three metamaterial elements and their equivalent circuits (distributed capacitance and inductance denoted by C_s and L_s, respectively, and lumped capacitance denoted by C_d), and photo of experimental setup showing resonator inserted between two monopole antennas*

a Schematics of three metamaterials
b Photo of experimental setup

To determine the *LC* parameters of these resonators, a set of measurements for each resonator is needed. As discussed earlier, the resonance frequency strongly depends on the effective capacitance and inductance of the structure. Hence, a series of lumped capacitors with different known values are loaded on the gap region of the resonator to change the resonance frequency. Only a single unit cell for each metamaterial design is used in the measurement to preclude the contributions from mutual electric and magnetic interactions among resonators. The equivalent circuit of each structure with additional lumped capacitors is shown in Fig. 1*a*. The resonance frequencies of M1, M2, M3 are, respectively, given by $f_1 = 1/2\pi\sqrt{\frac{1}{2}L_s(C_d + C_s)} = 1/\pi\sqrt{2L_s(C_d + C_s)}$, $f_2 = 1/2\pi\sqrt{2L_s(C_d + C_s)}$ and $f_3 = 1/2\pi\sqrt{L_s(C_d + C_s)}$.

Experimental setup: Each unit cell of the resonators is fabricated on a separate FR4 substrate with a thickness of 0.8 mm. The metal used for the resonators is gold-coated copper with a thickness of 35 µm. The structural parameters, as indicated in Fig. 1*a*, are as follows: $l =$ 13 mm, $a = 11$ mm, $b_1 = 3.6$ mm, $b_2 = 1.8$ mm, $g = 0.4$ mm, and $w = 0.8$ mm. For M1, M2, M3, these parameters result in an *LC* resonance at 3.4, 2.7 and 2.2 GHz, respectively. High-*Q* surface-mounted capacitors with various capacitances from 0.2 to 2.2 pF are chosen to connect across the gap regions of each resonator. Their capacitance tolerance is within 0.1 pF. All the capacitors are in standard 0603 packages. For any measurement, a capacitor-loaded resonator is inserted in the middle of a pair of monopole antennas, as shown in Fig. 1*b*. The antennas are then connected to an Agilent Technologies N5230A network analyser to measure the transmission coefficient S_{21} of the resonator normalised to that of free space. Several pairs of antennas are used to cover different resonance frequencies.

Results and discussion: The measured transmission magnitudes of the three resonators at different capacitance values are illustrated in Fig. 2. The resonance frequency shows a strong dependence on the mounted capacitor. As the loaded capacitance increases, the resonance frequency decreases from 2.48 to 1.00 GHz for M1, from 1.76 to 0.97 GHz for M2, and from 1.48 to 0.93 GHz for M3.

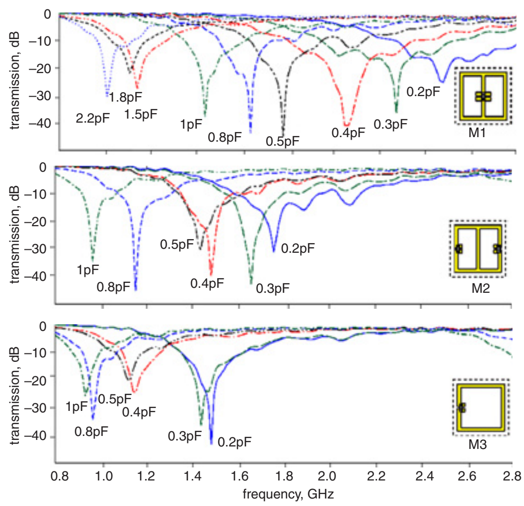

Fig. 2 *Transmission magnitude of three resonators with different loaded capacitors*

As shown in Fig. 3, the resonance frequencies from the experiment and numerical simulation with CST Microwave Studio are in good agreement. The small discrepancies are likely to be from the interaction between the resonator and the monopoles. From these results, C_s and L_s can be determined by fitting the resonance model of the equivalent circuit to the experimental relation. From the proposed method, it is found that $C_s = 0.19$ pF and $L_s = 21.03$ nH for M1, $C_s = 0.21$ pF and $L_s = 9.53$ nH for M2, and $C_s = 0.23$ pF and $L_s = 26.34$ nH for M3. These extracted values can be used to back-calculate the resonance frequency of the unloaded resonator. For M1, M2, and M3, the back-calculated resonance frequencies are 3.6, 2.53 and 2.05 GHz, respectively, which are comparable to 3.47, 2.58 and 2.06 GHz obtained from the experiment.

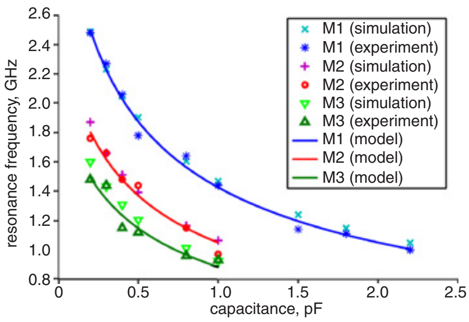

Fig. 3 *Measured and simulated resonance frequencies against loaded capacitance*

Fitting curves determined from equivalent circuit models

Conclusions: It has been demonstrated that the effective *LC* parameters of a metamaterial element can be determined by comparing the equivalent circuit model to the observed relation between the resonance frequency and the loaded capacitance. The parameters obtained from this method can be used for, e.g., proving theoretical models or designing and optimising metamaterials. The coupling between the resonator and the probing antennas will be addressed in future work. The proposed method is applicable to resonators not only in the microwave region, where lumped capacitors are available, but also in other frequency ranges via simulation.

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One or more of the Figures in this Letter are available in colour online.

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